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Schrödingerization Based Computationally Stable Algorithms for Ill-Posed Problems in Partial Differential Equations

Chuwen Ma

School of Mathematical Sciences, Shanghai Jiao Tong University, P. R. China

Email: chuwenii@sjtu.edu.cn

We introduce a simple and stable computational method for ill-posed partial differential equation (PDE) problems. The method is based on Schr\"odingerisation, introduced in [S. Jin, N. Liu and Y. Yu, arXiv:2212.13969][S. Jin, N. Liu and Y. Yu, Phys. Rev. A, 108 (2023), 032603], which maps all linear PDEs into Schr\"odinger-type equations in one higher dimension, for quantum simulations of these PDEs. Although the original problem is ill-posed, the Schr\"odingerised equations are Hamiltonian systems and time-reversible, allowing stable computation both forward and backward in time. The original variable can be recovered by data from suitably chosen domain in the extended dimension. We will use the (constant and variable coefficient) backward heat equation and the linear convection equation with imaginary wave speed as examples. Error analysis of these algorithms are conducted and verified numerically. The methods are applicable to both classical and quantum computers, and we also lay out quantum algorithms for these methods. Moreover, we introduce a smooth initialisation for the Schr\"odingerised equation which will lead to essentially spectral accuracy for the approximation in the extended space, if a spectral method is used. Consequently, the extra qubits needed due to the extra dimension, if a qubit based quantum algorithm is used, for both well-posed and ill-posed problems, becomes almost $\log\log {1/\varepsilon}$ where ε is the desired precision. This optimizes the complexity of the Schr\"odingerisation based quantum algorithms for any non-unitary dynamical system introduced in [S. Jin, N. Liu and Y. Yu, arXiv:2212.13969][S. Jin, N. Liu and Y. Yu, Phys. Rev. A, 108 (2023), 032603].

References:

[1] S. Jin, N. Liu and Y. Yu, arXiv:2212.13969.
[2] S.Jin, N. Liu and C. Ma, arXiv:2403.19123.