Physics-Inspired Convolutional Neural Network for Solving Inverse Scattering Problems

Xudong CHEN

Department of Electrical and Computer Engineering,

National University of Singapore

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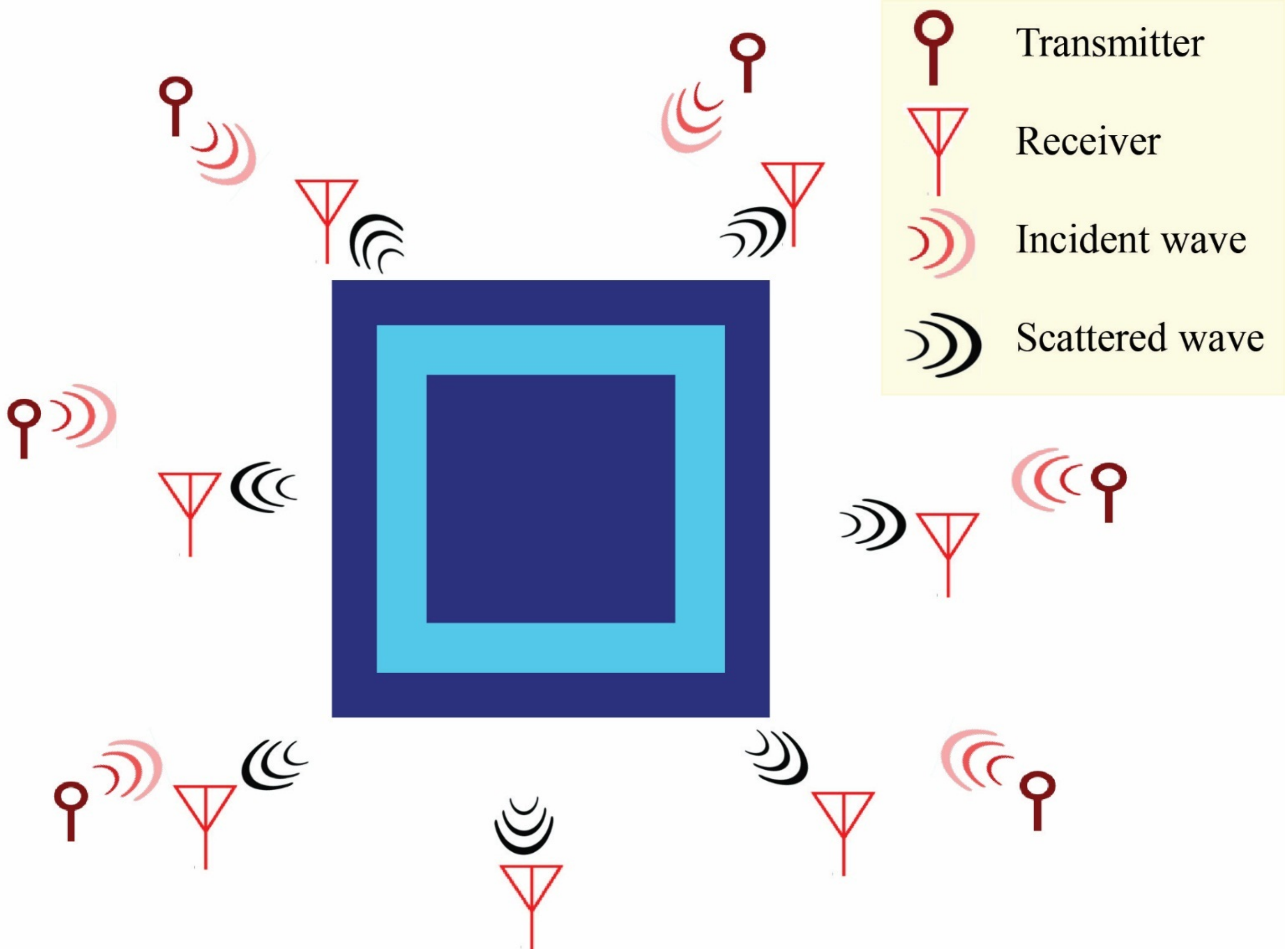
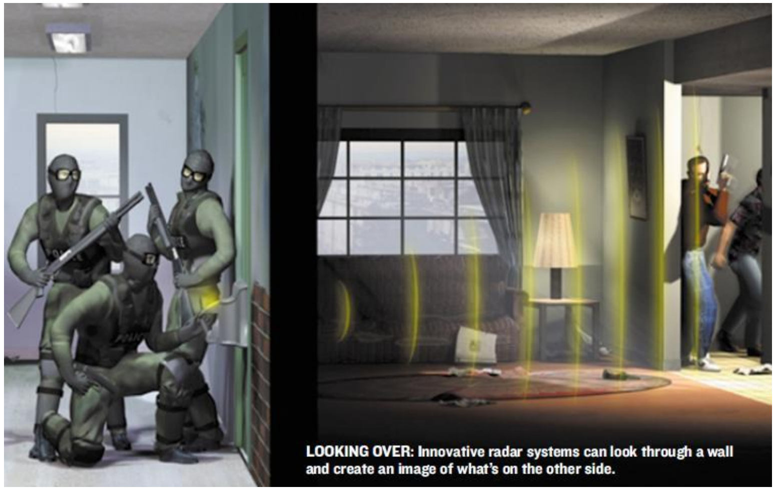
Introduction to Inverse scattering problems (ISPs)

- To find unknown scatterers (permittivities, sizes, locations) inside the wall from scattering data



