

Tiles of cyclic groups

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We say that (A, B) is a tiling pair of the abelian group G if every element g of G can uniquely be expressed as $a + b = g$, where $a \in A$ and $b \in B$. On his blog, Tao suggested to prove that every tile of a cyclic group of square-free order is a transversal. This statement follows from an old result of Tijdeman's as it was pointed out by Laba and Meyerowitz. This question arose in connection with the article by Coven and Meyerowitz [1] on subsets of cyclic groups, in which they formulated sufficient conditions that ensure for subsets to be tiles, and following Tao's blogpost the necessity of these conditions is what we call the Coven-Meyerowitz conjecture. The conjecture was confirmed when the order of the group has at most 2 different prime divisors and in a series of long papers written by Laba and Londner [2] it was proved for slightly larger class of integers.

We propose [3] a new approach to either prove the conjecture or disprove it using our new constructions of tiles. In the meantime we propose a weaker conjecture saying that if A and B are tiling pairs of the cyclic group \mathbb{Z}_M then either both of them or none of them satisfies the conditions of Coven and Meyerowitz.

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References

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