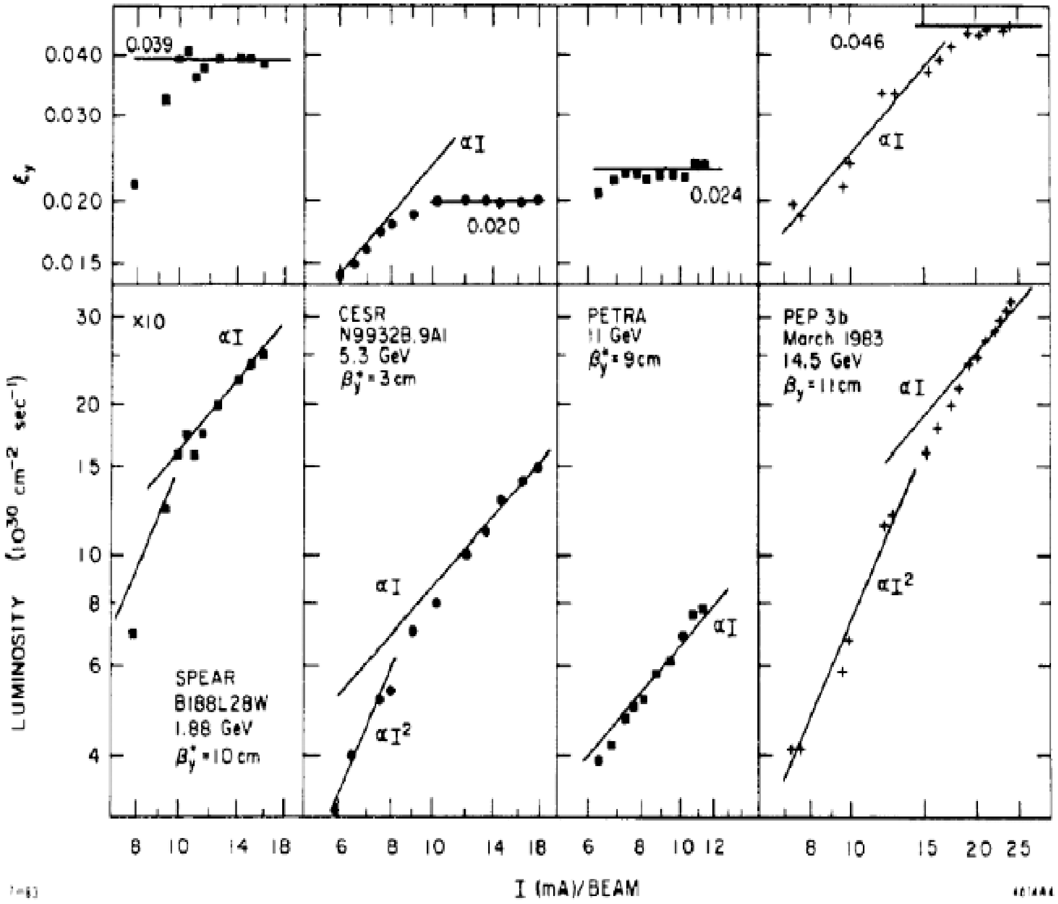
Some Issues on Beam-Beam Interaction at CEPC