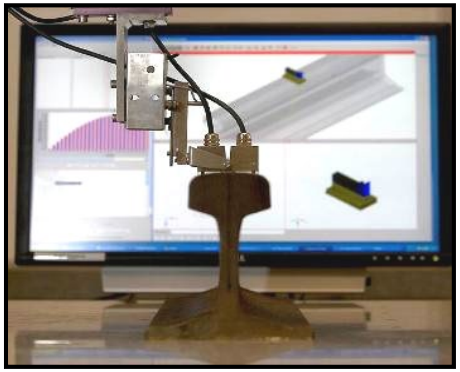
Real-world applications ISPs

Through Wall Imaging

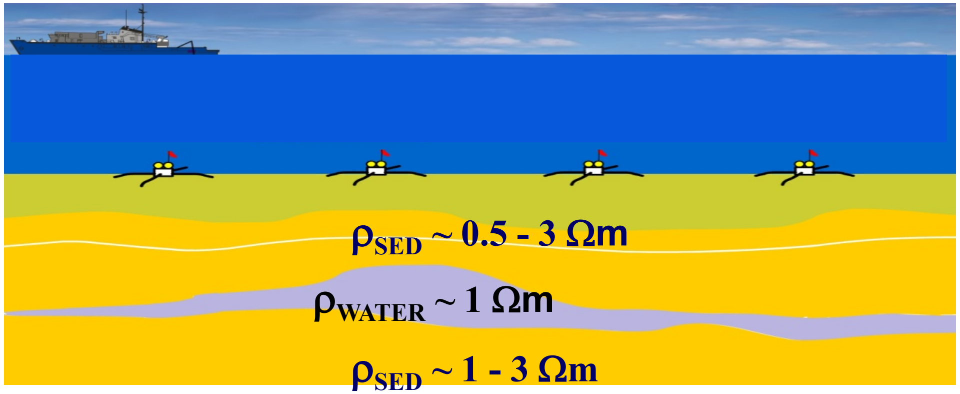


Ultrasonic scanning imaging

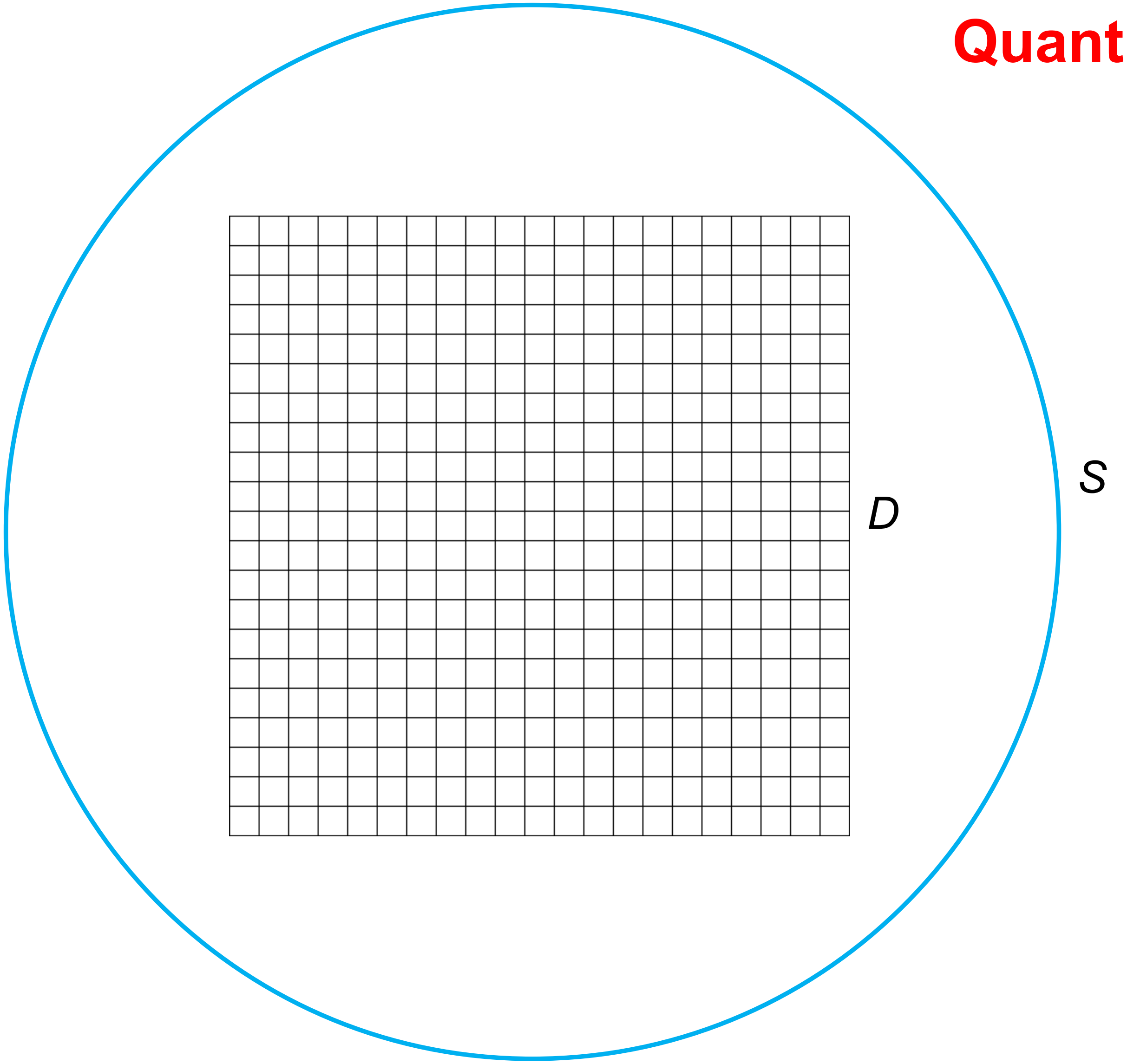
Non-destructive evaluation



Petroleum exploration

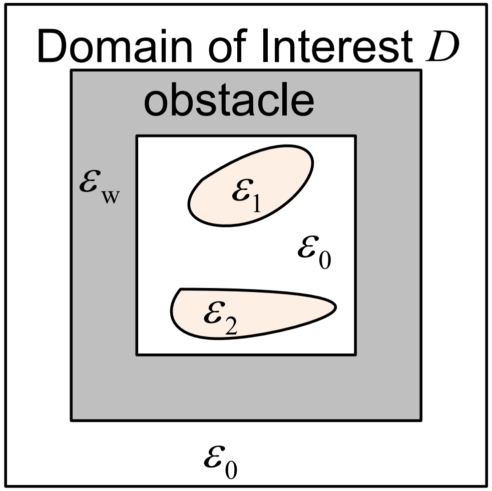


Introduction to ISPs



Quantitative imaging:

The value of ε_r will be recovered at each pixel of D



Motivation

- Apply neural network (NN) to solve ISPs, a regression problem.
- How to provide guidance to deep learning?
  - ➢ Find frameworks and links with mathematics (Optimization, Numerical differential equations, Control…)
  - ➢ Find insights from physics
    - In many real-world applications, data are collected by sensors (optical, acoustical, microwave wave, electric current/voltage, heat….), automatically governed by physical laws
    - Some physical laws present well-known mathematical properties (even analytical solutions), which do not need to be learnt by training with a lot of data.

Forward problem (2D)

- Background medium with parameters $\varepsilon_b(\mathbf{r})$ μ_0
- Distribution of permittivities $\varepsilon(\mathbf{r}) = \varepsilon_r(\mathbf{r}) \cdot \varepsilon_b(\mathbf{r})$
- Governing equation (single frequency):

$$\left[\nabla^2 + k_b^2(\mathbf{r})\right] E_z^{\text{inc}}(\mathbf{r}) = 0, \qquad \left[\nabla^2 + k^2(\mathbf{r})\right] E_z^{\text{tot}}(\mathbf{r}) = 0, \quad \mathbf{r} \in D$$

$$\Rightarrow \left[\nabla^2 + k_b^2(\mathbf{r})\right] E_z^{\text{sca}}(\mathbf{r}) = -[k^2(\mathbf{r}) - k_b^2(\mathbf{r})] E_z^{\text{tot}}(\mathbf{r}) = -J(\mathbf{r}) \text{ [defined]}$$

where

$$k_b = \omega\sqrt{\varepsilon_b \mu_0}$$

$$E_z^{\text{tot}} = E_z^{\text{sca}} + E_z^{\text{inc}}$$

- Lippmann-Schwinger equation:

$$E_z^{\text{tot}}(\mathbf{r}) = E_z^{\text{inc}}(\mathbf{r}) + \int_D g(k_b;\mathbf{r},\mathbf{r}')\, [k^2(\mathbf{r}') - k_b^2(\mathbf{r}')] E_z^{\text{tot}}(\mathbf{r}')\, d\mathbf{r}'$$

$$J(\mathbf{r}) = [k^2(\mathbf{r}) - k_b^2(\mathbf{r})] \cdot \left[E_z^{\text{inc}}(\mathbf{r}) + \int_D g(k_b;\mathbf{r},\mathbf{r}') J(\mathbf{r}') d\mathbf{r}' \right]$$

- Scattered field

$$E_z^{\text{sca}}(\mathbf{r}) = \int_D g(k_b;\mathbf{r},\mathbf{r}') J(\mathbf{r}') d\mathbf{r}' \qquad \mathbf{r} \in S$$

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Forward problem (2D)

- Introduce notation

$$g(k_b;\mathbf{r},\mathbf{r}') \text{ is denoted as } \begin{cases} g_S(k_b;\mathbf{r},\mathbf{r}') & \text{for } \mathbf{r} \in S \text{ (at receiver)} \\ g_D(k_b;\mathbf{r},\mathbf{r}') & \text{for } \mathbf{r} \in D \text{ (inside domain)} \end{cases}$$

- Scattering equations:

$$E_z^{\text{sca}}(\mathbf{r}) = \int_D g_S(k_b;\mathbf{r},\mathbf{r}') J(\mathbf{r}') d\mathbf{r}', \qquad \mathbf{r} \in S$$

$$J(\mathbf{r}) = k_b^2(\mathbf{r}) \left[\varepsilon_r(\mathbf{r}) - 1 \right] \cdot \left[E_z^{\text{inc}} + \int_D g_D(k_b;\mathbf{r},\mathbf{r}') J(\mathbf{r}') d\mathbf{r}' \right], \quad \mathbf{r} \in D$$

Forward Problem (2D)

After discretization, we have

- Data equation:

$$\overline{E}^{scat} = \overline{\overline{G}}_S \cdot \overline{J}$$

- Lippmann-Schwinger equation:

$$\overline{J} = \overline{\overline{\chi}} \cdot \left(\overline{E}^{inc} + \overline{\overline{G}}_D \cdot \overline{J} \right)$$

$\bar{\bar{\chi}}$ is the diagonal matrix consisting of $k_b^2(\varepsilon_r - 1)$, referred to as the contrast (with the background) [scattering potential].

- Method of solving forward problem: Eliminate $\bar{J}$ and obtain the Incidence-to-Scattering mapping

$$\overline{E}^{scat} = \overline{\overline{P}} \cdot \overline{E}^{inc}$$

where

$$\bar{\bar{P}} = \bar{\bar{G}}_S \cdot \left(\bar{\bar{I}} - \bar{\bar{\chi}} \cdot \bar{\bar{G}}_D\right)^{-1} \cdot \bar{\bar{\chi}}$$

Inverse Scattering Problem (ISP)

$$\overline{E}^{scat} = \overline{\overline{P}} \cdot \overline{E}^{inc}$$

- The relationship between $\bar{E}^{scat}$ and $\bar{\bar{\chi}}$ is nonlinear
- Linear approximation ISP (Born Approximation)

$$\overline{\overline{P}} = \overline{\overline{G}}_S \cdot \left(\overline{\overline{I}} - \overline{\overline{\chi}} \cdot \overline{\overline{G}}_D \right)^{-1} \cdot \overline{\overline{\chi}}$$

$$\approx \overline{\overline{G}}_S \cdot \overline{\overline{\chi}}$$

- Full-wave ISP (i.e., no linearization approximation is made)
  - Highly nonlinear (due to $\bar{\bar{G}}_D$)
  - Ill-posed (due to $\bar{\bar{G}}_S$)
  - High-dimensional (due to the total number of pixels)
- Special case: Far-field operator

Two approaches to inverse problems

Forward problem

$H\{x\} = y$

an image $x \in \mathcal{X}$, a vector of measurements $y \in \mathcal{Y}$

operator $H : \mathcal{X} \to \mathcal{Y}$

Inverse problem

recover the original image, x, from the measurements, y

$R : \mathcal{Y} \to \mathcal{X}$

Objective function approach

$$R_{\text{obj}}\{y\} = \underset{x \in \mathcal{X}}{\operatorname{argmin}}\, f(H\{x\}, y) + g(x)$$

$f : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}^+$ is an appropriate measure of error

$g : \mathcal{X} \to \mathbb{R}^+$ is a regularization functional

Two approaches to inverse problems

Learning approach

Construct an explicit inverse: $x = R_\theta(y)$, given a training set of ground-truth images and their corresponding measurements $\{(x_n, y_n)\}_{n=1}^{N}$

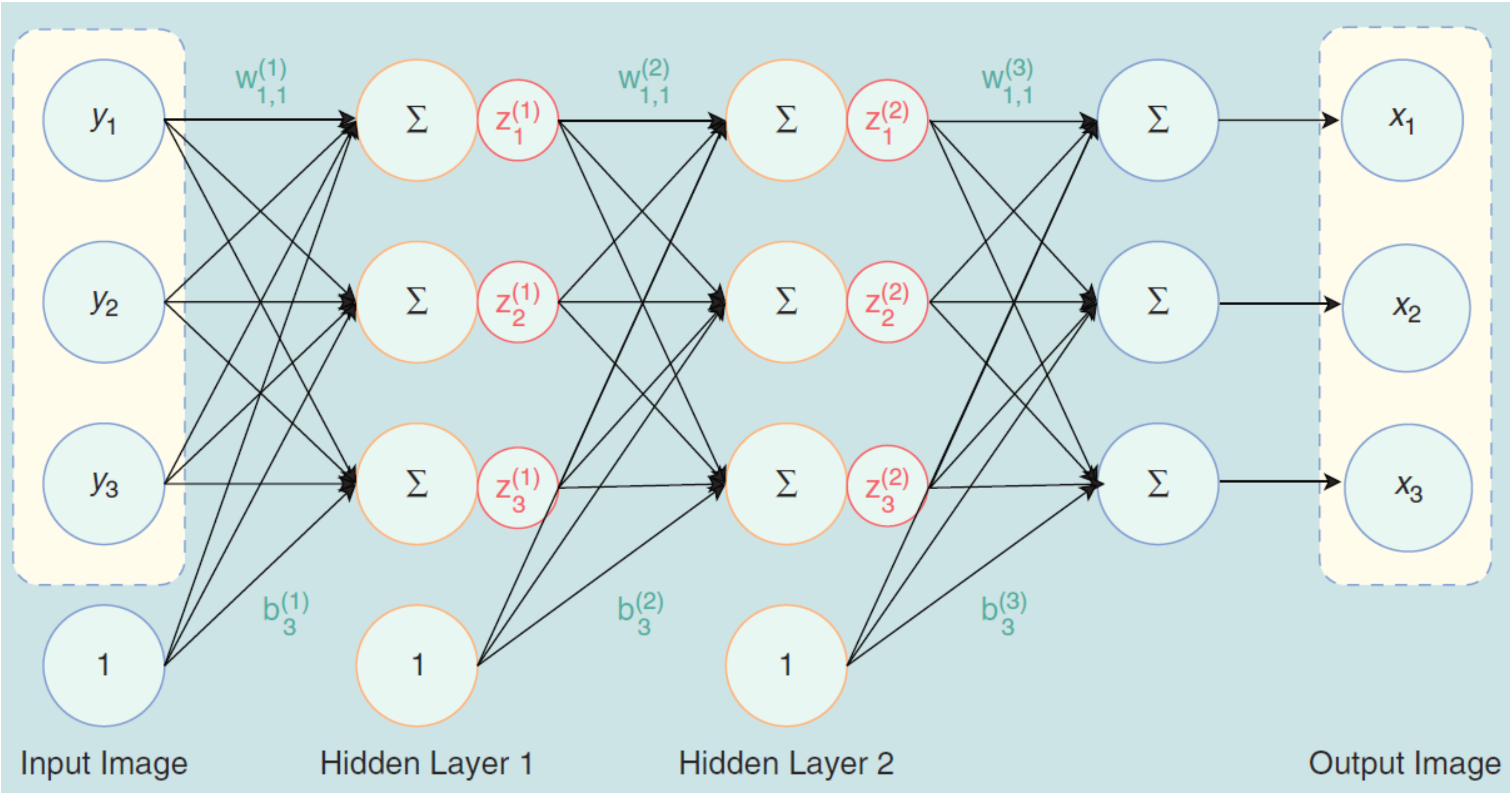
$$R_{\text{learn}} = \operatorname*{argmin}_{R_\theta, \theta \in \Theta} \sum_{n=1}^{N} f(x_n, R_\theta\{y_n\}) + g(\theta)$$

