

Hybrid Neural-Network FEM Approximation of Conductivity in Elliptic Problem

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In this talk, we investigate the numerical identification of the conductivity in elliptic problem using neural networks. The numerical scheme is based on the standard output least-squares formulation where the Galerkin finite element method (FEM) is employed to approximate the state and neural networks (NNs) act as a smoothness prior to approximate the unknown conductivity. The hybrid approach enjoys both rigorous mathematical foundation of the FEM and inductive approximation properties of NNs. We derive a priori error estimates in the standard $L^2(\mathcal{Q})$ norm for the numerical reconstruction, under a positivity condition which can be verified for a large class of problem data. The error bounds depend explicitly on the noise level, regularization parameter and discretization parameters. The numerical experiments indicate that the hybrid method is very robust for large noise when compared with the pure FEM approximation.