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Hadamard Integrator for Time-Dependent Wave Equations: Lagrangian Formulation via Ray Tracing

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Starting from the time-domain Kirchhoff-Huygens representation of wave solutions, we propose a novel Hadamard integrator for the self-adjoint time-dependent wave equation in an inhomogeneous medium. First, we create a new asymptotic series based on the Gelfand-Shilov function, dubbed Hadamard's ansatz, to approximate the Green's function of the time-dependent wave equation. Accordingly, the governing equations and related initializations for the eikonal and Hadamard coefficients are derived using the properties of the Gelfand-Shilov generalized function. Second, incorporating the leading term of Hadamard's ansatz into the Kirchhoff-Huygens representation, we develop an original Hadamard integrator for the Cauchy problem of the time-dependent wave equation and derive the corresponding Lagrangian formulation in geodesic polar coordinates. Third, to construct the Hadamard integrator in the Lagrangian formulation efficiently, we use a short-time ray tracing method to obtain equal-time wavefront locations accurately, and we further develop fast algorithms to compute Chebyshev-polynomial based low-rank representations of both wavefront locations and variants of Hadamard coefficients. Fourth, equipped with these low-rank representations, we apply the Hadamard integrator to efficiently solve time-dependent wave equations with highly oscillatory initial conditions, where the time step size is independent of the initial conditions. By judiciously choosing the medium-dependent time step, our new Hadamard integrator can propagate wave field beyond caustics implicitly and advance spatially overturning waves in time naturally. Moreover, since the integrator is independent of initial conditions, the Hadamard integrator can be applied to many different initial conditions once it is constructed. Both two-dimensional and three-dimensional numerical examples illustrate the accuracy and performance of the proposed method.