Θ is the set of all possible parameters

$f : \mathcal{X} \times \mathcal{X} \to \mathbb{R}^+$ is a measure of error

$g : \Theta \to \mathbb{R}^+$ is a regularizer

An example of a fully connected neural network with two hidden layers



The activation of the jth output neuron in layer l is defined as

$z^l_j = f\left(\sum_i w^l_{i,j} z^{l-1}_i + b^l_j\right)$, where $f(\cdot)$ is the chosen activation function.

All weights $\mathbf{w}$ and biases $\mathbf{b}$ are learned during the training phase.

Lucas et.al. IEEE Signal Processing Magazine, 2018

Two approaches to inverse problems

- How to bridge the gap between objective-function approach and machine-learning approach forms a fertile ground for future investigation for the whole imaging community
  - Unroll an iterative optimization algorithm, turning each iteration into a layer of network

McCann, Jin, Unser, "Convolutional Neural Networks for Inverse Problems in Imaging " *IEEE Signal Processing Magazine*, 2017

"Central to this work will be questions of how best to combine CNNs with knowledge of the underlying physics as well as objective-function techniques"

Learning approach to ISP

- There is room to improve: deep learning approach has not yet had the profound impact on inverse problems that they have had for object classification
- Early stage:
  - Caorsi, TGRS, 1999;
  - Rekanos, TM, 2002;
  - Bermani, TGRS, 2003
- Shallow NN (and support vector machine (SVM)) are used
- Non-pixel representation (a few parameters, i.e., positions, geometries, and homogeneous dielectric properties)
- Advantage: real-time
- Limitations: requirement on prior information on scatterers; piecewise constant permittivity

Learning approach to ISP

- More versatile pixel-based representation + Deep NN

Four categories:

- Direct learning: (x, y)

  Comment: Black-box: no insight

- Hybrid approach:
  - ➢ Still use objective function approach + NN in each iteration of optimization [Learn gradient: Guo, TAP, 2019; Adler, IP, 2017]

    Comment: Improve local efficiency, but not global non-linearity
  - ➢ NN obtains initial guess + objective function approach: Sanghvi, TCI, 2019 (learn induced current, followed by SOM)

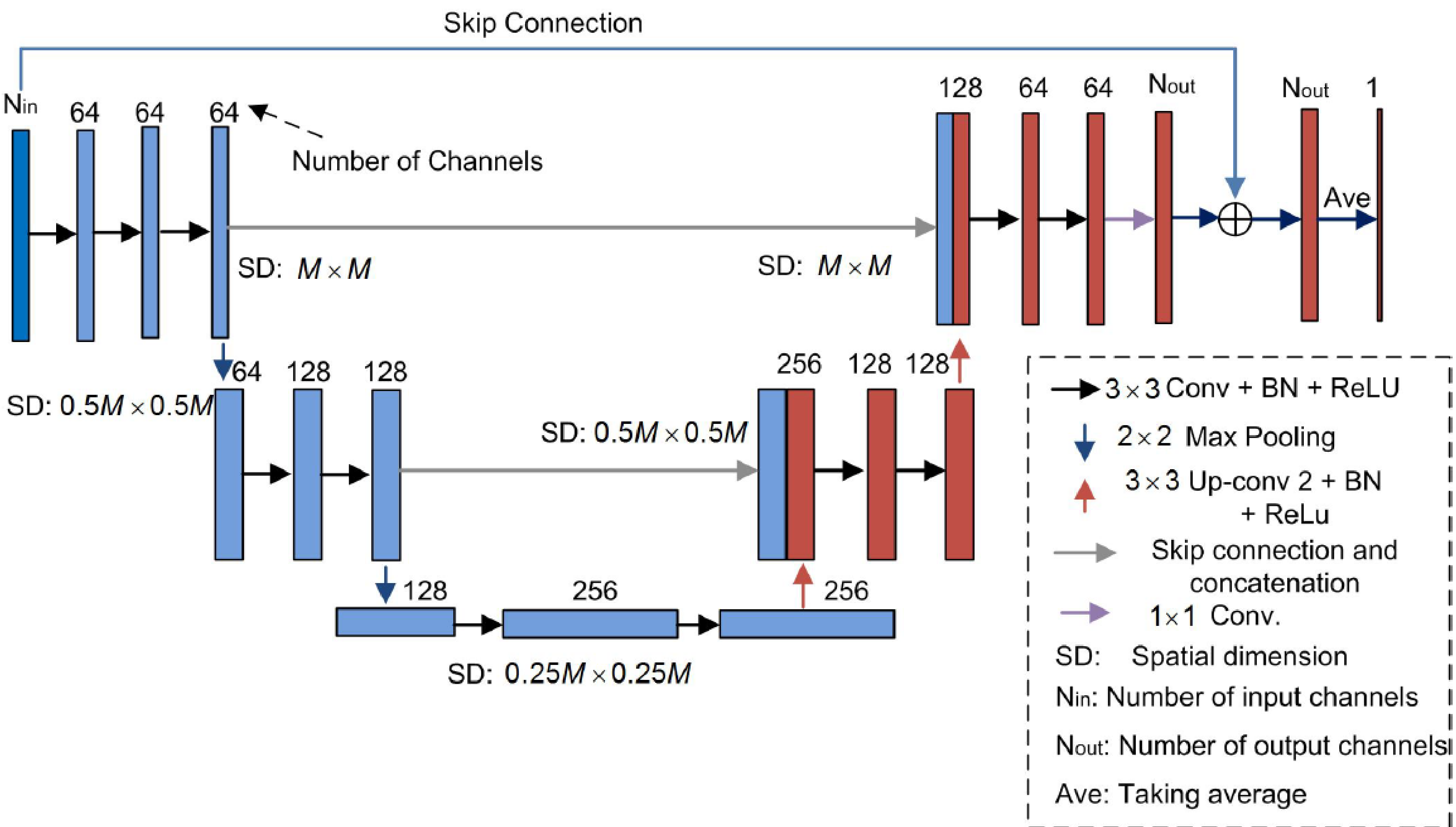
- New-representation: ($\hat{x}$, $\hat{y}$)

  New input & output [Li, TAP, 2019 (DeepNIS); Wei, TGRS, 2019 (DIS, BPS, DCS); Sun, OE, 2018 (BP)]

  Comment: need mathematical & physical insights; most promising

- Other advanced approaches

Examples Wei, IEEE-TGRS, 2019



The U-net architecture for the proposed three CNN schemes: DIS, BPS, and DCS

Three CNN schemes

1. Direct Inversion Scheme (DIS)

Inputs: Scattered field $\bar{E}^s$; Output: the contrasts $\bar{\bar{\chi}}$

2. Back-Propagation Scheme (BPS)

$$\overline{I}^b = \gamma \cdot \overline{\overline{G}}_S^H \cdot \overline{E}^s$$

$$\gamma = \frac{(\overline{E}^s)^T \cdot (\overline{\overline{G}}_S \cdot (\overline{\overline{G}}_S^H \cdot \overline{E}^s))^*}{||\overline{\overline{G}}_S \cdot (\overline{\overline{G}}_S^H \cdot \overline{E}^s)||}$$

$$\overline{\chi}^b(n) = \frac{\sum_{p=1}^{N_i} \overline{I}_p^b(n) \cdot [\overline{E}_p^{t,b}(n)]^*}{\sum_{p=1}^{N_i} ||\overline{E}_p^{t,b}(n)||^2}$$

Inputs: the BP contrasts $\bar{\bar{\chi}}^b$; Output: the contrasts $\bar{\bar{\chi}}$

3. Dominant Current Scheme (DCS)

- Recall the two equations

$$\overline{E}^{scat} = \overline{\overline{G}}_S \cdot \overline{J}$$

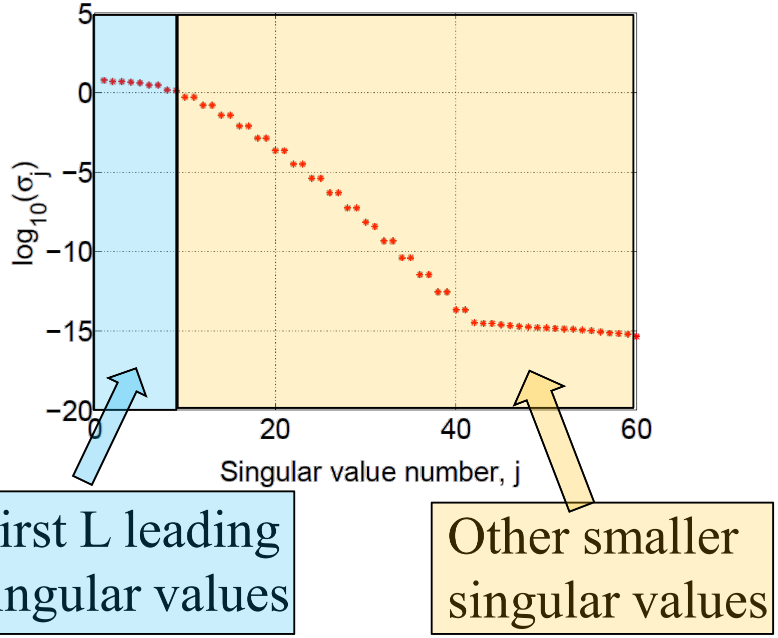
$$\overline{J} = \overline{\overline{\chi}} \cdot \left(\overline{E}^{inc} + \overline{\overline{G}}_D \cdot \overline{J} \right)$$