Y. Zhang
On behalf of CEPC Accelerator Study Group

Thanks: K. Ohmi, D. Shatilov, K. Oide, D. Zhou

Beam-beam parameter in e+/e- colliders



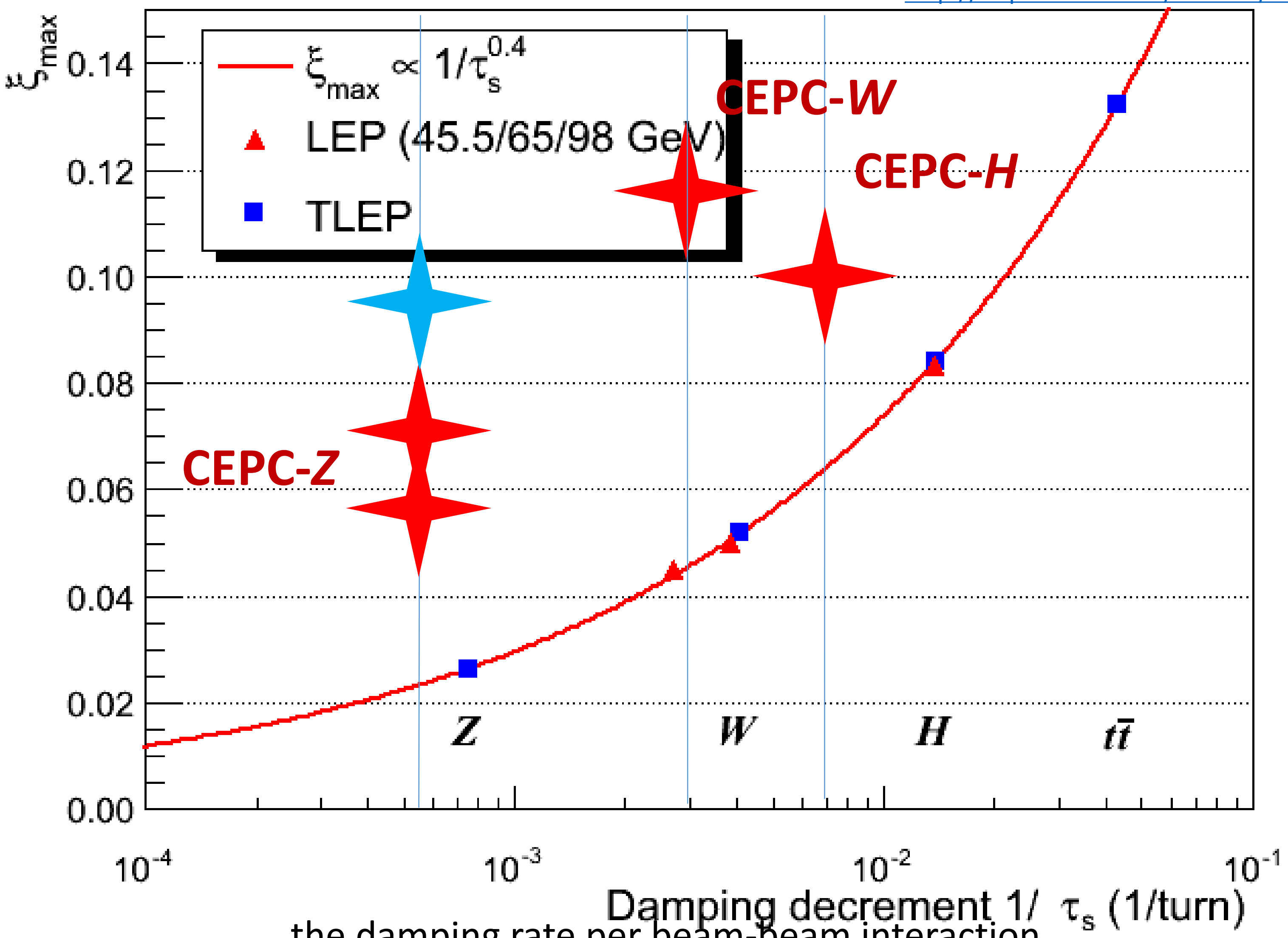
J. Seeman, “Observations of the beam–beam interaction”, 1985

Machine Parameters of the KEKB (June 17 2009)

	LER	HER	
Circumference	3016		m
RF Frequency	508.88		MHz
Horizontal Emittance	18	24	nm
Beam current	1637	1188	mA
Number of bunches	1584 + 1		
Bunch current	1.03	0.750	mA
Bunch spacing	1.84		m
Bunch trains	1		
Total RF volatage Vc	8.0	13.0	MV
Synchrotron tune ν_s	-0.0246	-0.0209	
Betatron tune ν_x / ν_y	45.506/43.561	44.511/41.585	
beta's at IP β_x^* / β_y^*	120/0.59	120/0.59	cm
momentum compaction a	3.31 x 10^{-4}	3.43 x 10^{-4}	
Estimated vertical beam size at IP from luminosity σ_y^*	0.94	0.94	µm
beam-beam parameters ξ_x / ξ_y	0.127/0.129	0.102/0.090	
Beam lifetime	133@1637	200@1188	min.@mA
Luminosity (Belle CsI)	21.08		10^{33}/cm^2/sec
Luminosity records per day / 7days/ 30days	1.479/8.428/30.208		/fb

Beam-Beam Parameter at CEPC & LEP2

http://tlep.web.cern.ch/content/accelerator-challenges



R. Assmann

the damping rate per beam-beam interaction

Crab-Waist Compensation

Collision with large Φ is not a new idea .....

