Cyclic complementary extensions of groups and skew morphisms

Kan Hu
Zhejiang Ocean University, Zhoushan, China
hukan@zjou.edu.cn

Based on joint works with Robert Jajcay, Istvan Kovacs and Young Soo Kwon

A finite group G is said to be a cyclic complementary extension of a subgroup A if G has a cyclic subgroup C such that $G = AC$ and $A \cap C = 1$. A chosen generator c of C determines a permutation φ on A and an integer-valued function $\Pi : A \to \mathbb{Z}_{|c|}$ via the identity $cx = \varphi(x)c^{\Pi(x)}$ $(x \in A)$ with the following properties:

(a) $\varphi(1_A) = 1_A$ and $\varphi(xy) = \varphi(x)\varphi^{\Pi(x)}(y)$ for all $x, y \in A$.

(b) $\Pi(1_A) = 1$ and $\Pi(xy) = \sum_{i=1}^{\Pi(x)} \Pi(\varphi^{i-1}(y))$ for all $x, y \in A$.

The permutation φ and the function Π are known as a skew morphism of A and an extended power function of φ, respectively. It turns out that every cyclic complementary extension of a given finite group A can be characterized by the corresponding skew morphism of A and the associated extended power function, which allows us to investigate certain combinatorial objects such as regular Cayley maps as well as generalized regular Cayley maps.

References

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