Representation of braid-like groups

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This is joint work with Maxim Dudar

Elements of the cactus group have a geometric representation by strands on a plane or in space. Braid group has a similar representation, therefore cactus groups can be considered as some analogue of braid groups.

The cactus group was appeared in the works of S. L. Devadoss [?] and M. Davis, T. Januszkiewicz [?]. A cactus group J_n is generated by a_i, $2 \leqslant i \leqslant n$, with defining relations:

$$a_i^2 = 1,\ 2 \leqslant i \leqslant n, \quad (a_i a_k a_j a_k)^2 = 1,\ i \leqslant j,\ i + j \leqslant k,$$
$$a_i a_k a_j a_k = a_{i+j-k} a_j a_{i+j-k} a_i,$$
$$4 \leqslant j + 2 \leqslant i \leqslant n; \quad j < k < i; \quad 2 \leqslant i + j - k \leqslant n; \quad 2k \leqslant i + j.$$

We construct a linear representation of J_n which is based on the work [?], where faithful linear representation of generalized cactus groups was constructed. The general construction of this linear representation does not give an explicit linear representation of the cactus group J_n.

Let $\mathbb{S} = \{\langle (i_1, i_2), (i_2, i_3) \ldots, (i_k, i_{k+1}) \rangle \mid i_j < i_{j+1},\ 1 \leqslant j \leqslant k \leqslant n-1,\ (i_j, i_{j+1}) \in S_n\}$. Let us define mappings $f_i : \mathbb{S} \to \mathbb{S}$ for $2 \leqslant i \leqslant n$, which acts on $K \in \mathbb{S}$ according to the rule $f_i(K) = \langle (j_1, j_2), \ldots, (j_k, j_{k+1}) \rangle$, where $i_l \leqslant i$ for some maximal l and

$$j_s = \begin{cases} i - i_{l-s+1} + 1, & 1 \leqslant s \leqslant l, \\ i_s, & l < s \leqslant k+1. \end{cases}$$

Theorem 1. *From the general construction of the R. Yu's linear representation for the group J_n we obtain a representation $\Pi : J_n \to \mathrm{Aut}(V)$ by automorphisms of the free $\mathbb{Z}[t]$-module V with basis e_K, $K \in \mathbb{S}$, in which the images of the generators a_i are the automorphisms φ_i acting as follows:*

$$\phi_i(e_K) = \begin{cases} -e_K, & K = W_i, \\ e_K, & i < i_1, \\ e_{f_i(K)} + 2te_{W_i}, & W_i \not\subset f_i(K) \text{ and } i_1 \leqslant i < i_{k+1}, \\ e_{f_i(K)}, & W_i \subset f_i(K) \text{ or } i_{k+1} \leqslant i. \end{cases}$$

It was proven that this linear representation is reducible. In the case $n = 3$, the resulting reduced representation is reducible for all $t \in \mathbb{R}$.

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References

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