Crab-Waist transformation is !

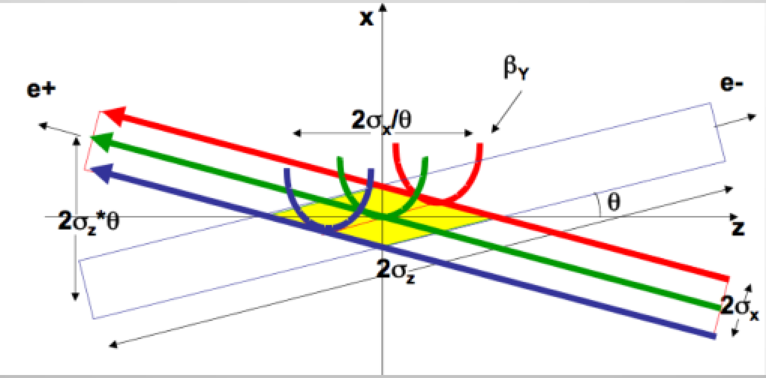
$$y = \frac{xy'}{2\theta}$$

sextupole (anti)sextupole

$$\Delta\nu_x = \pi$$
$$\Delta\nu_y = \frac{\pi}{2}$$

- $L_{geometric}$ gain
- x-y synchro-betatron and betatron resonance suppression

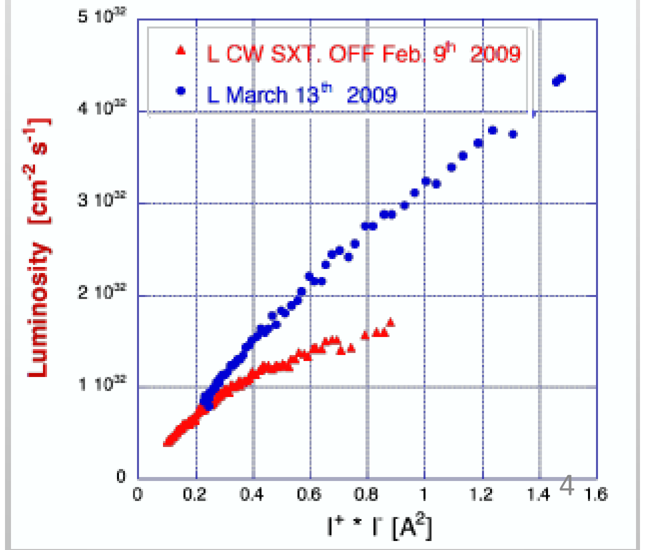
P. Raimondi , 2° *SuperB Workshop, March 2006*
P.Raimondi, D.Shatilov, M.Zobov, physics/0702033
C. Milardi et al., Int.J.Mod.Phys.A24, 2009
M. Zobov et al., Phys. Rev. Lett. 104, 2010



DAΦNE Luminosity and Tune Shift

	KLOE (Spt 2005)	FINUDA (Apr 2007)	SIDDHARTA *CW* (Jun 2009)
Luminosity [10^{32} cm^{-2}s^{-1}]	1.53	1.6	4.53 (5.0)
I(ele) [A]	1.38	1.50	1.52
I(pos) [A]	1.18	1.1	1
n_b	111	106	105
ε_x [mm mrad]	0.34	0.34	0.28
β_x [m]	1.5	2.	0.25
β_y [cm]	1.8	1.9	0.9
ξ	0.0245	0.0291	0.0443 (0.074)

