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Topological Matter Protected by Momentum Space Crystallographic Groups

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While crystallographic groups are typically considered in real space, momentum-space crystallographic groups (MCGs) have recently emerged as an active research area. This development is largely driven by the framework of projective crystal symmetry, where all non-symmorphic crystallographic groups arise from phase factors between real-space translations and point-group elements, according to Mackey's representation theory [1,2]. A key implication of non-symmorphic MCGs is that the momentum-space unit—traditionally regarded as a torus—can take the form of any compact flat manifold, known as the ten platycosms, which are the orbital spaces of the ten Bieberbach groups [3]. For each platycosm, the topological classification, specifically the reduced K-group, is isomorphic to the second integral cohomology group of the corresponding Bieberbach group [3]. We will further demonstrate that the cohomology groups of MCGs can exhaustively classify all Abelian crystalline topological insulators, as well as all twistings of point-group actions over the Brillouin torus [4]. By establishing an isomorphism between the integral cohomology and a one-degree-lower cohomology with U(1)-valued functions over momentum space as coefficients, we can algebraically formulate a complete set of topological invariants for classifying Abelian crystalline topological insulators and algebraically represent all twistings of point-group actions.

References:

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