- Important: both the $\overline{\overline{G}}_S$ and $\overline{\overline{G}}_D$ operators, containing most data in the two equations, are independent of unknown scatterers
  - Motivate us to analyze the property of these two operators before reconstructing the contrast $\bar{\bar{\chi}}$
  - The computational overhead of such analysis should not be large

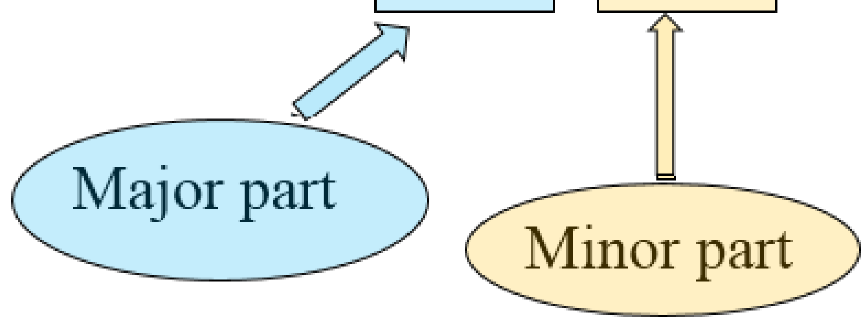
Dominant Current Scheme (DCS)

- The vector space, C^M, of the current $\bar{J}$ can be decomposed into subspaces.
- Singular Value Decomposition (SVD)

$$\bar{\bar{G}}_S = \bar{\bar{U}}_S \cdot \bar{\bar{\Sigma}}_S \cdot \bar{\bar{V}}_S^{*}$$



$$\bar{J} = \bar{\bar{V}}_S \cdot \bar{\alpha} = \bar{\bar{V}}_S^{+} \cdot \bar{\alpha}^{+} + \bar{\bar{V}}_S^{-} \cdot \bar{\alpha}^{-}$$



Dominant Current Scheme (DCS)

- Obtain the deterministic part using the linear relation

$$\bar{J}^{+} = \bar{\bar{V}}_S^{+} \cdot \bar{\alpha}^{+} = \sum_{j=1}^{L} \bar{v}_j^{+} \frac{\bar{u}_j^{+*} \cdot \overline{E}^{scat}}{\sigma_j}$$

- Dominant part current is defined as:

$$\bar{J}^{d} = \bar{J}^{+} + \bar{J}^{l}$$

$$\bar{J}^{l} = \bar{\bar{F}} \cdot \bar{\alpha}$$

Low-frequency Fourier components

$$\bar{\chi}_p^{d}(n) = \frac{\bar{J}_p^{d}(n) \cdot [\overline{E}_p^{t,d}(n)]^{*}}{||\overline{E}_p^{t,d}(n)||^{2}}$$

p: index of incidence

Wei, IEEE-TGRS, 2019

DCS: Inputs: the dominant contrasts $\bar{\bar{\chi}}_p^{d}$; Output: the contrasts $\bar{\bar{\chi}}$

Computational cost

- M pixels in the domain of interest

  N_r receivers: $M >> N_r$
- Computational cost of SVD

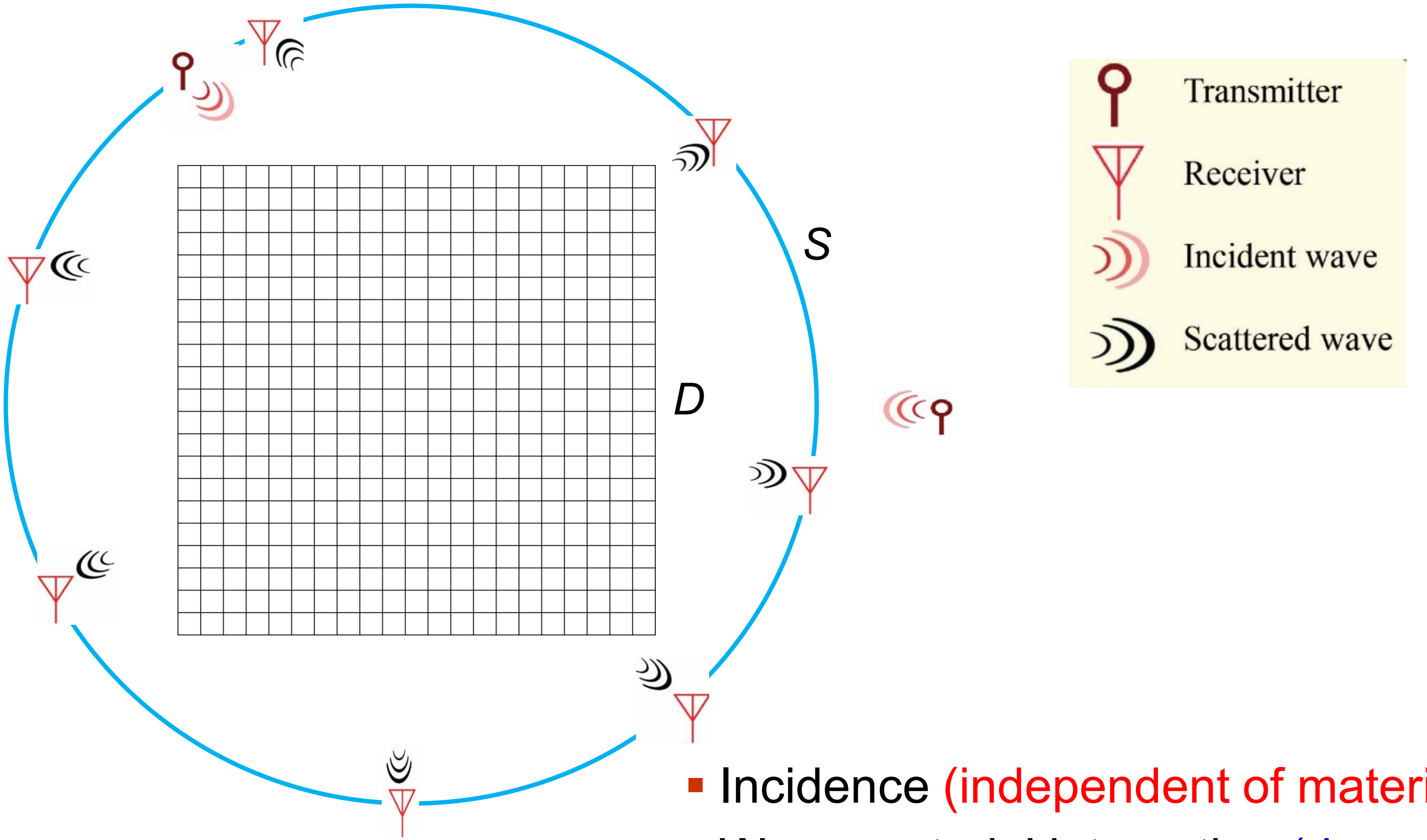
$$\overline{\overline{G}}_S : \quad O(M^2 N_r)$$

- Only a thin SVD of $\overline{\overline{G}}_S$ is needed (first L singular vectors)
- Computational cost of thin SVD:

$$\overline{\overline{G}}_S : \quad O(LMN_r^2) \propto O(M)$$

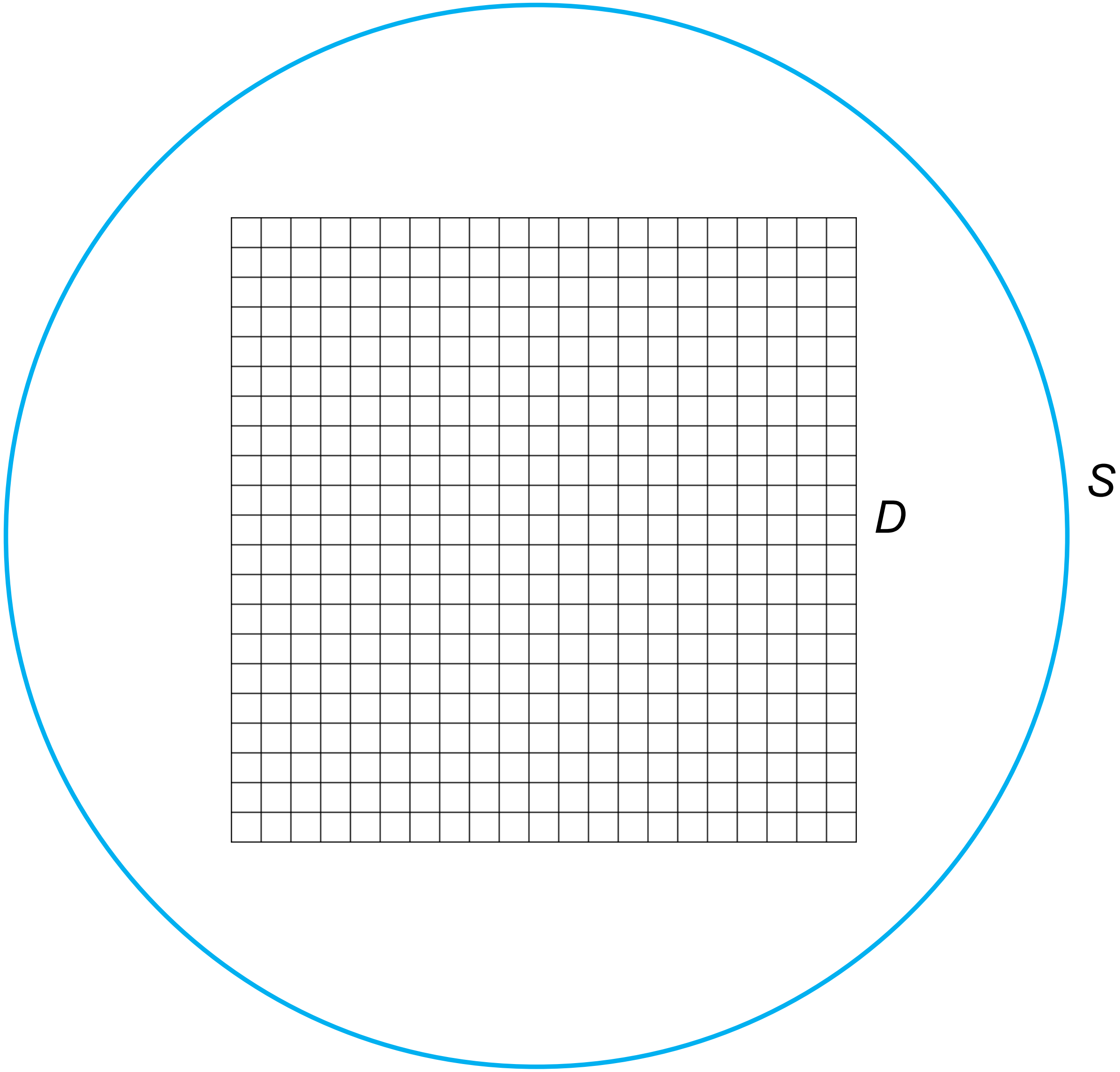
- $\overline{\overline{F}} \cdot \overline{\alpha}$ can be directly calculated by fast Fourier transform $O(M \log M)$

