Differential-Equation Constrained Optimization with Stochasticity

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Most inverse problems from physical sciences are formulated as PDE-constrained optimization problems. This involves identifying unknown parameters in equations by optimizing the model to generate PDE solutions that closely match measured data. The formulation is powerful and widely used in many sciences and engineering fields. However, one crucial assumption is that the unknown parameter must be deterministic. In reality, however, many problems are stochastic in nature, and the unknown parameter is random. The challenge then becomes recovering the full distribution of this unknown random parameter. It is a much more complex task. In this talk, we examine this problem in a general setting. In particular, we conceptualize the PDE solver as a push-forward map that pushes the parameter distribution to the generated data distribution. This way, the SDE-constrained optimization translates to minimizing the distance between the generated distribution and the measurement distribution. We then formulate a gradient-flow equation to seek the ground-truth parameter probability distribution. This opens up a new paradigm for extending many techniques in PDE-constrained optimization to that for systems with stochasticity. This is a joint work with Qin Li (University of Wisconsin Madison) and Li Wang (University of Minnesota Twin Cities) [1].

References:

[1] Li, Q., Wang, L., & Yang, Y. (2023). Differential-Equation Constrained Optimization With Stochasticity. arXiv preprint arXiv:2305.04024.