Analytical treatment of dynamic apertures and beam lifetimes due to beam beam effects in circular colliders

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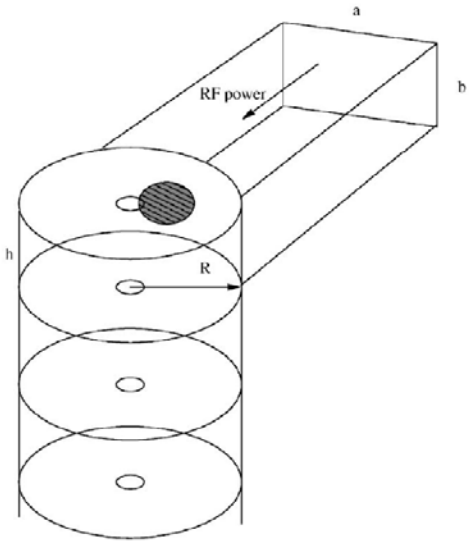

Fig. 2. A waveguide cavity coupling system type II.

Pre-words

Problem 1

Waveguide accelerating structure coupling problems:
How to determine the coupling coefficient β given couping dimension？

Usually so called Rf expersts use:
Experiments and/or numberical sumulation by computer program

$$\beta = \frac{N\pi Z_0 k_0 \Gamma_{10} l_1^6 e_0^4 \mathrm{e}^{-2\alpha d} \sin^2(2\pi L/\lambda_{\mathrm{g},10})}{9abRR_\mathrm{s}(R+h)(K(e_0)-E(e_0))^2(1+(Z_0 R/2R_\mathrm{s}(R+h))(v_\mathrm{g}/c))}\left(\frac{\pi}{a\Gamma_{10}}\right)^2 \qquad (36)$$

where R_s is the metal surface resistance. If the aperture is circular with radius r the attenuation coefficient should be expressed as $\alpha = (2\pi/\lambda)((\lambda/3.41r)^2 - 1)^{1/2}$.

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Nuclear Instruments and Methods in Physics Research A 481 (2002) 36–42

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

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Analytical determination of the coupling coefficient of waveguide cavity coupling systems

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Problem 2

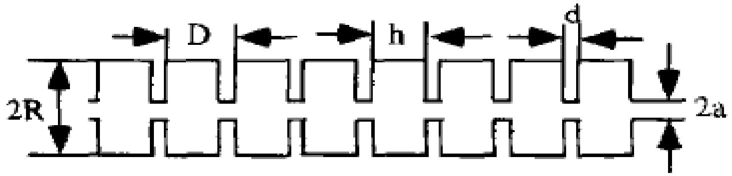
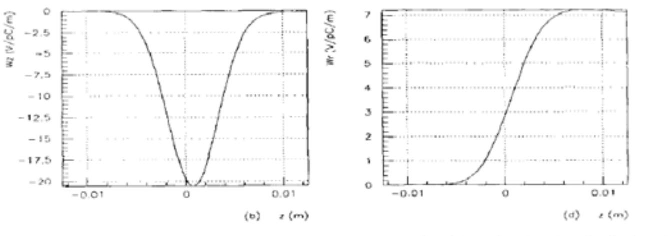


Fig. 1. Disk-loaded accelerating structure.



Nuclear Instruments and Methods in Physics Research A 381 (1996) 174–177

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

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Letter to the Editor

Analytical formulae and the scaling laws for the loss factors and the wakefields in disk-loaded periodic structures

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Received 19 February 1996

Wakefield problems by a charged bunch in a discloaded structure:

How to know the wakefields both longitudinal and transeverse？

Usually so called acclerator expersts use: numberical sumulation by computer program

$$k_{mnl} = \frac{2\xi h u_{mn}^2 J_m^2\left(\frac{u_{mn}}{R}a\right)}{\left(\left(\frac{u_{mn}}{R}\right)^2+\left(\frac{l\pi}{h}\right)^2\right)\epsilon_0 D\pi R^4 J_{m+1}^2(u_{mn})} \times \left(\frac{S(x_1)^2+S(x_2)^2}{4}\right), \qquad (16)$$

where

$$\xi = \begin{cases} 1, & m \neq 0, \\ \frac{1}{2}, & m = 0, \end{cases} \qquad (17)$$

$$S(x) = \frac{\sin(x)}{x}, \qquad (18)$$

and

$$x_1 = \frac{h}{2}\left(\left(\left(\frac{u_{mn}}{R}\right)^2+\left(\frac{l\pi}{h}\right)^2\right)^{1/2} - \frac{l\pi}{h}\right), \qquad (19)$$

$$x_2 = \frac{h}{2}\left(\left(\left(\frac{u_{mn}}{R}\right)^2+\left(\frac{l\pi}{h}\right)^2\right)^{1/2} + \frac{l\pi}{h}\right). \qquad (20)$$

Problem 3

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

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Analytical estimation of dynamic apertures limited by the wigglers in storage rings

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Dynamic aperture problems by a wiggler in a storage ring:

How to know the DA by a wiggler？

Usually so called acclerator physicists use: numberical sumulation by computer program

$$A_{N_w,y}(s) = \sqrt{\frac{3\beta(s)}{\beta_{y,m}^2}}\frac{\rho_w}{k_y\sqrt{L_w}}. \qquad (19)$$

Problem 4

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Letter to the Editor

Theory of single bunch transverse collective instabilities in electron storage rings