Three steps of imaging process



- Incidence (independent of material): $\bar{E}^{inc}$
- Wave-material interaction (dependent on material): $\bar{J} = \bar{\bar{\chi}} \cdot (\bar{E}^{inc} + \bar{\bar{G}}_D \cdot \bar{J})$
- Measurement of scattering data (independent of material): $\bar{E}^{scat} = \bar{\bar{G}}_S \cdot \bar{J}$

DIS v.s. BPS & DCS

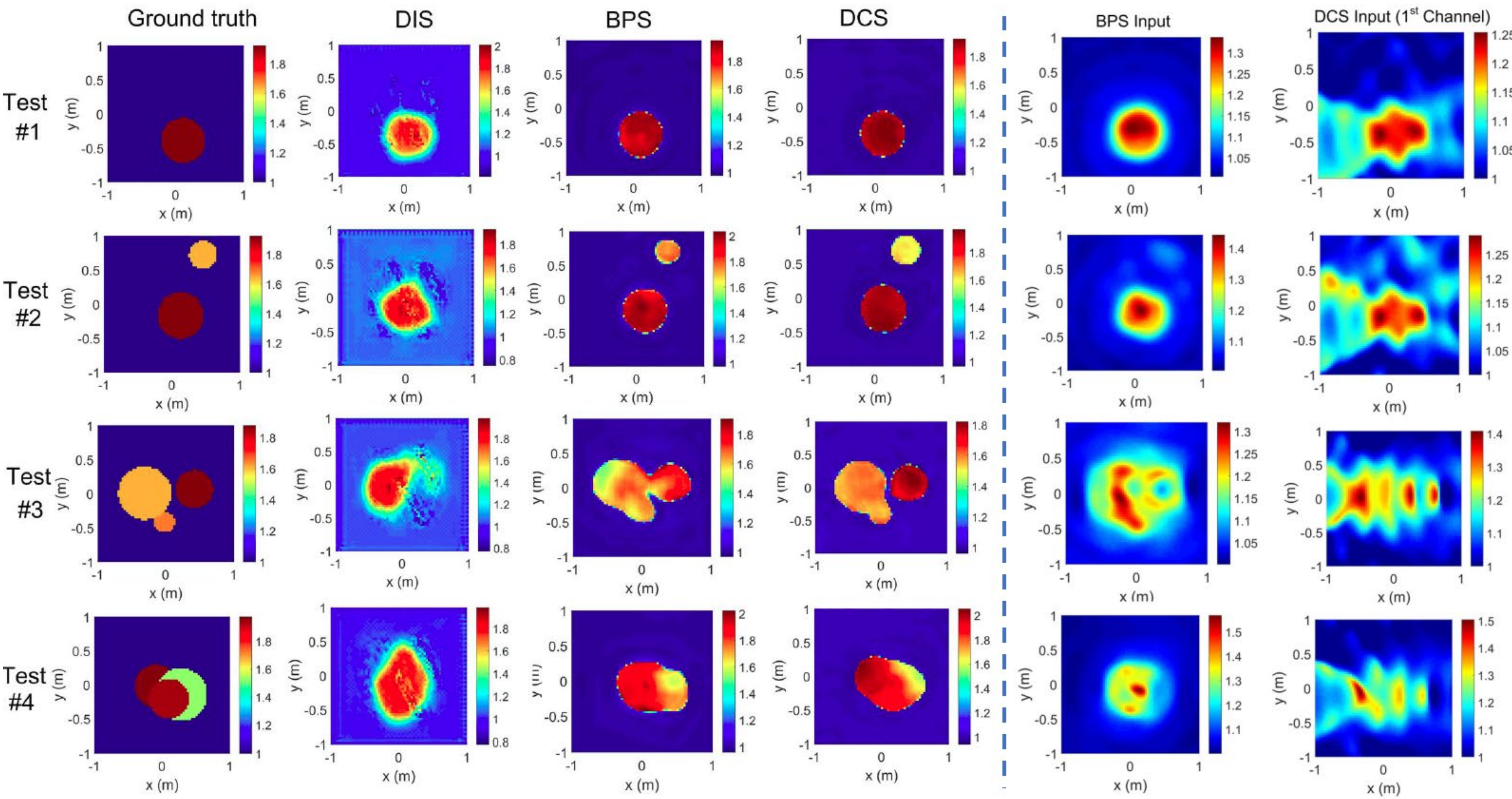


Wave physics between D and S is known:

$$G(\mathbf{r},\mathbf{r}') = \frac{i}{4} H_0^{(1)}(k|\mathbf{r}-\mathbf{r}'|)$$

<u>No need</u> to use lot of data to train the NN to learn it!

Numerical results



Example #1: One to three random circles

Size of Pixel: 64 x 64

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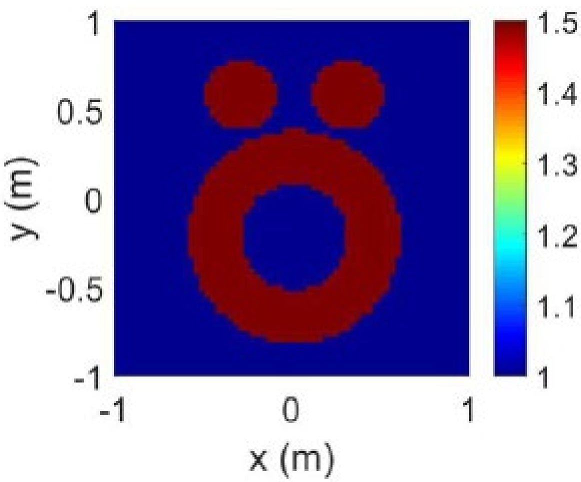
Numerical results

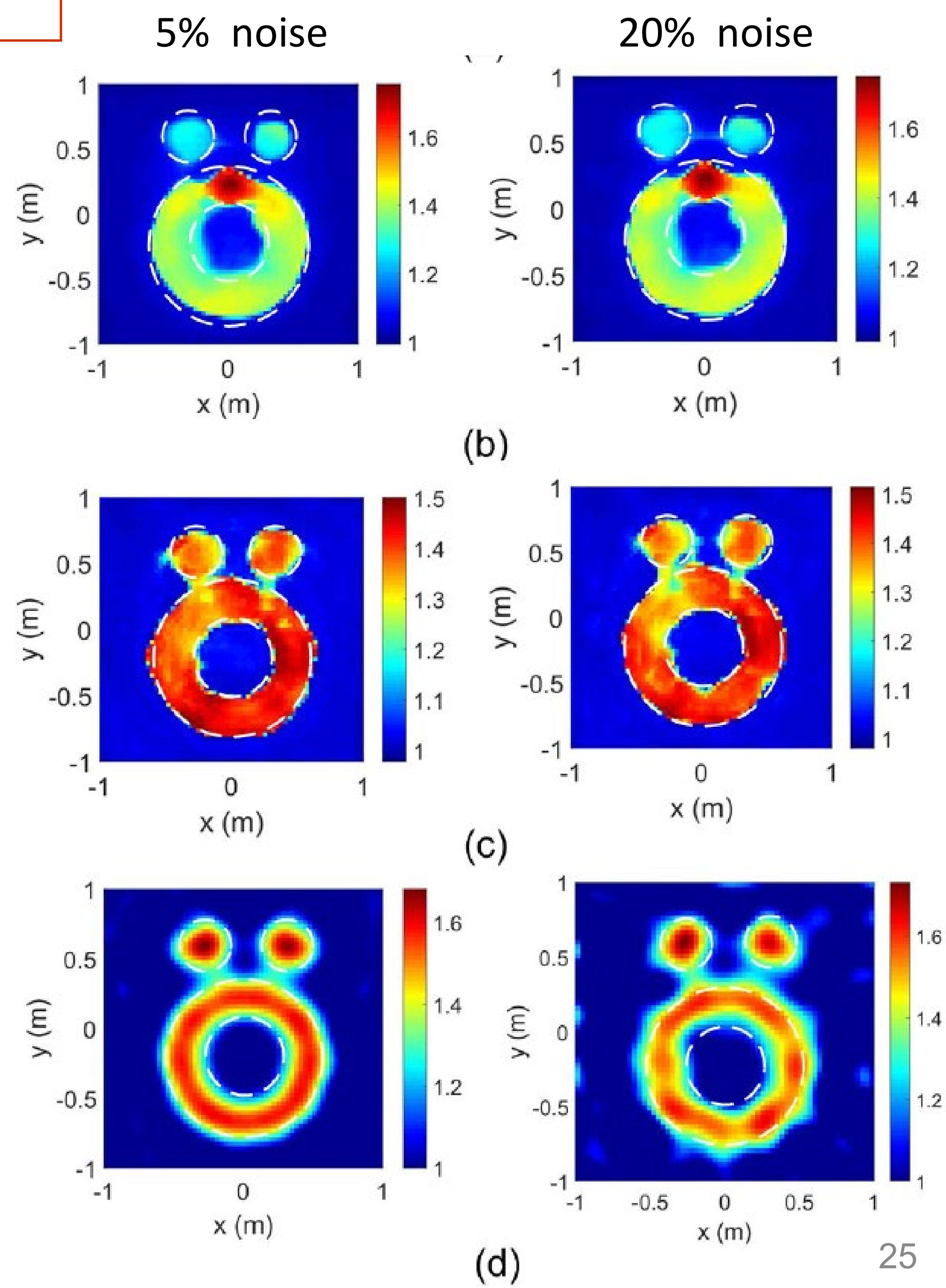


(a)

