Quantum Theory of Electric Polarization in Inhomogeneous Crystals

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In the talk, we study the polarization in a crystal induced by spatial gradients of slowly varying inhomogeneities, including as electromagnetic and strain fields. In a previous work, based on semiclassical theory, a formula for the electric polarization in such inhomogeneous crystals has been proposed [1]. One can naively expect that this result is related to electric quadrupoles in crystals, but in fact it is inconsistent with the expression for the electric quadrupole introduced from thermodynamic arguments [2]. In this talk, we address this problem based on the gradient expansion of Green's functions and the quantum kinetic equation. We analytically derive the formulae of the charge and current, up to second order in temporal and spatial gradients, and calculate the electric polarization and orbital magnetization [3]. This result reproduces the formulae of the charge polarizability and magnetoelectric susceptibility in previous papers. We also obtain the expressions for the higher-order contributions: the electric polarization induced by spatial modulation and the orbital magnetization induced by temporal modulation, and compare them with previous studies.

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[2] A. Daido, A. Shitade, and Y. Yanase, Phys. Rev. B 102, 235149 (2020).

[3] Y. Suzuki, N. Arai, M. Fujimoto, Y. Gao, D. Xiao, S. Murakami, in preparation, 2026.