

On second maximum intersecting families of Ree Unital

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This is joint work with Sergey Goryainov and Rhys Evans

In combinatorics, a unital is a set of $n^3 + 1$ points arranged into subsets of size $n + 1$ so that every pair of distinct points of the set are contained in exactly one subset. This is equivalent to saying that a unital is a $2 - (n^3 + 1, n + 1, 1)$ block design. One of the most important infinite families of unitals is the family of Ree unitals. Let $\Gamma = \text{Ree}(q)$ be the Ree group of type 2G_2 of order $(q^3 + 1)q^3(q - 1)$ where $q = 3^{2m+1}$. The Ree unital is a unital with points set the Sylow 3-subgroups of Γ and its blocks set those Sylow 3-subgroups normalized by an involution of Γ . It is a $2 - (q^3 + 1, q + 1, 1)$ design, with its automorphism group the full automorphism group of Γ .

An family of sets \mathcal{F} is called intersecting if any two sets of \mathcal{F} intersects non-trivially. The canonical intersecting family given by Erdős-Ko-Rado is all the sets containing a given point. Hilton-Milner construct an intersecting family, which is given by a single set S outside an canonical intersecting family \mathcal{F} , together with all sets in \mathcal{F} intersect with S , this family is usually known as the second largest intersecting family of different objects.

De Boeck proved in 2015 that the maximum intersecting family of blocks of Unitals are the canonical ones. We proved that Hilton-Milner type intersecting families of blocks of Ree Unitals are maximal intersecting families, and we wonder if they are second maximum intersecting families.