Luminosity as a function of colliding currents *CW-Sextupole* excitation



C. Milardi

Estimation of Beamstrahlung lifetime

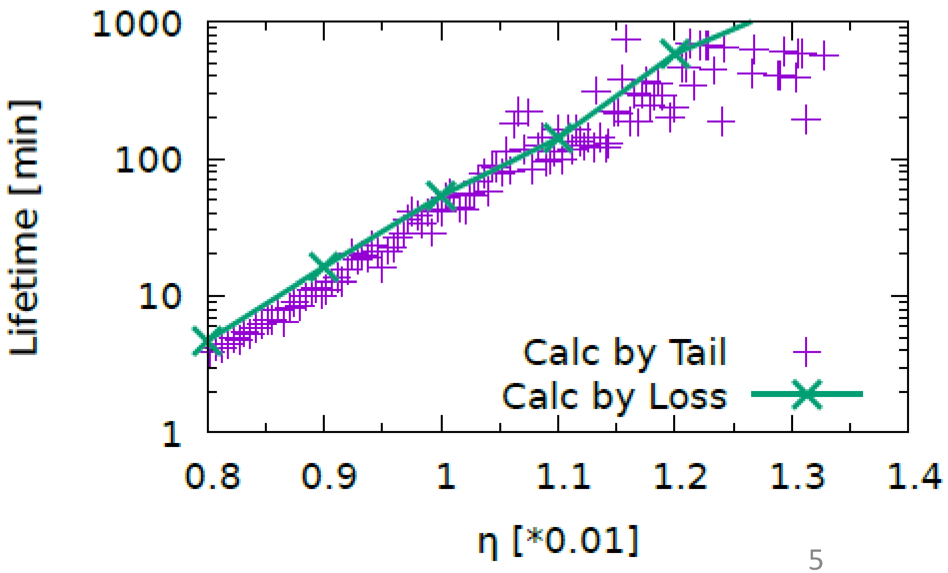
- Analysis [*V. Telnov, Phys. Rev. Letters 110 (2013) 114801*]

$$\tau_{BS} \approx \frac{1}{n_{IP} f_{rev}} \frac{4\sqrt{\pi}}{3} \sqrt{\frac{\delta_{acc}}{\alpha r_e}} \exp\left(\frac{2}{3} \frac{\delta_{acc}\alpha}{r_e \gamma^2} \frac{\gamma \sigma_x \sigma_z}{\sqrt{2} r_e N_b}\right) \frac{\sqrt{2}}{\sqrt{\pi}\sigma_z \gamma^2} \left(\frac{\gamma \sigma_x \sigma_z}{\sqrt{2} r_e N_b}\right)^{3/2}$$

- Calculated by beam distribution K. Ohmi

$$\tau_{bs} = \frac{\tau_z}{2Af(A)}$$

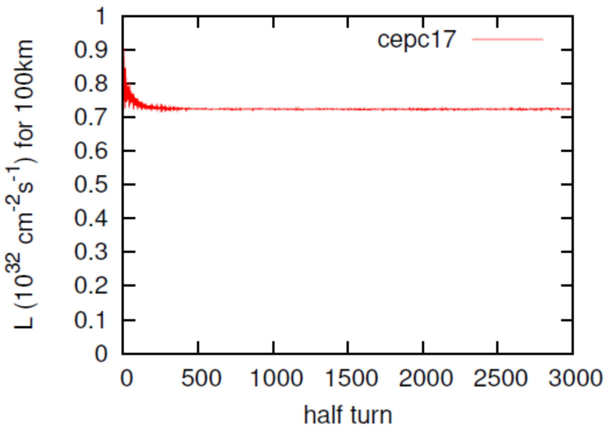
- A is the boundary of momentum acceptance in action,
- $f(J)$ is the distribution of action with beam-beam, $\int_0^\infty dJ\, f(J) = 1$
- τ_z is the longitudinal damping time