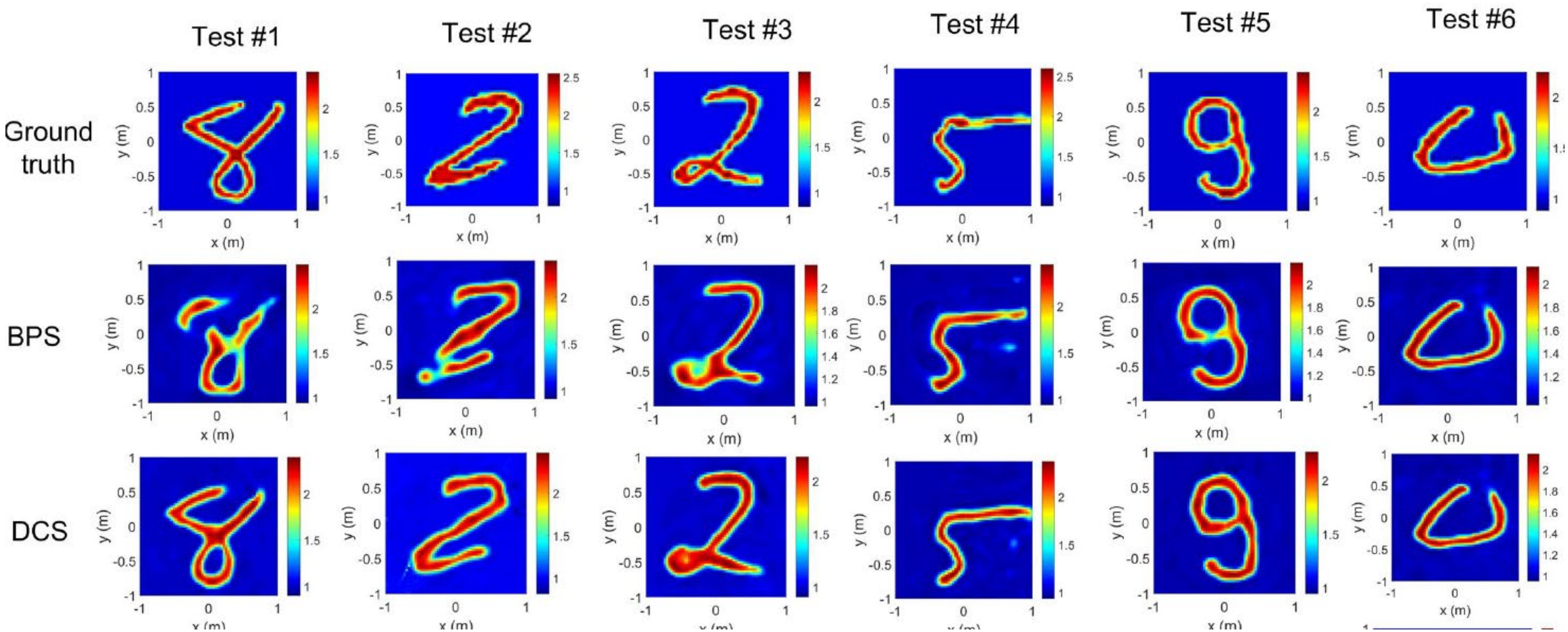
Example #2: **Test for four circles**

“Austria” tests:
(a) Ground-truth profile.

Reconstructed relative permittivity for
(b) BPS
(c) DCS
(d) iterative method (SOM)

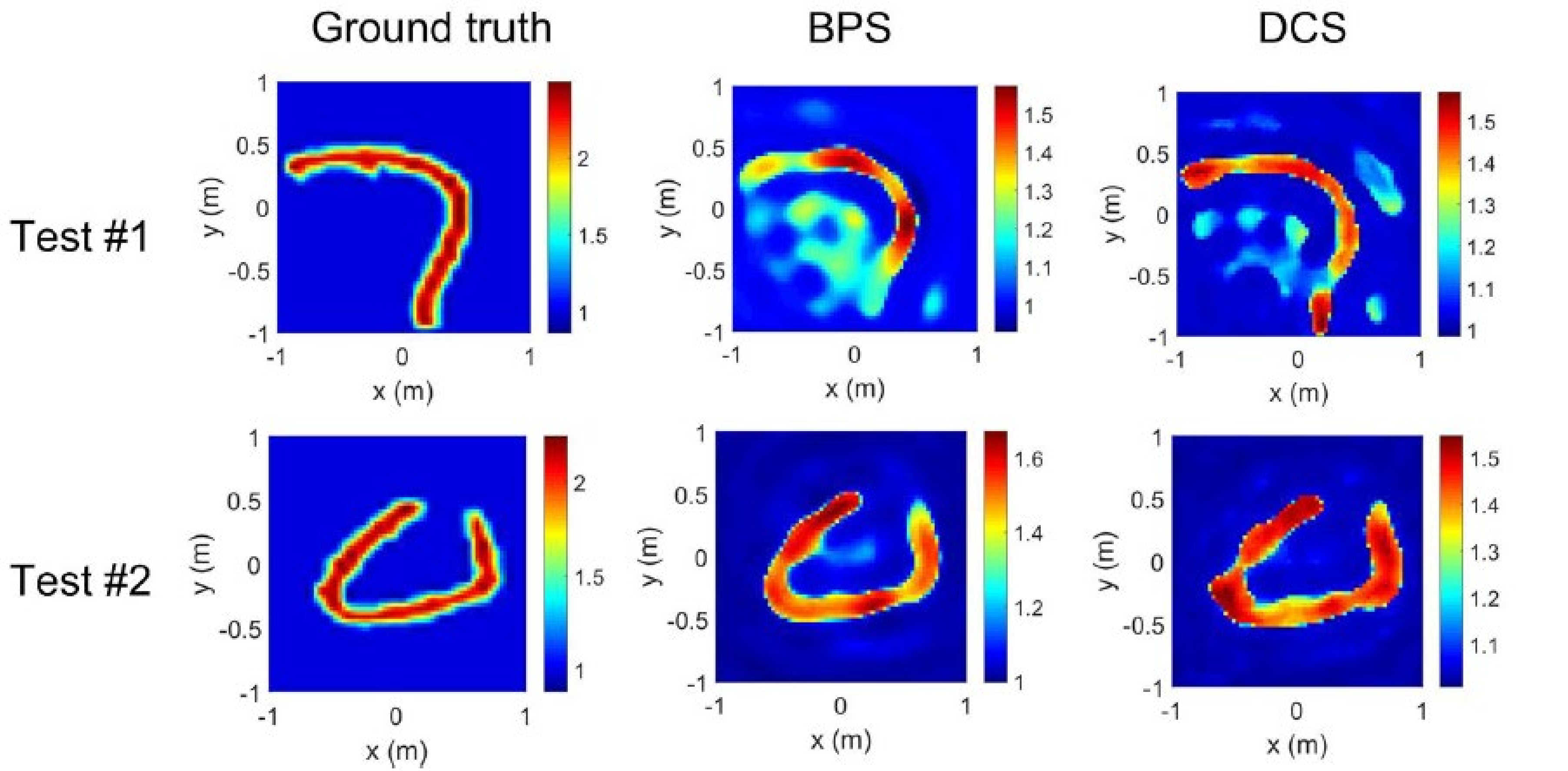


Numerical results



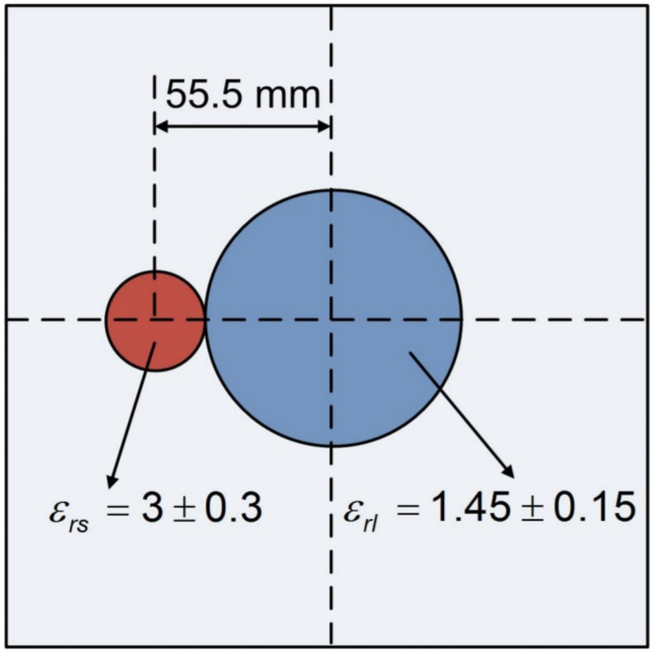
Example #3: Tests with MNIST database

Numerical results

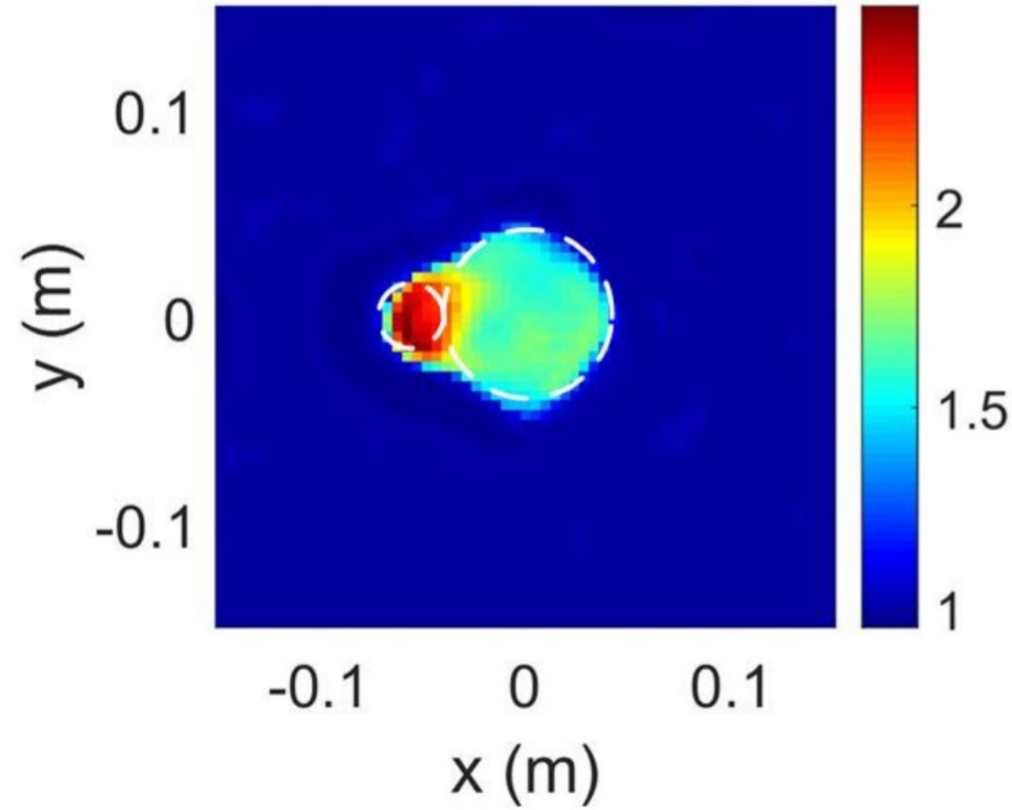


Use the network trained with **circular-cylinders in Example #1** to test **MNIST database in Example #3**