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Received 27 April 1998; received in revised form 30 June 1998; accepted 6 July 1998

Single bunch transvers instability threshold problem:

How to know the wakefields both longitudinal and transeverse？

Usually so called acclerator expersts use:

$$I_{\mathrm{b,coupling}}^{\mathrm{th}} = \frac{f_s E_0}{e\langle\beta_{y,c}\rangle \mathcal{K}_{\perp}^{\mathrm{tot}}(\sigma_z)},$$

Few people know that it should be expereesed as by Bruno Zotter (B. Zotter knows F~1 , but don't know the expression of F) :

$$I_{\mathrm{b,zotter}}^{\mathrm{th}} = \frac{F f_s E_0}{e\langle\beta_{y,c}\rangle \mathcal{K}_{\perp}^{\mathrm{tot}}(\sigma_z)}$$

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

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On the scaling law of single bunch transverse instability threshold current vs. the chromaticity in electron storage rings

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Almost no people know that F should be expereesed as:

$$I_{\mathrm{b,gao}}^{\mathrm{th}} = \frac{F' f_s E_0}{e\langle\beta_{y,c}\rangle \mathcal{K}_{\perp}^{\mathrm{tot}}(\sigma_z)}$$

with

$$F' = 4\boldsymbol{R}_{\varepsilon}|\xi_{c,y}|\frac{\nu_y \sigma_{\varepsilon 0}}{\nu_s E_0}.$$

Contents

- General theory of dynamic apertures from multipoles
- Dynamic aperture due to beam beam nonlinear effect induces dynamic apertures (head on, with crossing angle and parasitic crossing)
- The effects of number of interaction points in a circular collider
- Reference sources
- Conclusions

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Nuclear Instruments and Methods in Physics Research A 463 (2001) 50–61

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

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Analytical estimation of the beam–beam interaction limited dynamic apertures and lifetimes in e^+e^- circular colliders

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Received 30 July 2000; accepted 23 September 2000

Basic theroy of dynamic aperture in circular accelerator-1

$$H=\frac{p^2}{2}+\frac{K(s)}{2}x^2+\frac{1}{m!B_0\rho}\frac{\partial^{m-1}B_z}{\partial x^{m-1}}x^m L\sum_{k=-\infty}^{\infty}\delta(s-kL)$$

$$B_z=B_0(1+xb_1+x^2b_2+x^3b_3+\cdots+x^{m-1}b_{m-1}+\cdots)$$

Circular machine

A nonlinear multipole

For one multipole $B_z=B_0x^{m-1}b_{m-1}$ m≥3

$$\Psi=\int_0^s\frac{\mathrm{d}s'}{\beta_x(s')}+\phi_0$$

$$J=\frac{\varepsilon_x}{2}=\frac{1}{2\beta_x(s)}\left(x^2+\left(\beta_x(s)x'-\frac{\beta'_x x}{2}\right)^2\right)$$

$$H(J,\Psi)=\frac{J}{\beta_x(s)}.$$

$$\Psi_1=\Psi+\frac{2\pi v}{L}-\int_0^s\frac{\mathrm{d}s'}{\beta_x(s')}$$

$$J_1=J$$

$$H_1=\frac{2\pi v}{L}J_1.$$

$$I=\frac{x^2B_y|_{x=0,y=0}}{2\rho^2B_0}+\frac{1}{B_0\rho}\sum_{n=1}^{\infty}\frac{1}{n!}\frac{\partial^{n-1}B_y}{\partial x^{n-1}}\bigg|_{x=0,y=0}(x+\mathrm{i}y)^n-(1+x/\rho)\left(1+\frac{\Delta P}{P_0}-\left(\bar{p}_x-\frac{eA_x}{P_0}\right)^2-\left(\bar{p}_y-\frac{eA_y}{P_0}\right)^2\right)^{1/2}-\frac{e\Phi}{P_0}$$

$$x=\sqrt{2J_1\beta_x(s)}\cos\left(\Psi_1-\frac{2\pi v}{L}s+\int_0^s\frac{\mathrm{d}s'}{\beta_x(s')}\right).$$

$$\frac{\mathrm{d}J_1}{\mathrm{d}s}=-\frac{\partial H_1}{\partial\Psi_1}$$

$$\frac{\mathrm{d}\Psi_1}{\mathrm{d}s}=\frac{\partial H_1}{\partial J_1}$$

$$\overline{J_1}=\overline{J_1}(\Psi_1,J_1)$$

$$\overline{\Psi_1}=\overline{\Psi_1}(\Psi_1,J_1)$$

$$\bar{I}=I+K_0\sin\theta$$

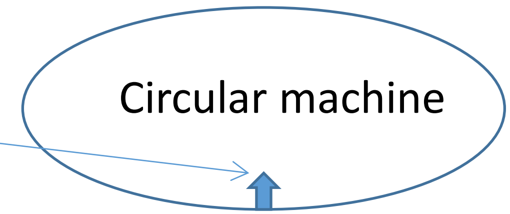
$$\bar{\theta}=\theta+\bar{I}$$

$|K_0|\leq 1$ (0.97164) Analytical DA expressions

J. Gao, "Analytical estimation of the dynamic apertures of circular accelerators", **Nuclear Instruments and Methods in Physics Research A** 451 (2000) 545-557.

