

On Brezis' First Open Question: A Complete Solution

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In 2023, H. Brezis published a list of his "favorite open problems", which he described as challenges he had "raised throughout his career and has resisted so far". We provide a complete resolution to the first one--Open Problem 1.1--in Brezis's favorite open problems list: the existence of solutions to the long-standing Brezis-Nirenberg problem on a three-dimensional ball

$$-\Delta u + \lambda u + u^5 = 0 \quad \text{in } B_1,$$

$$u = 0 \quad \text{on } \partial B_1, \quad 0 < \lambda \leq \frac{\pi^2}{4}$$

Furthermore, using the building blocks of Del Pino-Musso-Pacard-Pistoia sign-changing solutions to the Yamabe problem, we establish the existence of infinitely many sign-changing, nonradial solutions for the full range of the parameter $0 < \lambda < +\infty$. We also extend the results to cuboids or rectangular prisms. (Joint work with Liming Sun and Wen Yang.)