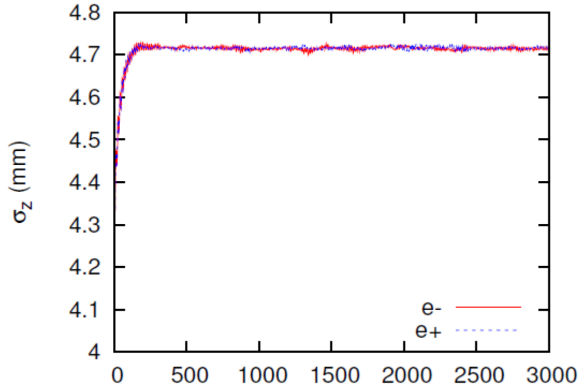


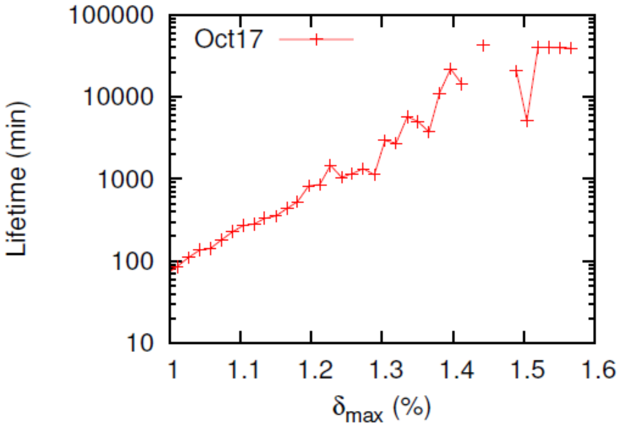
Cross check of Beam-Beam Code

Luminosity | ***Bunch Length*** | ***Beamstrahlung Lifetime***

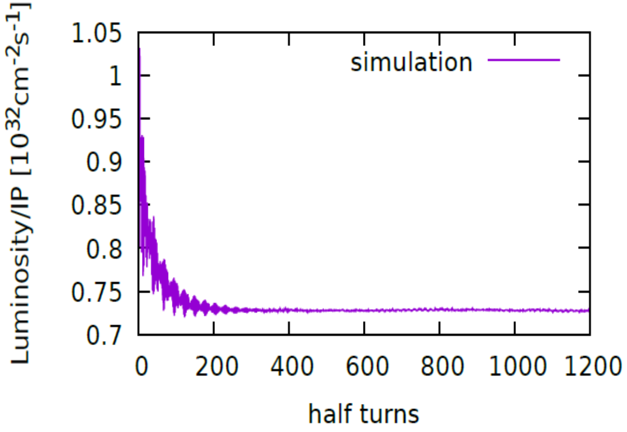
BBSS by K. Ohmi

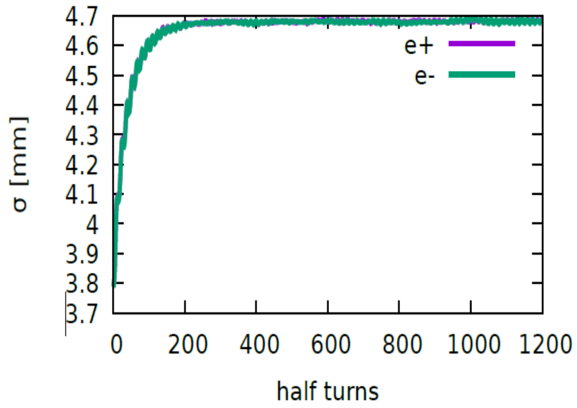


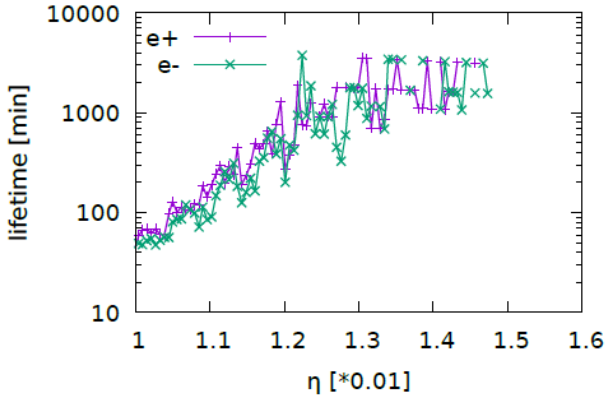




IBB by Y. Zhang







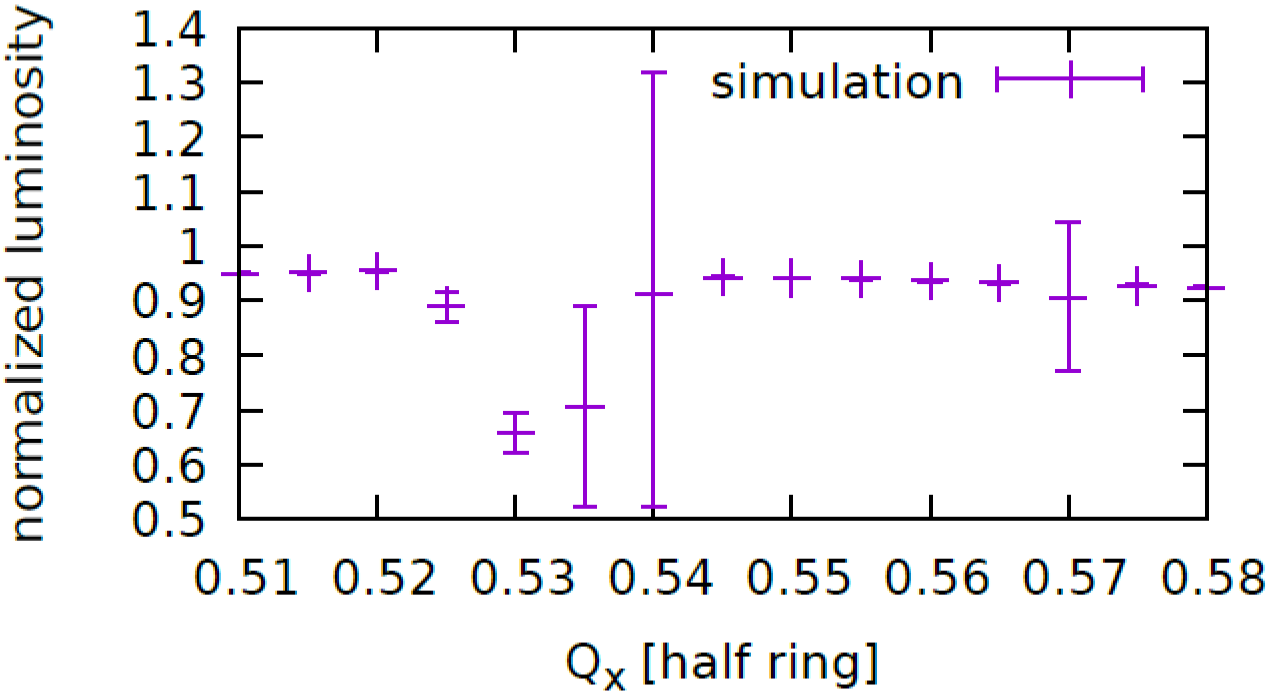
