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Optimal Convergence of the Arbitrary Lagrangian–Eulerian Interface Tracking Method for Two-phase Navier–Stokes Flow

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Optimal-order convergence in the H^1 norm is proved for an arbitrary Lagrangian-Eulerian interface tracking finite element method for the sharp interface model of two-phase Navier-Stokes flow using high-order curved evolving mesh. In this method, the interfacial mesh points move with the fluid's velocity to track the sharp interface between two phases of the fluid, and the interior mesh points move according to a harmonic extension of the interface velocity. The error of the semidiscrete arbitrary Lagrangian-Eulerian interface tracking finite element method is shown to be $O(h^k)$ in the $L^\infty(0, T; H^1(\Omega))$ norm for the Taylor-Hood finite elements of degree $k \ge 2$. This high-order convergence is achieved by utilizing the piecewise smoothness of the solution on each subdomain occupied by one phase of the fluid, relying on a low global regularity on the entire moving domain. The theoretical error analysis is consistent with the numerical experiments, which illustrate the convergence of the method in simulating benchmark examples of gas bubbles rising in liquid.