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Rigorous Computation for Eigenvalue Problems and Solution Verification for the Navier-Stokes Equations

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Recently, rigorous computation has emerged as a powerful tool for providing explicit error estimates for numerical solutions to boundary value problems and eigenvalue problems of differential operators. The author's recent book [1] surveys these developments, showcasing significant progress in this field. In this talk, I will summarize the latest results on rigorous computation, with a focus on rigorous eigenvalue estimation, and describe the underlying concepts in the verification of solutions to the Navier-Stokes equations, including techniques for handling three-dimensional domains.

Additionally, I will outline the objectives and current progress of an ongoing project aimed at verifying the existence of multiple solutions to the Navier-Stokes equations in 3D domains. This discussion will highlight several key challenges and intricate sub-problems to address.

References:

[1] Xuefeng LIU, Guaranteed Computational Methods for Self-Adjoint Differential Eigenvalue Problems, Springer Singapore, July 2024.

[2] Xuefeng Liu, Mitsuhiro T. Nakao, Shin'ichi Oishi, Computer-assisted proof for the stationary solution existence of the Navier–Stokes equation over 3D domains, Communications in Nonlinear Science and Numerical Simulation, Volume 108, 2022, 106223.