If the machine parameter is reasonable

- Limit of bunch population by beam-beam interaction
  - Beamstrahlung lifetime
  - If X-Z instability is suppressed
  - If asymmetric bunch current collision is stable
  - If there exist large enough stable working point space
  - If beam-beam parameter is safe enough

Tune Scan

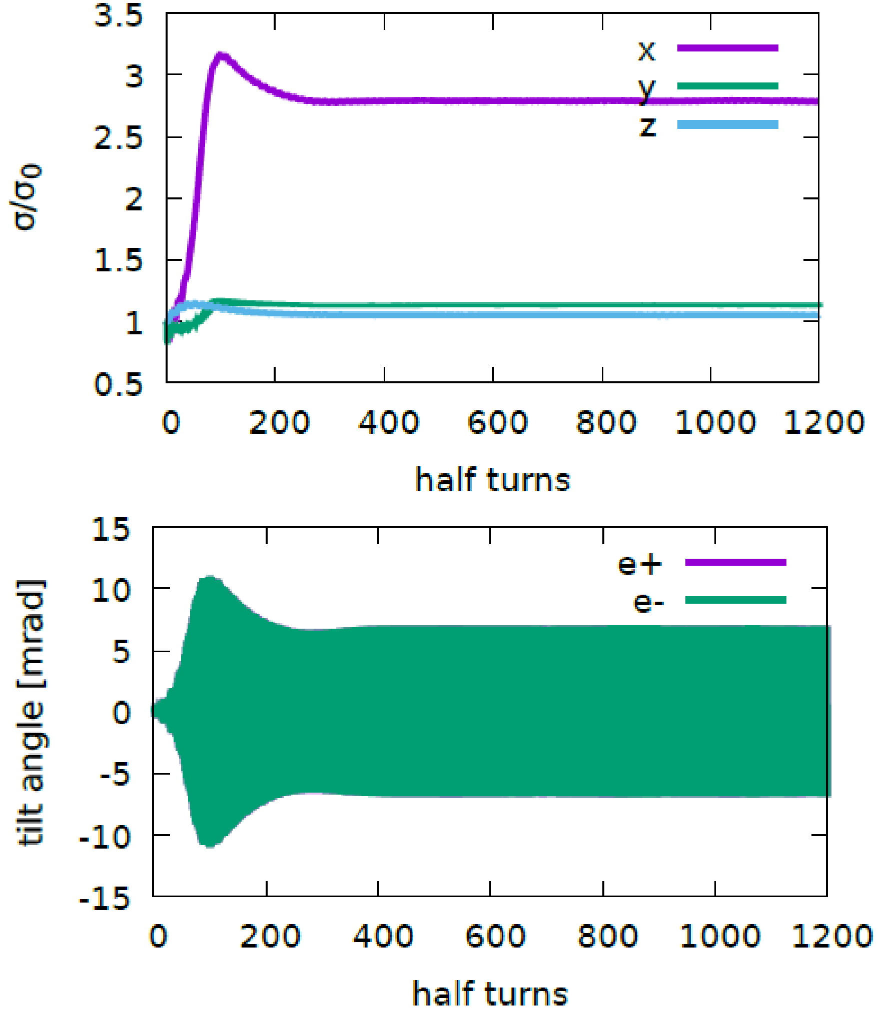
K. Ohmi and etal., DOI:10.1103/PhysRevLett.119.134801

