

**Asymptotic Behavior at Infinity of Phaseless Total-Field Operators and Applications to  
Phaseless Inverse Scattering at a Fixed Frequency**

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This paper is concerned with inverse acoustic scattering with phaseless total-field data at a fixed frequency in 2D and 3D cases with plane wave incidence or point source incidence. We propose a general framework for studying this phaseless inverse problem. Define the phaseless total-field operators as integral operators with the kernels given in terms of the phaseless total-field measured on the sphere  $\partial B_R$  with a large radius  $R$ . By making use of harmonic analysis techniques we establish the asymptotic property in the operator norm from  $L^2(S^{n-1})$  to  $L^2(S^{n-1})$  of the phaseless total-field operators for large enough  $R$ , that is, the phaseless total-field operator tends to the corresponding far-field operator defined in terms of the far-field pattern measured on the unit sphere  $S^{n-1}$ , as  $R$  goes to infinity, where  $n=2,3$ . As applications of the asymptotic property, we establish uniqueness results for inverse scattering problems with phaseless total-field data generated by incident plane waves or point sources at a fixed frequency and measured in a small domain. Further, we develop an approximate factorization method to numerically reconstruct both the location and shape of the unknown scatterer from the phaseless total-field data generated by incident plane waves or point sources at a fixed frequency and measured on the sphere  $\partial B_R$  with a sufficiently large radius  $R$ . Numerical examples in 2D are also carried out to demonstrate the effectiveness of our inversion method.