Basic theroy of dynamic aperture in circular accelerator-2

$$H = \frac{p^2}{2} + \frac{K(s)}{2}x^2 + \frac{1}{m!B_0\rho}\frac{\partial^{m-1}B_z}{\partial x^{m-1}}x^m L\sum_{k=-\infty}^{\infty}\delta(s-kL)$$



$$B_z = B_0(1 + xb_1 + x^2b_2 + x^3b_3 + \cdots + x^{m-1}b_{m-1} + \cdots)$$

A nonlinear multipole

For one multipole $B_z = B_0 x^{m-1} b_{m-1}$ m≥3

$$A_{\mathrm{dyna},2m} = \sqrt{2\beta_x(s)}\left(\frac{1}{m\beta_x^m(s(2m))}\right)^{\frac{1}{2(m-2)}}\left(\frac{\rho}{|b_{m-1}|L}\right)^{1/(m-2)}$$

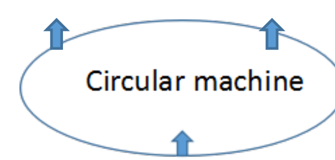
Standard Mapping
Chirikov Criterion

Relation between X and Y $A_{\mathrm{dyna},2m,y} = \sqrt{\frac{\beta_x(s(2m))}{\beta_y(s(2m))}(A^2_{\mathrm{dyna},2m,x} - x^2)}$

Hénon and Heiles problem

For more independent multipoles

$$A_{\mathrm{dyna,total}} = \frac{1}{\sqrt{\sum_i \frac{1}{A^2_{\mathrm{dyna,sext},i}} + \sum_j \frac{1}{A^2_{\mathrm{dyna,oct},j}} + \sum_k \frac{1}{A^2_{\mathrm{dyna,deca},k}} + \cdots}}$$



Many multipoles

J. Gao, “Analytical estimation of the dynamic apertures of circular accelerators”, **Nuclear Instruments and Methods in Physics Research A** 451 (2000) 545-557.

Nonlinear beam-beam effects-1 (e+e-)

Bsseti-Erskine formula for beam-beam induced transverse kicks

$$\delta y' + \mathrm{i}\,\delta x' = -\frac{N_e r_e}{\gamma_*} f(x, y, \sigma_x, \sigma_y)$$

$f(x, y, \sigma_x, \sigma_y)$ can be expressed by Basseti–Erskine formula [9]:

$$f(x,y,\sigma_x,\sigma_y) = \sqrt{\frac{2\pi}{\sigma_x^2-\sigma_y^2}}\left(w\left(\frac{x+\mathrm{i}y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - \exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right) w\left(\frac{(\sigma_y/\sigma_x)x+\mathrm{i}(\sigma_x/\sigma_y)y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right)$$

where w is the complex error function expressed as

$$w(z) = \exp(-z^2)(1-\mathrm{erf}(-\mathrm{i}z)).$$

J. Gao, “Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders”, **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Nonlinear beam-beam effects-2 (e+e-)

$$\delta'_y = \frac{N_e r_e}{\gamma_*}\left(\frac{1}{2\sigma^2}y - \boxed{\frac{1}{16\sigma^4}y^3} + \frac{1}{192\sigma^6}y^5 - \frac{1}{3072\sigma^8}y^7 + \frac{1}{61440\sigma^{10}}y^9 - \frac{1}{1474560\sigma^{12}}y^{11}\right.$$
$$\left. + \frac{1}{41287680\sigma^{14}}y^{13} - \cdots\right) \qquad \text{(RB)}$$

$$\delta'_x = -\frac{N_e r_e}{2\gamma_*}\left(\frac{2}{\sigma_x^2}x - \boxed{\frac{1}{3\sigma_x^4}x^3} + \frac{1}{30\sigma_x^6}x^5 - \frac{1}{420\sigma_x^8}x^7 + \frac{1}{7560\sigma_x^{10}}y^9 - \frac{1}{166320\sigma_x^{12}}x^{11}\right.$$
$$\left. + \frac{1}{4324320\sigma_x^{14}}x^{13} - \cdots\right) \qquad \text{(FB)}$$

$$\delta'_y = -\frac{N_e r_e}{\sqrt{2}\gamma_*}\left(\frac{2}{\sigma_x\sigma_y}y - \boxed{\frac{1}{3\sigma_x\sigma_y^3}y^3} + \frac{1}{20\sigma_x\sigma_y^5}y^5 - \frac{1}{168\sigma_x\sigma_y^7}y^7 + \frac{1}{1728\sigma_x\sigma_y^9}y^9 - \frac{1}{21120\sigma_x\sigma_y^{11}}y^{11}\right.$$
$$\left. + \frac{1}{299520\sigma_x\sigma_y^{13}}y^{13} - \cdots\right) \qquad \text{(FB).}$$