Experimental results



Ground truth profile:
"FoamDielExt", Institue Fresnel



Reconstructed results

Summary of BPS & DCS

- Inputs are approximate contrasts $\bar{\bar{\chi}}^a$ obtained at low computational cost; Outputs are exact contrasts $\bar{\bar{\chi}}$
  - A poor $\bar{\bar{\chi}}^a$ increases the difficulty of the problem
  - A more accurate $\bar{\bar{\chi}}^a$ increases computational cost
- Motivate us to propose new models
- Reexamine Lippmann-Schwinger equation

$$\bar{J} = \bar{\bar{\chi}} \cdot (\bar{E}^i + \bar{\bar{G}}_D \cdot \bar{J})$$

$$\bar{J}^+ + \bar{J}^- = \bar{\bar{\chi}} \cdot (\bar{E}^i + \bar{\bar{G}}_D \cdot \bar{J}^+ + \bar{\bar{G}}_D \cdot \bar{J}^-)$$

- Input: $\bar{J}^+$ & $\bar{E}^+ (= \bar{E}^i + \bar{\bar{G}}_D \cdot \bar{J}^+)$;   Output: $\bar{J}$
  - χ is the element-wise ratio of $\bar{J}$ to $(\bar{E}^i + \bar{\bar{G}}_D \cdot \bar{J})$

CNN architecture: Induced current learning method (ICLM)

- Lippmann-Schwinger equation

$$\bar{J}^{+} + \bar{J}^{-} = \bar{\bar{\chi}} \cdot (\bar{E}^{+} + \bar{\bar{G}}_D \cdot \bar{J}^{-})$$

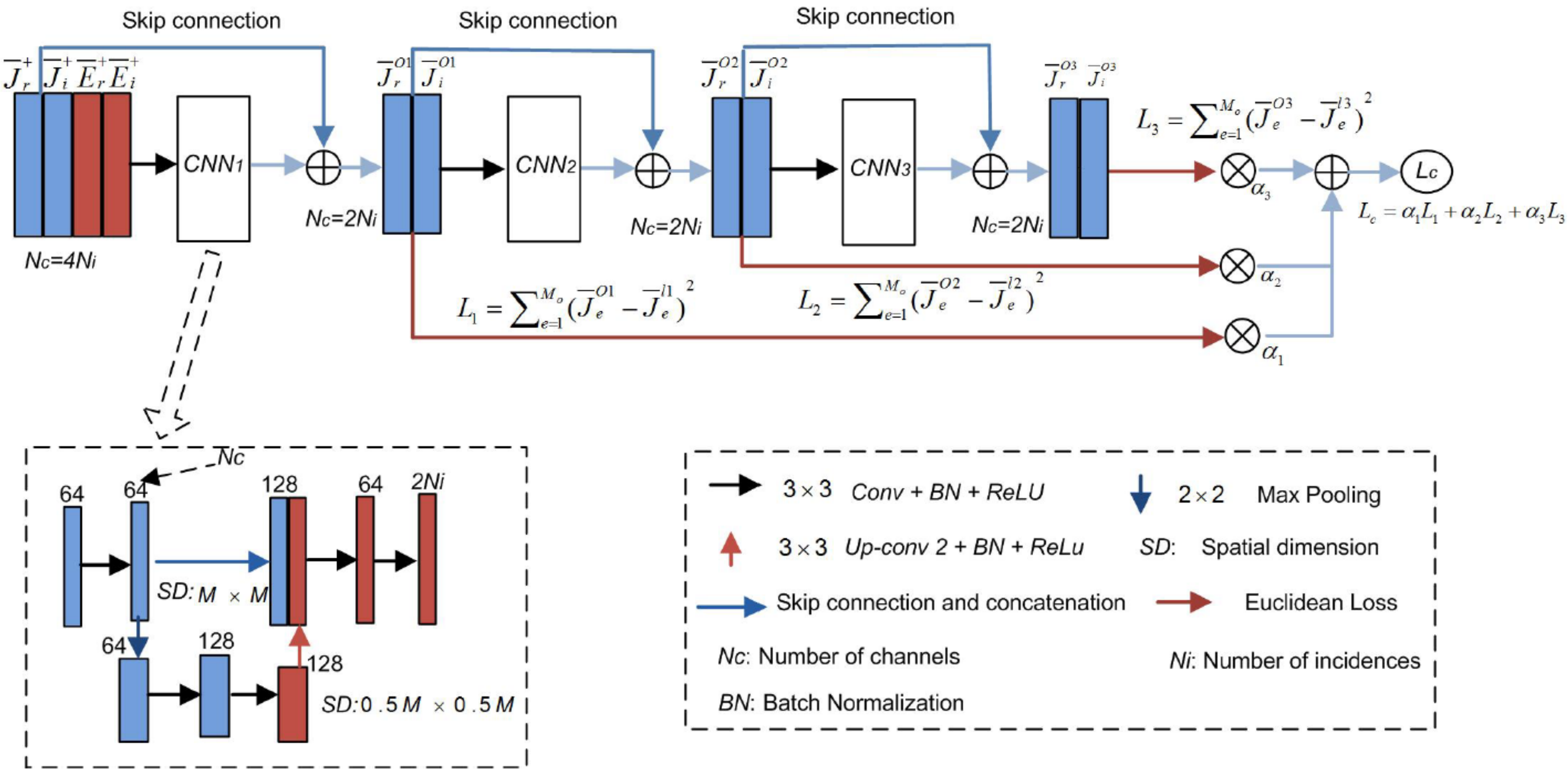
- Input: $\bar{J}^{+}$ & $\bar{E}^{+}$; Output: $\bar{J}$; what is learnt is in fact $\bar{J}^{-}$
  - Residue type: Skip connection is added
- Multiple labels are used, motivated by multi-resolution of $\bar{J}^{-}$
  - Traditional optimization approach: low-frequency features are recovered at initial iterations; high-frequency features come at later iterations
  - Some non-learning models have explicitly used basis-expansion methods

The *s*-th label is chosen as

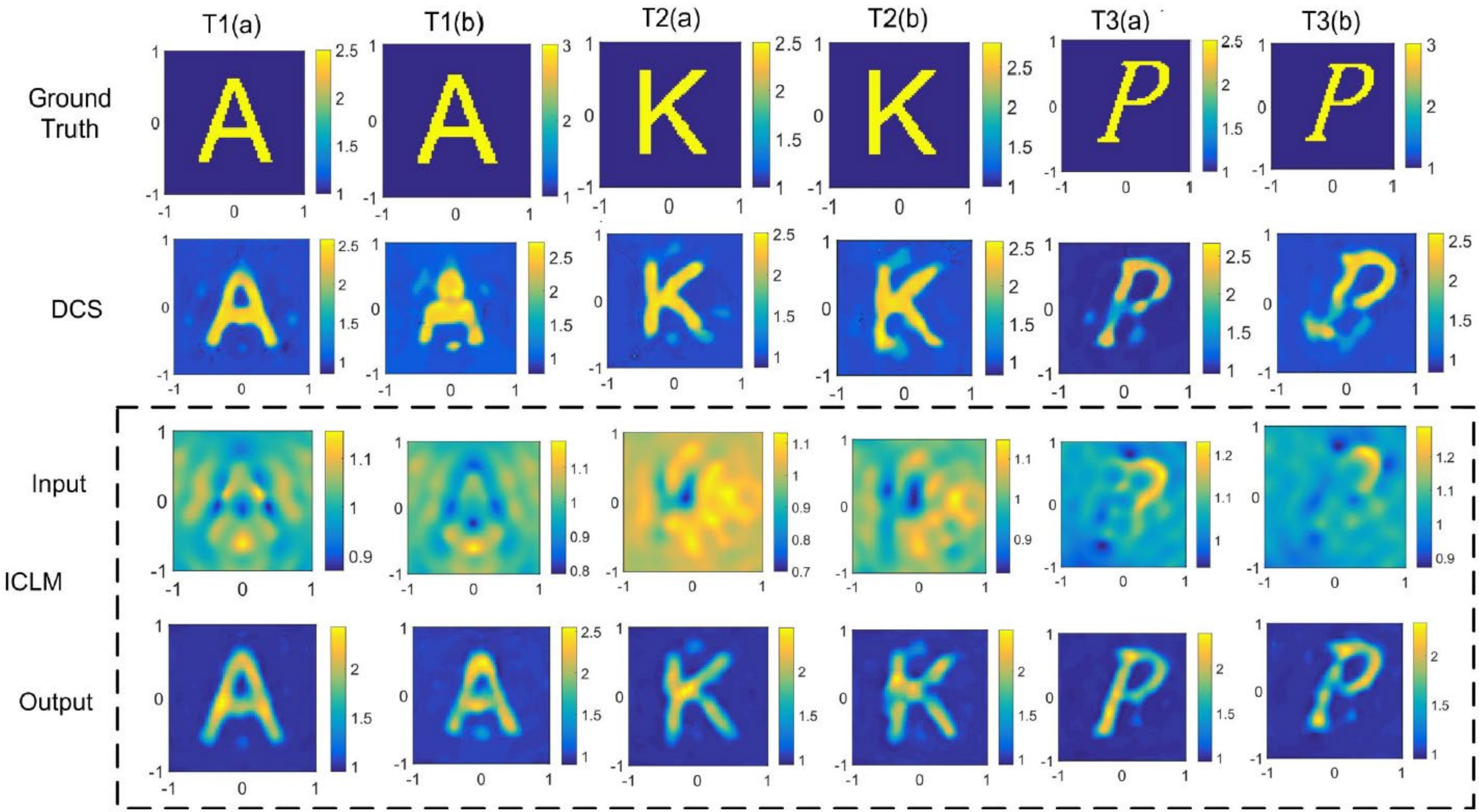
$$\overline{J}^{ls} = \overline{J}^{+} + \overline{\overline{F}}^{H} \cdot [\overline{\overline{M}}_s \circ (\overline{\overline{F}} \cdot \overline{J}^{-})]$$

Physics-Inspired Cascaded end-to-end CNN

- Z. Wei and X. Chen, "[Physics-Inspired]() Convolutional Neural Network for Solving Full-Wave Inverse Scattering Problems," *IEEE TAP*, Accepted, 2019



Numerical examples



Tests on Latin letters with the network trained by MIMIST dataset, with 15% Gaussian noise added

Promising research direction

- This research work has won the Ulrich L. Rohde Innovative Conference Paper Award in IEEE ICCEM 2019 conference
- Desay SV Automotive Singapore Pte. Ltd.

