Quantum Geometry in Nonlinear Responses

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Abstract: The concept of quantum geometry has profoundly reshaped our understanding of condensed matter phenomena. Quantum geometry consists of two key components: the Berry curvature, characterizing the density of Berry phase associated with a cyclic evolution in parameter space, and the quantum metric, characterizing the distance between neighboring Bloch states. In the linear response regime, the Berry curvature and quantum metric underlie various Hall effects and the optical conductivity, respectively.

Nonlinear responses have much richer structures and are attracting intense interest recently. In this talk, I focus on the role of quantum geometry — specifically the dipoles of Berry curvature and quantum metric — in nonlinear responses. Within the semiclassical framework, the nonlinear Hall effect is related to quantum geometry in two distinct ways: the intrinsic contribution is determined by the quantum metric dipole, requiring the breaking of both time-reversal and inversion symmetry, while the extrinsic contribution is determined by the Berry curvature dipole, requiring only inversion-symmetry breaking. This picture generalizes naturally to higher-order responses and to nonlinear responses of other observables. For dynamical second-order responses, the injection currents generated by linearly and circularly polarized light are related to the quantum metric dipole and Berry curvature dipole, respectively. Reducing dimensionality further enriches this landscape, where these geometric effects combine with intrinsic structural chirality and topological states to produce unique phenomena. Spatially dispersive responses represent another class of nonlinear effects, characterizing the response to slow spatial variation of external fields. The optical current driven by a spatially varying electric field determines how a crystal distinguishes the opposite propagation direction of light, and is governed by the Berry curvature dipole or quantum metric dipole depending on whether the light is circularly or linearly polarized.

In summary, quantum geometry provides a unifying framework for a broad range of nonlinear responses. Its tunability then offers a promising route toward next-generation device design.