J. Gao, “Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders”, **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Nonlinear beam-beam effects-2 (e+e-)

$$\frac{d^2x}{ds^2}+K_x(s)x = -\frac{N_e r_e}{2\gamma_*}\left(\frac{2}{\sigma_x^2}x - \boxed{\frac{1}{3\sigma_x^4}x^3} + \frac{1}{30\sigma_x^6}x^5 - \frac{1}{420\sigma_x 8}x^7 + \frac{1}{7560\sigma_x^{10}}x^9 - \frac{1}{166320\sigma_x^{12}}x^{11} + \frac{1}{4324320\sigma_x^{14}}x^{13} - \cdots\right) \sum_{k=-\infty}^{\infty}\delta(s-kL) \qquad \text{(FB)}$$

$$\frac{d^2y}{ds^2}+K_y(s)y = -\frac{N_e r_e}{\sqrt{2}\gamma_*}\left(\frac{2}{\sigma_x\sigma_y}y - \boxed{\frac{1}{3\sigma_x\sigma_y^3}y^3} + \frac{1}{20\sigma_x\sigma_y^5}y^5 - \frac{1}{168\sigma_x\sigma_y^7}y^7 + \frac{1}{1728\sigma_x\sigma_y^9}y^9 - \frac{1}{21120\sigma_x\sigma_y^{11}}y^{11} + \frac{1}{299520\sigma_x\sigma_y^{13}}y^{13} - \cdots\right) \sum_{k=-\infty}^{\infty}\delta(s-kL) \qquad \text{(FB)}$$

J. Gao, “Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders”, **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Nonlinear beam-beam effects-3 (e+e-)

$$H_x = \frac{p_x^2}{2} + \frac{K_x(s)}{2}x^2 + \frac{N_e r_e}{2\gamma_*}\left(\frac{1}{\sigma_x^2}x^2 - \boxed{\frac{1}{12\sigma_x^4}x^4} + \frac{1}{180\sigma_x^6}x^6 - \frac{1}{3360\sigma_x^8}x^8 + \cdots\right)$$

$$\times \sum_{k=-\infty}^{\infty} \delta(s - kL) \qquad \text{(FB)}$$

$$H_y = \frac{p_y^2}{2} + \frac{K_y(s)}{2}y^2 + \frac{N_e r_e}{\sqrt{2}\gamma_*}\left(\frac{1}{\sigma_x\sigma_y}y^2 - \boxed{\frac{1}{12\sigma_x\sigma_y^3}y^4} + \frac{1}{120\sigma_x\sigma_y^5}y^6 - \frac{1}{1344\sigma_x\sigma_y^7}y^8 + \cdots\right)$$

$$\times \sum_{k=-\infty}^{\infty} \delta(s - kL) \qquad \text{(FB)}$$

where $p_x = \mathrm{d}x/\mathrm{d}s$ and $p_y = \mathrm{d}y/\mathrm{d}s$.

J. Gao, “Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders”, **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Nonlinear beam-beam effects-4 (e+e-)

$$H = \frac{p^2}{2} + \frac{K(s)}{2}x^2 + \frac{1}{3!B\rho}\frac{\partial^2 B_z}{\partial x^2}x^3 L \sum_{k=-\infty}^{\infty} \delta(s-kL) + \frac{1}{4!B\rho}\frac{\partial^3 B_z}{\partial x^3}x^4 L \sum_{k=-\infty}^{\infty} \delta(s-kL) + \cdots$$

where

$$B_z = B_0(1 + xb_1 + x^2 b_2 + x^3 b_3 + x^4 b_4 + \cdots + x^{m-1} b_{m-1} + \cdots).$$

The dynamic aperture corresponding to each multipole is given as

$$\boxed{A_{\mathrm{dyna},2m,x}(s) = \sqrt{2\beta_x(s)}\left(\frac{1}{m\beta_x^m(s_{2m})}\right)^{1/2(m-2)}\left(\frac{\rho}{|b_{m-1}|L}\right)^{1/(m-2)}}$$

$$\frac{b_{m-1}}{\rho}L = \frac{N_e r_e}{C_{m,\mathrm{FB},x} 2\gamma_* \sigma_x^m} \qquad (\mathrm{FB}, x)$$

$$\frac{b_{m-1}}{\rho}L = \frac{N_e r_e}{C_{m,\mathrm{FB},y}\sqrt{2}\gamma_* \sigma_y \sigma_x^{m-1}} \qquad (\mathrm{FB}, y)$$

Table 1
Summary of multipole coefficients

m	4	6	8	10	12	14
$C_{m,\mathrm{RB}}$	16	192	3072	61 440	1 474 560	41 287 680
$C_{m,\mathrm{FB},x}$	3	30	420	7560	166 320	4 324 320
$C_{m,\mathrm{FB},y}$	3	20	168	1728	21 120	299 520

J. Gao, "Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders", **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Nonlinear beam-beam effects-5 (e+e-)

$$A_{\mathrm{dyna},8,y}(s) = \frac{\sqrt{\beta_y(s)}}{\beta_y(s_{\mathrm{IP}})}\sqrt{\frac{\rho}{|b_3|L}} = \frac{\sqrt{\beta_y(s)}}{\beta_y(s_{\mathrm{IP}})}\left(\frac{16\gamma_* \sigma^4}{N_e r_e}\right)^{1/2} \quad \text{(RB)} \tag{25}$$

$$A_{\mathrm{dyna},8,x}(s) = \frac{\sqrt{\beta_y(s)}}{\beta_y(s_{\mathrm{IP}})}\sqrt{\frac{\rho}{|b_3|L}} = \frac{\sqrt{\beta_x(s)}}{\beta_x(s_{\mathrm{IP}})}\left(\frac{6\gamma_* \sigma_x^4}{N_e r_e}\right)^{1/2} \quad \text{(FB)} \tag{26}$$

