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Geodesic X-Ray Transform and Boundary Rigidity in Low Regularity

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We prove that the geodesic X-ray transform is injective on $L^2(M)$ when the Riemannian metric is simple but the metric tensor is only finitely differentiable. The number of derivatives needed depends explicitly on dimension, and in dimension 2 we assume $g \in C^{10}$. Our proof is based on microlocal analysis of the normal operator: we establish ellipticity and a smoothing property in a suitable sense and then use a recent injectivity result on Lipschitz functions [1]. The proof of this injectivity result in the smooth setting [2] uses the parametrix construction for elliptic pseudodifferential operators which is a standard result in PDE theory [3]; when the metric tensor is C^k, the Schwartz kernel is not smooth but C^{k-2} off the diagonal, which makes standard smooth microlocal analysis inapplicable as the corresponding symbol only satisfies the pseudodifferential operators symbol estimates up to a finite degree.

As an application we use the injectivity of the geodesic X-ray transform to prove that even for metrics at low regularity - the scattering relation on a compact two-dimensional simple manifolds determines the Dirichlet-to-Neumann map - a major component of the proof of boundary rigidity for simple metrics proved in [4] for smooth geometry.

References:

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