# Quantum-mechanical formulation of current-generating local orbital magnetization

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Magnetism is one of the most fundamental physical phenomena and has been studied extensively for many years. While many magnetic materials are characterized by arrangements of electron spins, magnetic states can also originate from the orbital motion of electrons. Recent examples include orbital ferromagnetism in moiré Chern insulators [1] and charge order accompanied by time-reversal-symmetry breaking in kagome superconductors [2]. In spin magnetism, magnetic structures can be described both intuitively and quantitatively in terms of the real-space distribution of the spin density. To obtain an analogous real-space description of orbital magnetism, however, one must formulate a local orbital magnetization quantum mechanically. This is a nontrivial problem, and various definitions of local magnetization have been proposed. One of the well-known definitions is the Bianco–Resta local marker [3], which is natural in the sense that its bulk value is independent of the microscopic details of the system boundary and reproduces the modern theory of orbital magnetization [4]. A limitation of this definition, however, is that it is not necessarily guaranteed to microscopically reproduce the familiar relation between current and magnetization in classical electrodynamics, $j(x)=\nabla\times m(x)$. Although various alternative definitions have been proposed since the work of Bianco and Resta, none has fully resolved this issue.

In this talk, we define the local orbital magnetization as the response of the grand potential to a local magnetic flux and derive quantum-mechanical formulas for both continuum and lattice models [5]. The resulting three-point formula not only yields the local current density as its rotation, but also reproduces the modern theory of orbital magnetization when averaged over a unit cell in the bulk. Although an ambiguity in the longitudinal component remains in three dimensions, we show that, in two-dimensional systems, the three-point formula uniquely determines a local orbital magnetization satisfying the relevant physical conditions. For lattice models, we explicitly establish the relation between the three-point formula and bond currents, demonstrating that orbital magnetization in two-dimensional systems can be interpreted as a spatial distribution of microscopic ferromagnetic domains. We further introduce a natural local marker on lattice sites through a coarse-graining procedure and show that this marker reproduces the coarse-grained current density up to third-order gradient corrections. As an application, we demonstrate that the spatial moment of the local marker agrees exactly with the orbital magnetic quadrupole moment of a finite system, thereby providing a basis for describing surface magnetization and higher-order magnetic multipoles. Finally, using the Haldane model [6], we illustrate the obtained formulas and relationship between orbital magnetization, bond currents, edge magnetization, and the orbital magnetic quadrupole moment.

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