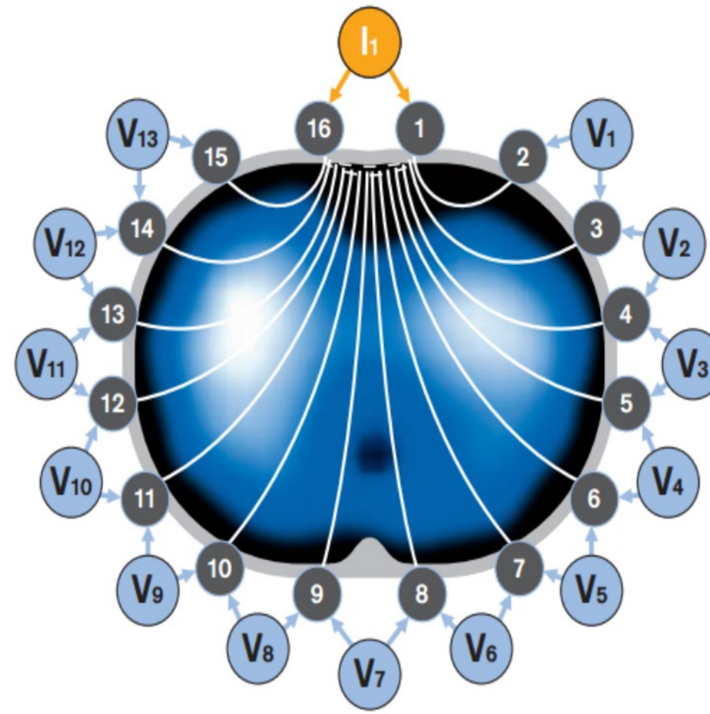
  Radar target classification in Advanced Driver-Assistance Systems (ADAS)

- **Applicable to inverse problems based on other physical principles**
  - Vector wave equation (Maxwell equations in 3-D)
  - Transport equation
  - Electrical impedance tomography (EIT)

Electrical impedance tomography (EIT)

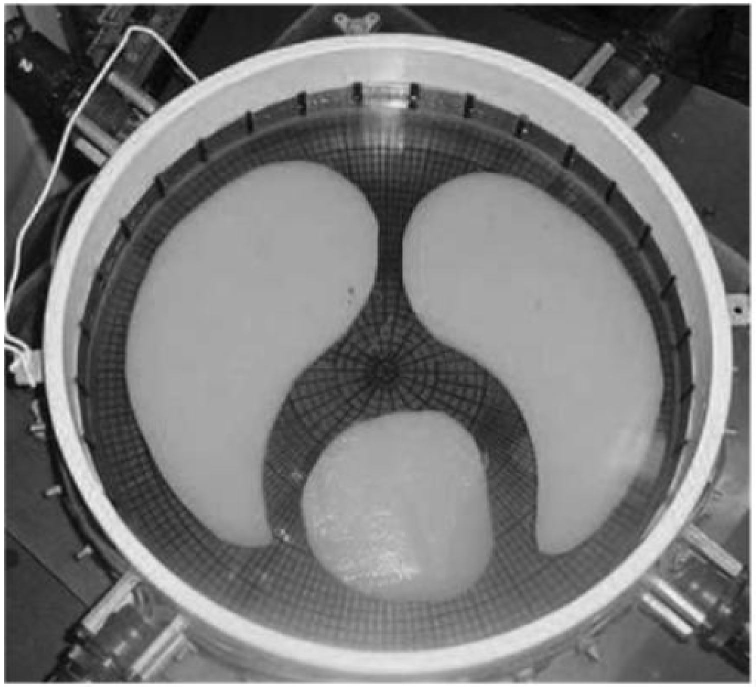
- EIT provides higher contrast at low frequency, widely used in biomedical applications

Hill-Rom
Enhancing outcomes for patients and their caregivers.

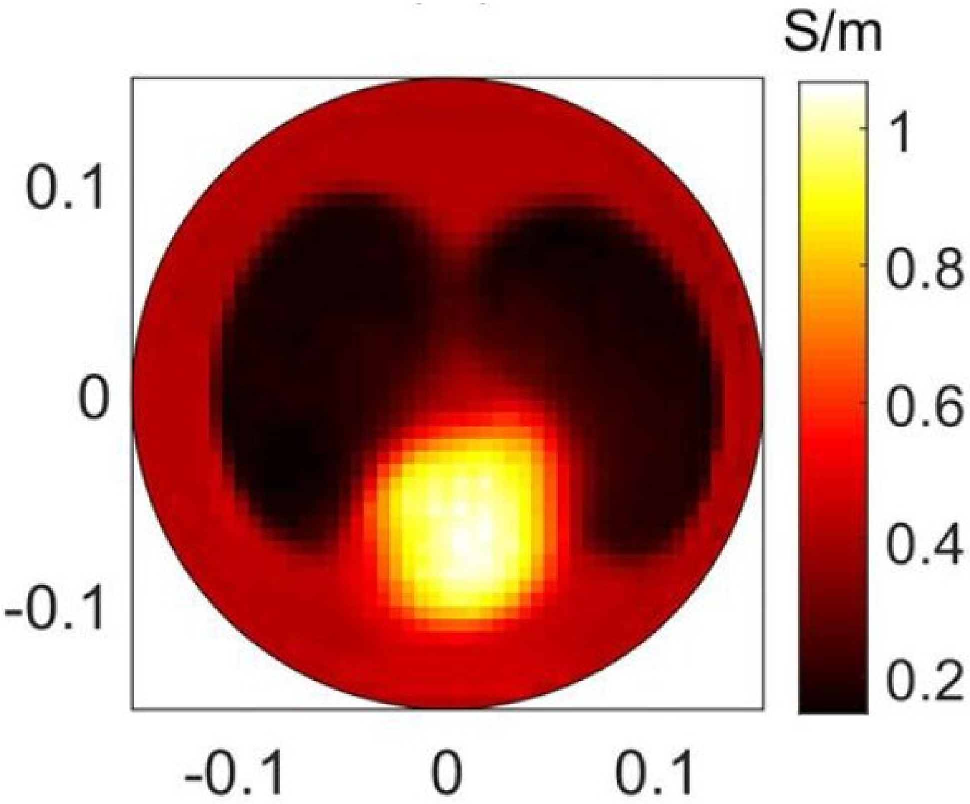


- Hamilton *TMI,* 2018 (Deep D-Bar)
- Wei, *TBE,* 2019
- Wei, in preparation, 2019

Reconstructions of agar heart and lung phantom contained in a saline filled tank



Phantom



Reconstructed

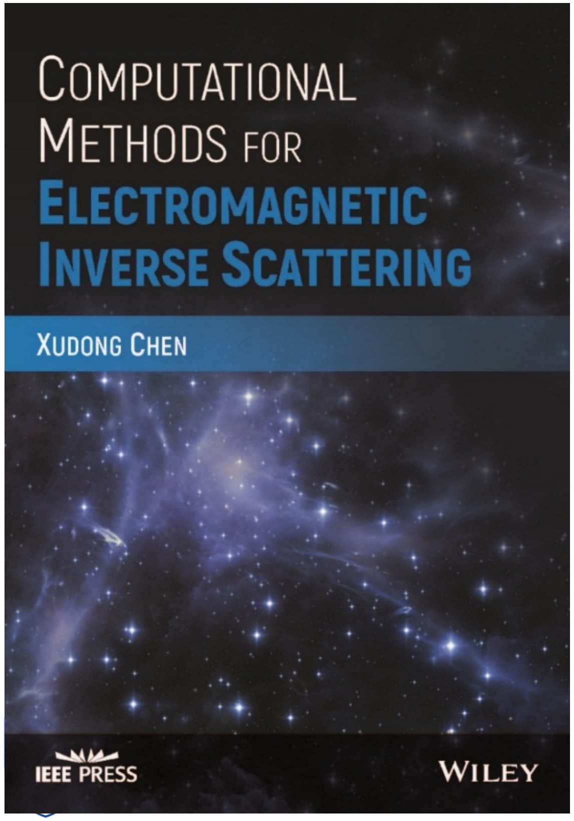
Take-home message

- No matter using objective function approach or learning approach, the key is to construct corresponding target function in a way such that it depends in a much less nonlinear way on unknowns.
- Avoid directly dealing with measurement data, where CNN has to spend unnecessary cost to train and learn underlying wave physics. Extract out as much as possible what people can do and leave the remaining to machine.
- The above two need a fairly good understanding of the forward problem (physical and mathematical insights)

Reference:

Z. Wei and X. Chen, "Deep Learning Schemes for Full-Wave Nonlinear Inverse Scattering Problems, " *IEEE Trans. on Geoscience and Remote Sensing*, vol. 57, pp. 1849-1860, 2019

Z. Wei and X. Chen, "Physics-Inspired Convolutional Neural Network for Solving Full-Wave Inverse Scattering Problems," *IEEE Trans. on Antennas and Propagation*, Accepted, 2019



X. Chen,
Computational Methods for Electromagnetic Inverse Scattering.
Wiley, 2018

Chapter 6:
Reconstructing Dielectric Scatterers

Thank you!

IAS Workshop on Inverse Problems, Imaging and Partial Differential Equations
Institute for Advanced Study, Hong Kong University of Science and Technology
Hong Kong, May 20-24, 2019