A generalization of Ramanujan's sum over finite groups

Monu Kadyan

Shenzhen Institutes of Advanced Technology, Chinese Academy of Sciences.

monu4math@gmail.com

This is joint work with Priya Dhankhar and Sanjay Kumar Singh

Abstract

Let G be a finite group, and let $x \in G$. Define $[x^G] := \{y \in G : \langle x^G \rangle = \langle y^G \rangle\}$, where $\langle x^G \rangle$ denotes the normal subgroup of G generated by the conjugacy class of x. In this paper, we determine an explicit formula for the eigenvalues of the normal Cayley graph $\mathrm{Cay}(G, [x^G])$. These eigenvalues can be viewed as a generalization of classical Ramanujan's sum in the setting of finite groups. Surprisingly, the formula we derive for the eigenvalues of $\mathrm{Cay}(G, [x^G])$ extends the known formula of classical Ramanujan's sum to the context of finite groups. This generalization not only enrich the theory of Ramanujan's sum but also provide new tools in spectral graph theory, representation theory, and algebraic number theory.