

Most curvilinear tilings and Hurwitz theorem

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This is joint work with Min Yan

The classical Hurwitz realisation problem was largely solved by Edmond, Kulkarni, Stong in 1980's. Some of their work was done by relating to tilings of surfaces. In this talk, we revisit the problem from the fresh angle of edge-to-edge tilings of surfaces by congruent curvilinear polygons, where by curvilinear we mean the edges are not necessarily straight. We concentrate on the most extreme curvilinear case, where the polygon is equivalent to a planar diagram D .

Let V_1, V_2, \dots, V_k be vertices in the surface S_D determined by the planar diagram D . Then the vertices in a tiling by D are $V_1^{k_1}, V_2^{k_2}, \dots, V_p^{k_p}$. The tiling gives a branched cover over S_D , with k_i as the branching data. Our first result is the necessary and sufficient condition for a branched data to come from a tiling.

The classification of tilings of all surfaces by the given planar diagram D is the same as finding “minimal tilings”, which are tilings that do not cover other tilings. We have the following results.

Theorem 1. *Given natural numbers $k_1, \dots, k_p \geq 2$ with $p \geq 2$, suppose D is a planar diagram with p vertices such that $\chi(S_D) \leq 0$. Then for s big enough prime number, there exists minimal tilings by S_D with sd' tiles. (Let $\lambda = \text{LCM}(k_1, \dots, k_p)$, define $d' = 2\lambda$ if λ is even, but λ/k_i is odd for exactly an odd number of k_i ; otherwise define $d' = \lambda$.)*

Theorem 2. *For natural numbers $k_1 \geq 3$, suppose D is a planar diagram with 1 vertex such that $\chi(S_D) \leq 0$. Then for any integer $s \geq 2$, there exists minimal tilings by S_D with vertices V^{k_1} and sd' tiles. For $k_1 = 2$, then for any integer $s = 2^m$, $m \geq 1$ is a natural number, there exists minimal tilings by S_D with vertices V^{k_1} and sd' tiles.*

So, for D satisfying $\chi(S_D) \leq 0$, the number of minimal tilings with the given vertices $V_1^{k_1}, V_2^{k_2}, \dots, V_p^{k_p}$ is always infinite. This shows the classification problem can be hard to solve for all surfaces. We also discuss minimal surfaces for surfaces of fixed genus.

References

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