

From Topology to Thermodynamic Paradoxes in Electromagnetic Systems

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I will present an overview of my group's work on topological effects in photonics. Since the pioneering work of Haldane and Raghu, there has been rapid growth in interest in how topology shapes the physics of electromagnetic systems, especially in systems that show nonreciprocity through broken time-reversal symmetry. This talk will connect several key ideas, such as nonlocality, dispersion, and periodicity, into a unified picture of the topological behavior of photonic systems.

At the core is the special nature of dispersive electromagnetic systems, where frequency-dependent responses can lead to surprising effects. A notable example is particle-hole symmetry: unlike in electronic systems, this symmetry can produce an electromagnetic spectrum without a lower bound and support an infinite ladder of edge states, giving rise to new forms of boundary phenomena [1, 2]. Dispersion in periodic systems can also cause bands to accumulate around resonance frequencies, which challenges standard classifications and creates topologies that are not well defined.

I will show that such ill-defined topologies are closely tied to real-space singularities, where electromagnetic fields diverge and the bulk–edge correspondence fails, even in strongly dissipative systems [3, 4]. Most strikingly, in the absence of suitable spatial cut-offs, electromagnetic systems may even seem to violate basic principles of thermodynamic equilibrium, leading to thermodynamic paradoxes. The resolution comes from including nonlocal effects, which prevent unphysical localization and restore consistency by setting spatial scales through physical processes such as drift-diffusion in plasmas or exchange interactions in ferrites [4].

Finally, I will discuss spacetime crystals, structures with time-varying modulation that imitate motion. In particular, traveling-wave spacetime modulation and physical motion open new ways to create topological phases by combining spatial and temporal periodicity [5], and even allow the Hall effect to be imitated in electromagnetic platforms [6].

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