

Realized Copula of Volatility

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We study a new measure of codependency in the second moment of a continuous-time multivariate asset price process, which we name the realized copula of volatility. The statistic is based on local volatility estimates constructed from high-frequency asset returns and affords a nonparametric estimator of the empirical copula of the latent stochastic volatility. We show the consistency of our estimator with in-fill asymptotic theory, either with a fixed or increasing time span. In the latter setting, we derive a feasible functional central limit theorem for the empirical process associated with the measurement error of the time-invariant marginal copula of volatility. We also develop a goodness-of-fit test to evaluate hypotheses about the shape of the latter. In a simulation study, we demonstrate that our estimator is a good proxy of both the empirical and marginal copula of volatility, even with a moderate amount of high-frequency data recorded over a relatively short sample. The goodness-of-fit test is found to exhibit size control and excellent power. We implement our framework on high-frequency transaction data from futures contracts that track the U.S. equity and treasury bond markets. A Gumbel copula is found to offer a near-perfect bind between the realized variance processes in these data. This is joint work with Kim Christensen, Wenjing Liu, and Yoann Potiron.