Cylindrical set conjecture

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This is joint work with Gergely Kiss, Ádám Markó and Zoltán Lóránt Nagy

Rédei conjectured that for any tile of an elementary abelian p-group of rank 3 ($A \bigoplus B = \mathbb{F}_p^3$) either A or B are contained in a plane. Simeon Ball outlined the following strategy of proving the conjecture, see also [2]. Prove the cylindrical set conjecture and prove that a subset of $\mathbb{F}_p^3$ of size p is contained in a plane or determines at least p directions.

A set of points $S \subseteq \mathbb{F}_p^n$ is called *p-divisible* if every affine hyperplane in $\mathbb{F}_p^n$ intersects S in 0 (mod p) points. The cylindrical set conjecture of Ball asserts that if S is a p-divisible set of p^2 points in $\mathbb{F}_p^3$, then S is a cylinder. We proved [1] that every p-divisible multiset S is both a $\mathbb{F}_p$-linear and $\mathbb{Z}$-linear combination of characteristic functions of cylinders. In addition, the multisets of size p^2 are $\mathbb{Z}$-linear combinations of a plane and weighted differences of parallel lines.

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References

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[2] Sam Mattheus, , (2022). Finite geometry and friends: Tilings of abelian groups and combinatorics in spherical buildings.