Towards a Theory of Recurrent Neural Networks for Nonlinear Time Series

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Recurrent Neural Networks (RNNs) are a class of artificial neural networks designed to model sequential data, including text, speech, and time series. This paper investigates the internal mechanisms of RNNs and provides theoretical guarantees within the framework of time series models. Specifically, we analyze a nonlinear autoregressive and moving-average model with exogenous variables model and derive a statistical error bound for RNN prediction performance. The error bound comprises two components: the approximation error, influenced by the smoothness of the target function and input dimensionality, and the estimation error, determined by the RNN architecture and the mixing properties of the underlying process. Our analysis demonstrates that RNNs excel at capturing nonlinear dependencies in noise, offering a distinct advantage over nonparametric methods such as feed-forward neural networks.