Q_y =0.61, Q_s = 0.035



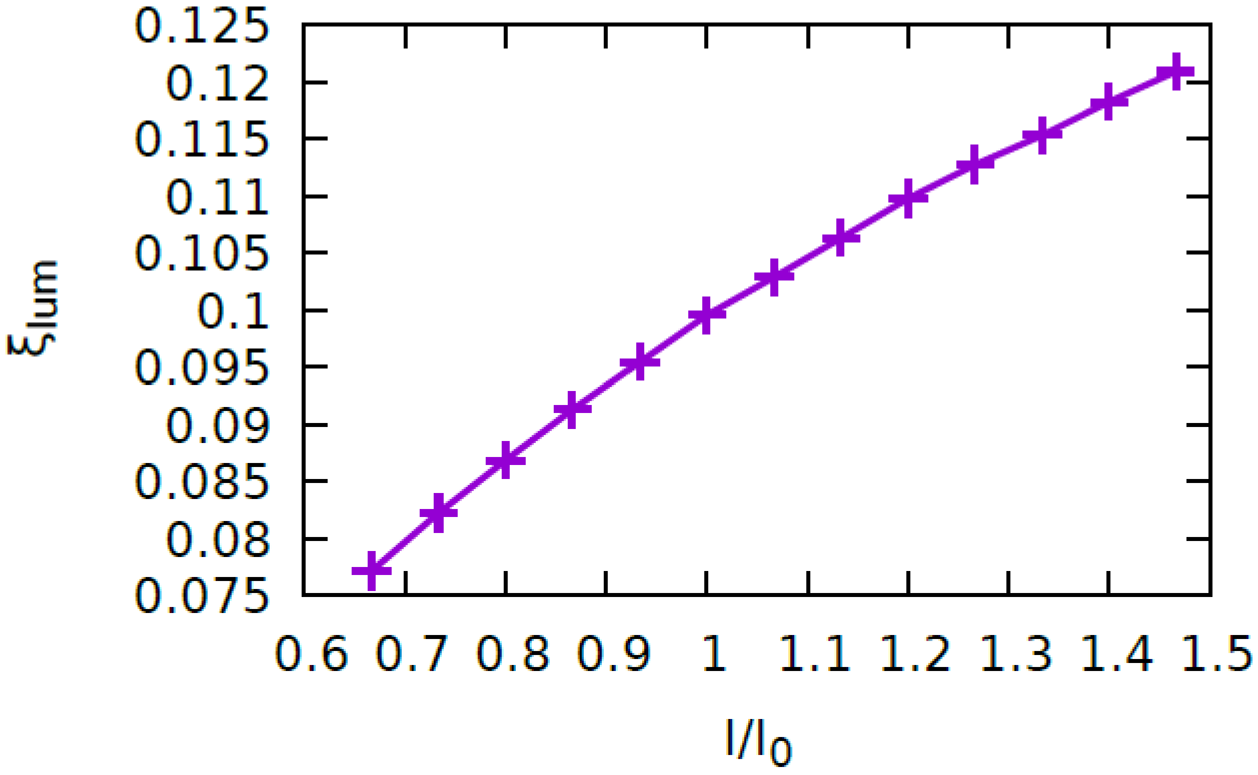
The error bar shows the turn-by-turn luminosity difference.