$$A_{\mathrm{dyna},8,y}(s) = \frac{\sqrt{\beta_y(s)}}{\beta_y(s_{\mathrm{IP}})}\sqrt{\frac{\rho}{|b_3|L}} = \frac{\sqrt{\beta_y(s)}}{\beta_y(s_{\mathrm{IP}})}\left(\frac{3\sqrt{2}\gamma_* \sigma_x \sigma_y^3}{N_e r_e}\right)^{1/2} \quad \text{(FB)} \tag{27}$$

$$\boxed{\begin{aligned} \mathscr{R}_{x,8} &= \frac{A_{\mathrm{dyna},8,x}(s)}{\sigma_{*,x}(s)} = \left(\frac{6\gamma_* \sigma_x^2}{N_e r_e \beta_x(s_{\mathrm{IP}})}\right)^{1/2} \quad \text{(FB)} \\ \mathscr{R}_{y,8} &= \frac{A_{\mathrm{dyna},8,y}(s)}{\sigma_{*,y}(s)} = \left(\frac{3\sqrt{2}\gamma_* \sigma_x \sigma_y}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right)^{1/2} \quad \text{(FB).} \end{aligned}}$$

J. Gao, “Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders”, **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Nonlinear beam-beam effects-6 (e+e-)

$$\xi_x^* = \frac{N_e r_e \beta_{x,\mathrm{IP}}}{2\pi\gamma^* \sigma_x(\sigma_x + \sigma_y)}$$

$$\xi_y^* = \frac{N_e r_e \beta_{y,\mathrm{IP}}}{2\pi\gamma^* \sigma_y(\sigma_x + \sigma_y)}$$

$$\mathscr{R}_{x,2m} = \frac{A_{\mathrm{dyna},2m,x}(s)}{\sigma_{*,x}(s)} = \left(\frac{2^{(m-2)/2} C_{m,\mathrm{FB},x}}{2\sqrt{m}\pi\xi_x^*}\right)^{1/m-2} \quad \text{(FB)}$$

$$\mathscr{R}_{y,2m} = \frac{A_{\mathrm{dyna},2m,y}(s)}{\sigma_{*,y}(s)} = \left(\frac{2^{(m-2)/2} C_{m,\mathrm{FB},y}}{\sqrt{2m}\pi\xi_y^*}\right)^{1/m-2} \quad \text{(FB).}$$

Obviously, *the normalized beam–beam effect limited dynamic apertures are determined only by the beam–beam tune shifts*. The impact of this discovery will be more appreciated later. When the higher order multipoles effects ($2m > 8$) can be neglected, Eqs. (25)–(27) give very good approximations dynamic apertures limited by one beam–beam IP. If there are N_{IP} interaction points in a ring the dynamic apertures described in Eqs. (25) and (27) will be reduced by a factor of $\sqrt{N_{\mathrm{IP}}}$ (if these N_{IP} interaction points can be regarded as independent).

J. Gao, "Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders", **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Nonlinear beam-beam effects-7 (e+e-)

$$\tau_{bb} = \frac{\tau_y}{2}\left(\frac{\langle y^2\rangle}{y_{\max}^2}\right)\exp\left(\frac{y_{\max}^2}{\langle y^2\rangle}\right) = \frac{\tau_y}{2}\left(\frac{\sigma_y(s)^2}{A_{\mathrm{dyna},y}(s)^2}\right)\exp\left(\frac{A_{\mathrm{dyna},y}(s)^2}{\sigma_y(s)^2}\right)$$

or

$$\tau^*_{bb,y} = \frac{\tau^*_y}{2}\left(\frac{16\gamma_*\sigma^2}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right)^{-1}\exp\left(\frac{16\gamma_*\sigma^2}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right) \quad \text{(RB)}$$

$$\tau^*_{bb,x} = \frac{\tau^*_x}{2}\left(\frac{6\gamma_*\sigma_x^2}{N_e r_e \beta_x(s_{\mathrm{IP}})}\right)^{-1}\exp\left(\frac{6\gamma_*\sigma_x^2}{N_e r_e \beta_x(s_{\mathrm{IP}})}\right) \quad \text{(FB)}$$

$$\tau^*_{bb,y} = \frac{\tau^*_y}{2}\left(\frac{3\sqrt{2}\gamma_*\sigma_x\sigma_y}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right)^{-1}\exp\left(\frac{3\sqrt{2}\gamma_*\sigma_x\sigma_y}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right) \quad \text{(FB)}$$

$$\tau^*_{bb,y} = \frac{\tau^*_y}{2}\left(\frac{4}{\pi\xi^*_y}\right)^{-1}\exp\left(\frac{4}{\pi\xi^*_y}\right) \quad \text{(RB)}$$

$$\tau^*_{bb,x} = \frac{\tau^*_x}{2}\left(\frac{3}{\pi\xi^*_x}\right)^{-1}\exp\left(\frac{3}{\pi\xi^*_x}\right) \quad \text{(FB)}$$

$$\tau^*_{bb,y} = \frac{\tau^*_y}{2}\left(\frac{3}{\sqrt{2}\pi\xi^*_y}\right)^{-1}\exp\left(\frac{3}{\sqrt{2}\pi\xi^*_y}\right) \quad \text{(FB).}$$

More generally, one has

$$\tau^*_{bb,2m,y} = \frac{\tau^*_y}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{RB}}}{4\pi\sqrt{m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{RB}}}{4\pi\sqrt{m}\xi^*_y}\right)^{2/m-2}\right) \quad \text{(RB)}$$

$$\tau^*_{bb,2m,x} = \frac{\tau^*_x}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},x}}{\pi 2\sqrt{m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},x}}{\pi 2\sqrt{m}\xi^*_x}\right)^{2/m-2}\right) \quad \text{(FB)}$$

