

## Recent Progress in the Classification of Primitive Flag-Transitive Generalized Quadrangles

Jianbing Lu

*National University of Defense Technology*

Finite generalized quadrangles (GQs) are point-line incidence geometries whose bipartite incidence graphs have diameter 4 and girth 8. Classical examples arise from polar spaces of rank 2. Their automorphism groups are closely related to finite simple groups of Lie type. Specifically, for these classical GQs, the full automorphism group is almost simple, with simple socle a classical group of Lie type, and it acts primitively on both points and lines, and transitively on flags.

Bamberg et al. (2012) proved that if a finite thick GQ admits an automorphism group acting primitively on both points and lines, then the group must be almost simple. Their work handled the cases where the socle is a sporadic or an alternating simple group. By the Classification of Finite Simple Groups, the only remaining possibility is that the socle is a simple group of Lie type.

In this talk, we introduce new methods to classify the low-dimensional Lie type socles for point-primitive and line-primitive actions. We completely determine the possibilities when the socle is  $\mathrm{PSL}_2(q)$  (which forces  $q = 9$  and the GQ to be  $W(3, 2)$ ),  $\mathrm{PSU}_3(q)$  (no such GQ exists), and  $\mathrm{PSp}_4(q)$  (the GQ is the classical symplectic quadrangle  $W(3, q)$ ). We then additionally assume flag-transitivity while retaining point-primitivity and line-primitivity, and consider general classical socles. Using Aschbacher's classification of maximal subgroups of classical groups, we prove that under these hypotheses the only possibilities are the classical generalized quadrangles.