

Extremal eigenvalues with respect to graph minors

Huiqiu Lin

East China University of Science and Technology, Shanghai, China

huiqiulin(at)126.com

Minors play a crucial role in various branches of graph theory, including structural graph theory, extremal graph theory, and topological graph theory, and have garnered significant interest in these areas.

In this talk we explore the maximal spectral radius, denoted $\text{spex}(n, \mathbb{H}_{\text{minor}})$, of n -vertex graphs that exclude any graph from a fixed family \mathbb{H} as a minor.

We derive the asymptotic value for $\text{spex}(n, \mathbb{H}_{\text{minor}})$ and establish a unified stable structure for extremal graphs by introducing a novel application of the absorbing method to eigenvalue analysis, along with models and partitions with respect to a general H minor. In particular, we prove three central theorems, the most fundamental of which asserts that every graph with spectral radius $\rho \geq \text{spex}(n, \{H\}_{\text{minor}})$ contains either an H minor or a spanning book $B_{\gamma_H, n-\gamma_H}$, where $\gamma_H = |H| - \alpha_H - 1$ and α_H is the independence number of H .

These three theorems, combined with detailed combinatorial analysis, enable us to determine $\text{spex}(n, \{H\}_{\text{minor}})$ for every complete r -partite graph H . This extends the result of Tait for $\text{spex}(n, \{K_r\}_{\text{minor}})$ and provides a stronger solution to his conjecture for $\text{spex}(n, \{K_{s,t}\}_{\text{minor}})$ [J. Combin. Theory Ser. A 166, 2019]. Additionally, these theorems imply or strengthen other existing eigenvalue-extremal results on minors, such as planar graphs by Tait and Tobin, $K_r - E(H)$ minors by Chen, Liu and Zhang, and friendship graph minors by He, Li and Feng.