Phylogenetic tree reconstruction from parsimony layered filters

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Let X be a finite set. A phylogenetic X-tree is a finite tree with leaf set X that contains no degree-2 vertex. Two phylogenetic X-trees are X-isomorphic if there is a graph isomorphism f between them fixing X pointwise. An unordered subset pair $\{A, B\}$ of X is called an X-split if $A \cap B = \emptyset$ and $A \cup B = X$. Let T be a phylogenetic X-tree. For any X-split $\{A, B\}$, its parsimony score on T is the maximum number of edge-disjoint paths in T with one endpoint in A and the other in B. For a nonnegative integer ℓ, let $\mathcal{P}_{T,\ell}$ be the set of all X-splits with parsimony score ℓ on T, and define $\mathcal{P}_{T,\leq\ell} \doteq \bigcup_{i=0}^{\ell} \mathcal{P}_{T,i}$.

Let $\mathbb{N}$ denote the set of all positive integers. For any $n \in \mathbb{N}$, let $\mathcal{T}_n$ be the set of all phylogenetic $[n]$-trees (up to $[n]$-isomorphism) and let $\mathcal{S}_n$ represent the set of all $[n]$-splits. A map f from $(\bigcup_{n\in\mathbb{N}} \mathcal{T}_n) \times \mathbb{N}$ to $\bigcup_{n\in\mathbb{N}} \mathcal{S}_n$ is called a parsimony layered filter if for every $(T, \ell) \in (\bigcup_{n\in\mathbb{N}} \mathcal{T}_n) \times \mathbb{N}$ we have $\mathcal{P}_{T,\ell} \subseteq f(T, \ell) \subseteq \mathcal{P}_{T,\leq\ell}$. Fix positive integers ℓ and n with $n \geq 2\ell + 5$, and let T and T' be phylogenetic $[n]$-trees. For any parsimony layered filter f, we show that the following three statements are equivalent: 1) T and T' are $[n]$-isomorphic; 2) $f(T, \ell) = f(T', \ell)$; 3) $\mathcal{P}_{T,\ell} \cup \mathcal{P}_{T',\ell} \subseteq \mathcal{P}_{T,\leq\ell} \cap \mathcal{P}_{T',\leq\ell}$.