Nonlinear beam-beam effects-8 (e+e-)

$$\frac{A_{\mathrm{dyna},8,y}(s)}{\sigma_*(s)} = \left(\frac{16\gamma_*\sigma^2}{N_\mathrm{e}r_\mathrm{e}\beta_y(s_\mathrm{IP})}\right)^{1/2} \quad \text{(RB)} \qquad = \left(\frac{4}{\pi\xi_y^*}\right)^{1/2}\frac{1}{\sqrt{N_{IP}}}$$

$$\frac{A_{\mathrm{dyna},8,x}(s)}{\sigma_{*,x}(s)} = \left(\frac{6\gamma_*\sigma_x^2}{N_\mathrm{e}r_\mathrm{e}\beta_x(s_\mathrm{IP})}\right)^{1/2} \quad \text{(FB)} \qquad = \left(\frac{3}{\pi\xi_x^*}\right)^{1/2}\frac{1}{\sqrt{N_{IP}}}$$

$$\frac{A_{\mathrm{dyna},8,y}(s)}{\sigma_{*,y}(s)} = \left(\frac{3\sqrt{2}\gamma_*\sigma_x\sigma_y}{N_\mathrm{e}r_\mathrm{e}\beta_y(s_\mathrm{IP})}\right)^{1/2} \quad \text{(FB).} \quad = \left(\frac{3}{\sqrt{2}\pi\xi_y^*}\right)^{1/2}\frac{1}{\sqrt{N_{IP}}}$$

$$\xi_{y,\max}^{\mathrm{RB}} = \frac{4\sqrt{2}}{3}\xi_{y,\max}^{\mathrm{FB}} = 1.89\xi_{y,\max}^{\mathrm{FB}}$$

N_{IP} is interaction number

Round beam vs flat beam

and

$$\xi_{x,\max}^{\mathrm{FB}} = \sqrt{2}\xi_{y,\max}^{\mathrm{FB}}.$$

J. Gao, “Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders”, **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Nonlinear beam-beam effects-4 (e+e-)

More generally, one has

$$\tau^*_{bb,2m,y} = \frac{\tau^*_y}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{RB}}}{4\pi\sqrt{m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{RB}}}{4\pi\sqrt{m}\xi^*_y}\right)^{2/m-2}\right) \quad \text{(RB)}$$

$$\tau^*_{bb,2m,x} = \frac{\tau^*_x}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},x}}{\pi 2\sqrt{m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},x}}{\pi 2\sqrt{m}\xi^*_x}\right)^{2/m-2}\right) \quad \text{(FB)}$$

$$\tau^*_{bb,2m,y} = \frac{\tau^*_y}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},y}}{\pi\sqrt{2m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},y}}{\pi\sqrt{2m}\xi^*_y}\right)^{2/m-2}\right) \quad \text{(FB).}$$

$$\xi^{\mathrm{RB}}_{y,\max} = \frac{4\sqrt{2}}{3}\xi^{\mathrm{FB}}_{y,\max} = 1.89\xi^{\mathrm{FB}}_{y,\max}$$

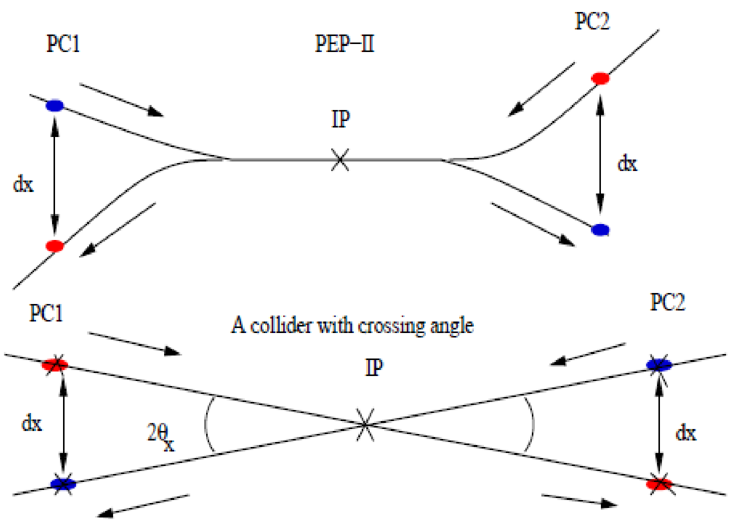
Round beam vs flat beam

and

$$\xi^{\mathrm{FB}}_{x,\max} = \sqrt{2}\xi^{\mathrm{FB}}_{y,\max}.$$

J. Gao, "Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders", **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Parasitic crossing beam-beam effects



with

$$\tau_{PC,y,RB} = \frac{\tau_y}{2}\left(\mathcal{R}_{y,PC,RB}\right)^{-1}\exp\left(\mathcal{R}_{y,PC,RB}\right)$$

$$= \frac{\tau_y}{2}\left(\frac{4}{\pi\xi_{PC,y}}\right)^{-1}\exp\left(\frac{4}{\pi\xi_{PC,y}}\right)$$

