

A symmetry characterization of t -designs in the Boolean lattice

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This is joint work with Yaokun Wu

Let n, a, b be positive integers with $a \leq b < n/2$, and write $[n] \doteq \{1, \dots, n\}$. For a family $\mathcal{F} \subseteq \binom{[n]}{b}$, we prove that $\mathcal{F}$ is a $\min\{2a-1, b\}$ -design if and only if the following symmetry condition holds: for any $S_1, S_2 \in \binom{[n]}{a}$, the number of members of $\mathcal{F}$ containing S_1 and disjoint from S_2 equals the number of members of $\mathcal{F}$ containing S_2 and disjoint from S_1 . We are led to this result in our investigation of a forbidden subposet problem in the Boolean lattice, and intend to explore possible extension/application of this symmetry principle in a broader algebraic combinatorics framework.