

An Eventown Result for Permutations

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A family of permutations $\mathcal{F} \subseteq S_n$ is *even-cycle-intersecting* if $\sigma\pi^{-1}$ has an even cycle for all $\sigma, \pi \in \mathcal{F}$. We show that if $\mathcal{F} \subseteq S_n$ is an even-cycle-intersecting family of permutations, then $|\mathcal{F}| \leq 2^{n-1}$, and that equality holds when n is a power of 2 and \mathcal{F} is a double-translate of a Sylow 2-subgroup of S_n . This result can be seen as an analogue of the classical eventown problem for subsets and it confirms a conjecture of János Körner on maximum reversing families of the symmetric group. Along the way, we show that the canonically intersecting families of S_n are also the extremal *odd-cycle-intersecting* families of S_n for all even n . While the latter result has less combinatorial significance, its proof uses an interesting new character-theoretic identity that might be of independent interest in algebraic combinatorics.