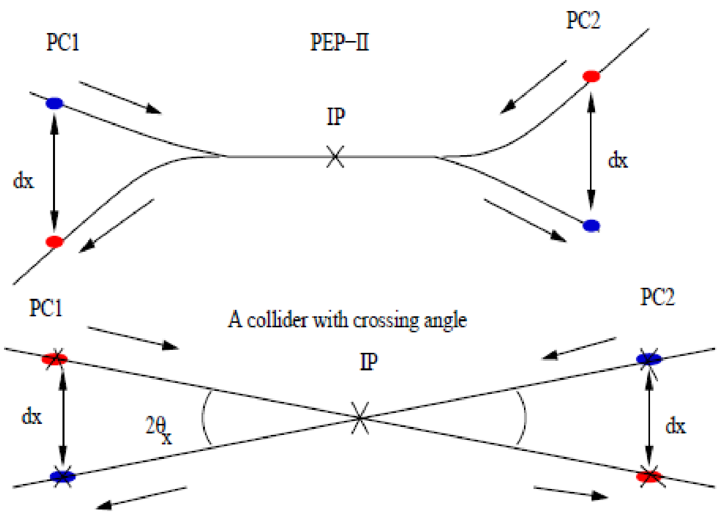
$$\xi_{PC,y} = \frac{r_e N_e \beta_{PC,x}}{2\pi\gamma_* \Sigma_{PC}^2} = \frac{r_e N_e \beta_{PC,y}}{2\pi\gamma_* d_x^2}$$

$$\Sigma_{PC}, \Sigma_{PC} = \sqrt{d_x^2 + d_y^2}$$

J. Gao, ON PARASITIC CROSSINGS AND THEIR LIMITATIONS TO E+E− STORAGE RING COLLIDERS, **Proceedings of EPAC 2004**, Lucerne, Switzerland, p. 671-673 (2004)

J. Gao, “Analytical treatment of the nonlinear electron cloud effect and the combined effects with beam-beam and space charge nonlinear forces in storage rings”, **Chinese Physics C** Vol. 33, No. 2, Feb., 2009, 135-144

Beam-beam effects with crossing angle



$$\mathscr{R}_{\text{syn-beta},x} = \frac{A_{\text{syn-beta},x}(s)^2}{\sigma_x(s)^2} = \frac{2}{3\pi^2}\left(\frac{1}{\xi_x^* \Phi}\right)^2$$

where $\Phi = (\sigma_z/\sigma_x)\phi$ is Piwinski angle.

J. Gao, "Analytical estimation of the effects of crossing angle on the luminosity of an e+e- circular collider", **Nuclear Instruments and Methods in Physics Research A** 481 (2002) 756–759

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Emittance growth and beam lifetime limitations due to beam–beam effects in e^+e^- storage ring colliders

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$$\xi_y \leqslant \xi_{y,\max,\mathrm{em,flat}} = \frac{h\mathscr{H}_0\gamma}{F}\sqrt{\frac{r_e}{6\pi R N_{\mathrm{IP}}}}\ F_{cw} \tag{16}$$

or, in general case, one has

$F_{cw} > 1$, is a factor describing crab-waist effect

$$\xi_y \leqslant \xi_{y,\max,\mathrm{em,flat}} = \frac{h\mathscr{H}_0}{2\pi F}\sqrt{\frac{T_0}{\tau_y\gamma N_{\mathrm{IP}}}}\ F_{cw} \tag{17}$$

where h is a constant used to quantify how the denominator in Eq. (11) is approaching to zero, defining $H_0 = h\mathscr{H}_0$, one has $H_0 \approx 2845$, which is not a derived value, but obtained by comparing with experimental results, R is the local dipole bending radius, and F is expressed as follows:

$$F = \frac{\sigma_s}{\sqrt{2}\beta_{y,*}}\left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right)^{1/2}. \tag{18}$$

The subscript em in Eqs. (16) and (17) denotes the emittance blow-up limited beam–beam parameter. When $\sigma_s = \beta_{y,*}$ one has $F = 1$.

$$\tau_{\mathrm{bb},y,\mathrm{flat}} = \frac{\tau_y}{2}\left(\frac{3}{\sqrt{2}\pi\xi_y N_{\mathrm{IP}}}\right)^{-1} \exp\left(\frac{3}{\sqrt{2}\pi\xi_y N_{\mathrm{IP}}}\right) \tag{19}$$

$$\tau_{\mathrm{bb},x,\mathrm{flat}} = \frac{\tau_x}{2}\left(\frac{3}{\pi\xi_x N_{\mathrm{IP}}}\right)^{-1} \exp\left(\frac{3}{\pi\xi_x N_{\mathrm{IP}}}\right) \tag{20}$$

and a rigid round beam

$$\tau_{\mathrm{bb},y,\mathrm{round}} = \frac{\tau_y}{2}\left(\frac{4}{\pi\xi_y N_{\mathrm{IP}}}\right)^{-1} \exp\left(\frac{4}{\pi\xi_y N_{\mathrm{IP}}}\right). \tag{21}$$

$$\tau_{\mathrm{bb},y,\mathrm{flat}} = \frac{\tau_y}{2}\left(\frac{3\xi_{y,\mathrm{max,em,flat}}}{\sqrt{2}\pi\xi_{y,\mathrm{max},0}\xi_y N_{\mathrm{IP}}}\right)^{-1} \times \exp\left(\frac{3\xi_{y,\mathrm{max,em,flat}}}{\sqrt{2}\pi\xi_{y,\mathrm{max},0}\xi_y N_{\mathrm{IP}}}\right) \tag{22}$$

