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Non-minimizing and Min-max Solutions to Bernoulli Problems

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Bernoulli type free boundary problems have a well-developed existence and regularity theory. Much of this, however, is restricted to the case of minimizers of the natural energy (the Alt-Caffarelli functional). I will describe a compactness and regularity theorem that applies to any critical point instead, based on a nonlinear frequency formula and Naber-Valtorta estimates. Then I will explain, via an example involving gravity water waves, how to use this theorem to find min-max type (mountain pass) solutions. This is based on joint work with Georg Weiss.