

Quantum Geometry in Nonlinear Transport and Magnetization

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Quantum geometry interprets the Hilbert space of quantum states and has attracted tremendous attention in the research quantum materials [5]. In this talk, I will cover two topics on how quantum geometry can be probed by nonlinear transport [7-11] and nonlinear magnetization [3]. In realistic materials where disorder scatterings are everywhere, how to identify the geometric effects remains a challenge. We develop a comprehensive theory for the second-order [6] and third-order [1,4] nonlinear transport, by treating the geometric effects and disorder scattering on an equal footing. More importantly, we show how the geometric and disorder mechanisms could be distinguished, by deriving a scaling law that expresses the nonlinear Hall conductivity as a polynomial function of the linear longitudinal conductivity. We apply the theory to 2D materials, topological materials, and altermagnets. This theory further promotes nonlinear transport as a quantitative probe of geometric effects in quantum materials. The Christoffel symbol is an essential quantity in Einstein's general theory of relativity. We discover that an electric field can induce a nonlinear magnetization in quantum materials, described by the Christoffel symbol [2]. Quite different from the previous scenarios, this orbital magnetization does not need spin-orbit coupling. We identify many 2D material candidates (e.g., BiF₃, ZnI₂, and Ru₄Se₅) that host this quantum Christoffel nonlinear magnetization. This nonlinear Christoffel magnetization can be probed by optical techniques such as magneto-optical Kerr spectroscopy. More importantly, this quantum Christoffel nonlinear magnetization gives a paradigm of how geometry dictates physics.

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