and

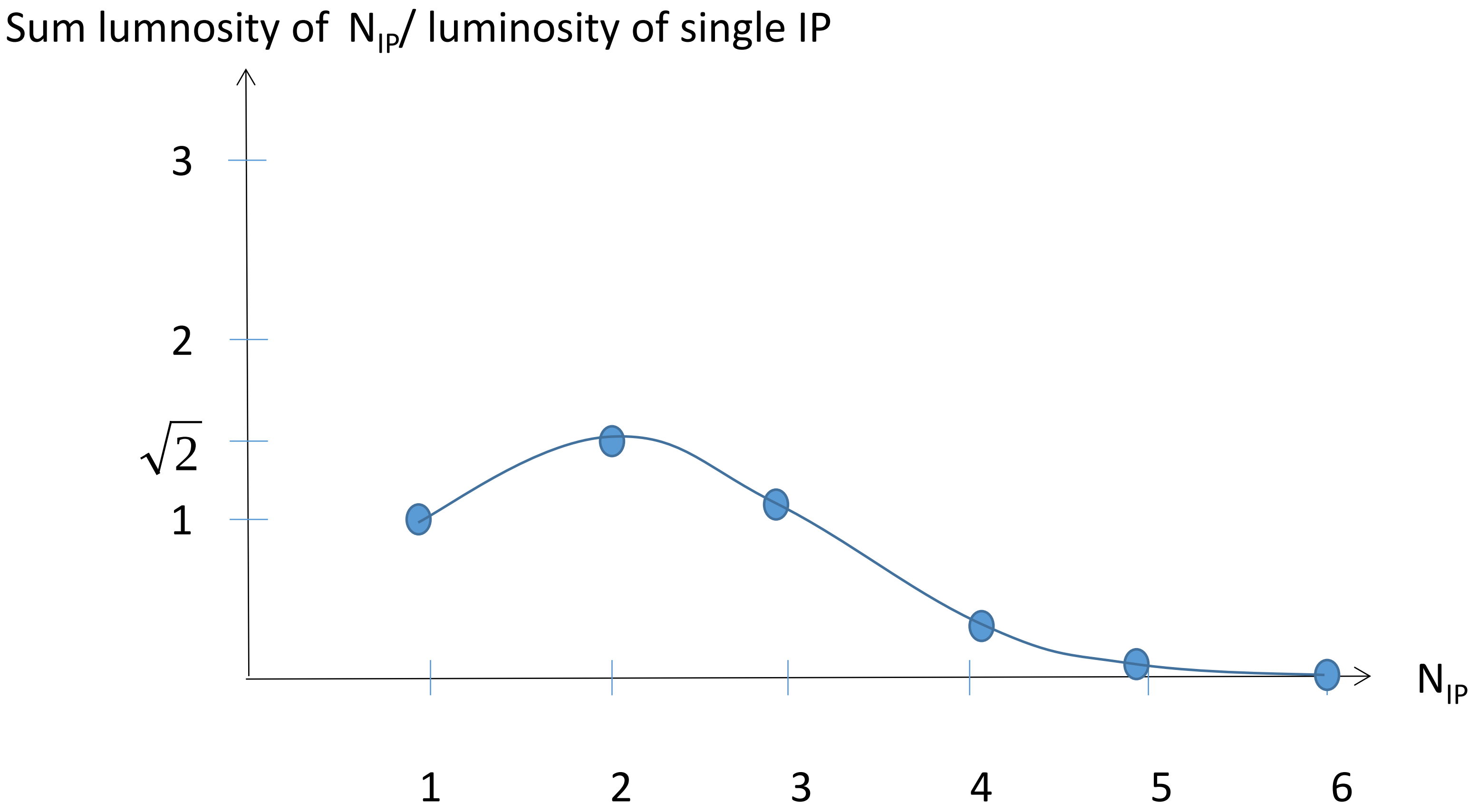
$$\tau_{\mathrm{bb},y,\mathrm{round}} = \frac{\tau_y}{2}\left(\frac{3\xi_{y,\mathrm{max,em,round}}}{\sqrt{2}\pi\xi_{y,\mathrm{max},0}\xi_y N_{\mathrm{IP}}}\right)^{-1} \times \exp\left(\frac{3\xi_{y,\mathrm{max,em,round}}}{\sqrt{2}\pi\xi_{y,\mathrm{max},0}\xi_y N_{\mathrm{IP}}}\right) \tag{23}$$

with

$$\xi_{y,\mathrm{max,em,round}} = 1.89\xi_{y,\mathrm{max,em,flat}} \tag{24}$$

J. Gao, “Emittance growth and beam lifetime limitations due to beam–beam effects in e+e- storage ring colliders, **Nuclear Instruments and Methods in Physics Research A** 533 (2004) 270–274

Discussion on CEPC interaction point number N_{IP}



Conlusions

1) Theoretical understanding of nonlinear motion in accelerators are very important

2) Analytical formulae of dynamic apertures and beam liffetimes due to beam beam nonlinear effects in circular colliders have been established

3) Aanalytical formulae have find their applications in real machine designs and analysises

4) Increasing interaction points will results in decreasing beam beam limited dynamic aperture and beam lifetime and limiting the sum luminosity of N_{IP}

5) It is not recommended to have more IPs than 2 for CEPC

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