On a Classification of Steady Solutions to Two-dimensional Euler Equations

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In this talk, I shall provide a classification of steady solutions to two-dimensional incompressible Euler equations in terms of the set of flow angles. The first main result asserts that the set of flow angles of any bounded steady flow in the whole plane must be the whole circle unless the flow is a parallel shear flow. In an infinitely long horizontal strip or the upper half-plane supplemented with slip boundary conditions, besides the two types of flows appeared in the whole space case, there exists an additional class of steady flows for which the set of flow angles is either the upper or lower closed semicircles. This type of flows is proved to be the class of non-shear flows that have the least total curvature. A further classification of this type of solutions will also be discussed. As consequences, we obtain Liouville-type theorems for two-dimensional semilinear elliptic equations with only bounded and measurable nonlinearity, and the structural stability of shear flows whose all stagnation points are not inflection points, including Poiseuille flow as a special case. Our proof relies on the analysis of some quantities related to the curvature of the streamlines. This talk is based on joint works with David Ruiz, Chunjing Xie and Huan Xu.