Isomorphic factorizations of the complete graphs into Cayley graphs on CI-groups

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Abstract

Isomorphic factorizations of complete graphs originate from the seminal work of Frank Harary and collaborators spanning [1] to [2], who initiated the systematic study of decompositions of complete graphs into pairwise isomorphic spanning subgraphs. In this paper, we investigate isomorphic factorizations of complete graphs into Cayley graphs on CI-groups.

Let $\Gamma = \text{Cay}\ (G, S)$ denote the Cayley graph of finite group G. We obtain a necessary and sufficient condition on CI-group G so that the complete graph on $|G|$ vertices can be edge-partitioned into k-copies of Cayley graph of the same CI-group G each isomorphic to Cay (G, S) for some inverse-closed subset $S \subset G \setminus \{1\}$. Further we give a construction of isomorphic factorizations of the complete graphs into Cayley graphs on CI-group.

Let Γ be a graph, and let $\mathcal{P} := \{P_0, P_1, \cdots, P_{k-1}\}$ be a partition of edge set $E(\Gamma)$ with $k \geq 2$. Then $(\Gamma, \mathcal{P})$ is called a *decomposition* of the graph Γ. Moreover, if the subgraph induced by each of the set P_i are all spanning subgraph of Γ, then the decomposition $(\Gamma, \mathcal{P})$ is called a *factorization* of the graph Γ, and each spanning subgraph Γ_i is called a *factor* of Γ, where $0 \leq i \leq k-1$. A factorization $(\Gamma, \mathcal{P})$ is termed *isomorphic factorization* if all its factors are pairwise isomorphic. For such a factorization, we denote the common factor (up to isomorphism) by Γ/k. In the specific case of a complete graph K_n, each factor K_n/k in an isomorphic factorization is called a k-if graph (where 'if' abbreviates isomorphic factorization). A finite group G is said to have *k-if property* if there exists a Cayley graph $X = \text{Cay}\ (G, S)$ of G is a k-if Cayley graph.

Further, we get the following theorem:

Theorem 0.1 *Let G be a finite CI-group. Then G has k-if property if and only if G is the direct product of elementary abelian group, and the order of each Sylow subgroup G_p of G satisfy $2k \mid |G_p| - 1$ when p is odd; $k \mid |G_p| - 1$ when $p = 2$.*

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References

[1] F. Harary, R. W. Robinson and N. C. Wormald, Isomorphic factorisations. I. Complete graphs, Trans. Amer. Math. Soc. **242** (1978), 243–260.

[2] F. Harary and R. W. Robinson, Isomorphic factorizations. X. Unsolved problems ,J. Graph Theory 9 (1985), 67-86.