On Some Properties of Archimedean Tiling Graphs

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A plane tiling $\mathcal{T}$ is a countable family of closed sets with non-empty interiors $\{T_1, T_2, \cdots\}$, which cover the plane without gaps or overlaps. Every closed set $T_i \in \mathcal{T}$ is called a tile of $\mathcal{T}$. We consider the special case in which each tile is a polygon. If the corners and sides of a polygon coincide with the vertices and edges of the tiling, we call the tiling edge-to-edge. A so-called type describes the neighbourhood of any vertex of the tiling. We consider plane edge-to-edge tilings in which all tiles are regular polygons, and all vertices are of the same type. There exist exactly eleven such tilings, which are called Archimedean tilings. A graph formed by an Archimedean tiling is called an Archimedean tiling graph.

Pick's theorem gives the area of a simple lattice polygon P using the number of lattice points in the interior and boundary of P. We generalize Pick's theorem to some lattice polygons in each of Archimedean tilings. For the (3.3.4.3.4) tiling, we obtain that if the boundary of a lattice polygon P is made up of the lattice segments which are parallel to the edges of the tiling, or of the symmetrical lattice segments, then the area of P is given by

$$A(P) = \frac{1}{8}[(2+\sqrt{3})b + (4+2\sqrt{3})i + (2-\sqrt{3})c + 8\sqrt{3} - 24],$$

where b is the number of lattice points on its boundary and i is the number of lattice points in its interior, c is the boundary characteristic of P. Moreover, we obtain a unified Pick-type formula of some special lattice polygons for all of the 11 Archimedean tilings.

For a graph $G = (V, E)$, a labeling $\partial\colon V \cup E \to \{1, 2, \ldots, k\}$ is called an edge irregular total k-labeling of G if the weights of any two different edges are distinct, where the weight of the edge xy under ∂ is defined to be $wt(xy) = \partial(x) + \partial(xy) + \partial(y)$. The total edge irregularity strength $\text{tes}(G)$ of G is the minimum k for which G has an edge irregular total k-labeling. In this talk, we will determine the exact value of the total edge irregularity strength for the hexagonal grid graph H_n^m by giving an edge irregular total $\left\lceil \frac{3mn+2(m+n)+1}{3} \right\rceil$-labeling.