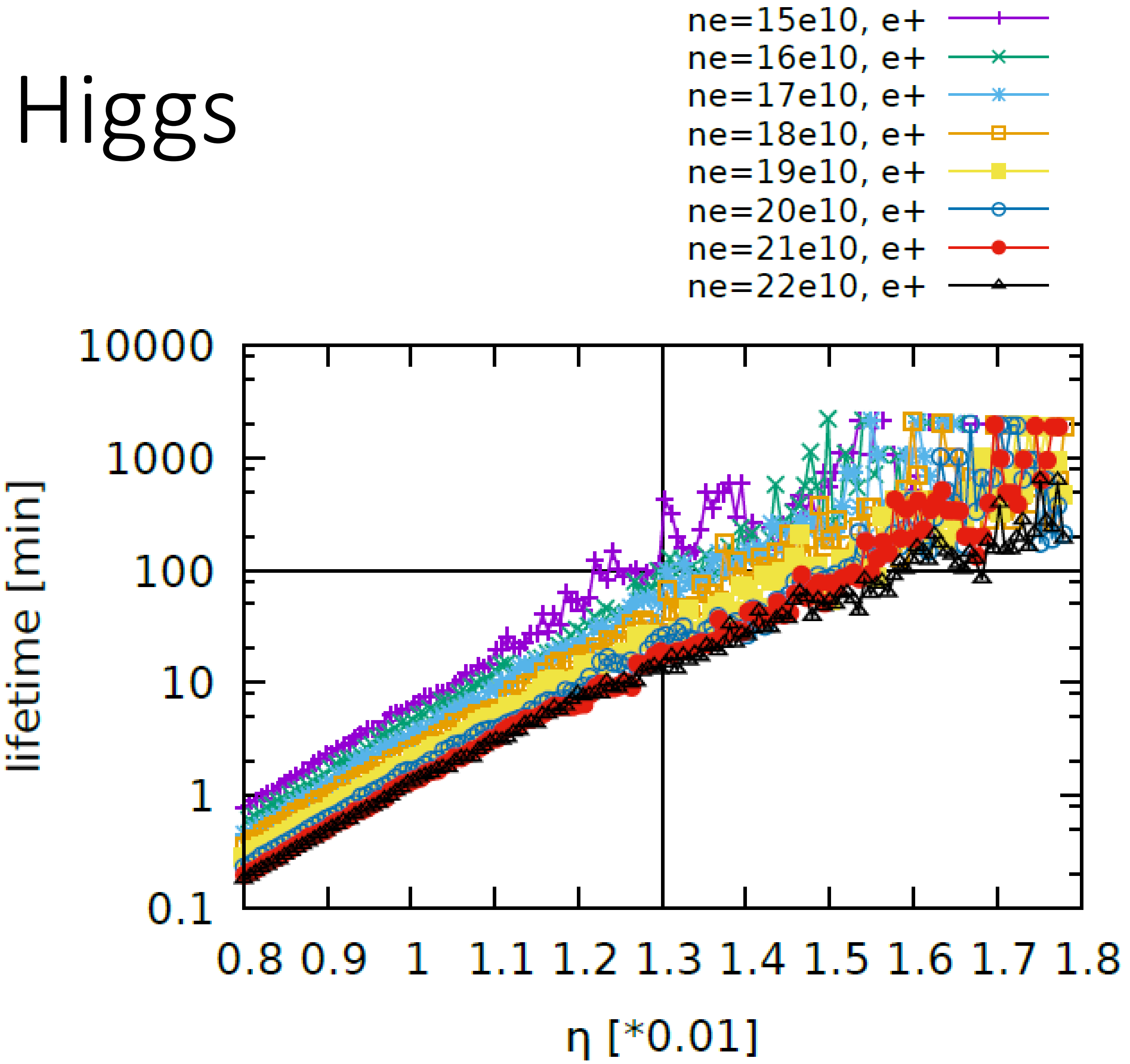
X-Z instability @(0.535,0.61)



Bunch Current Limit @ Higgs

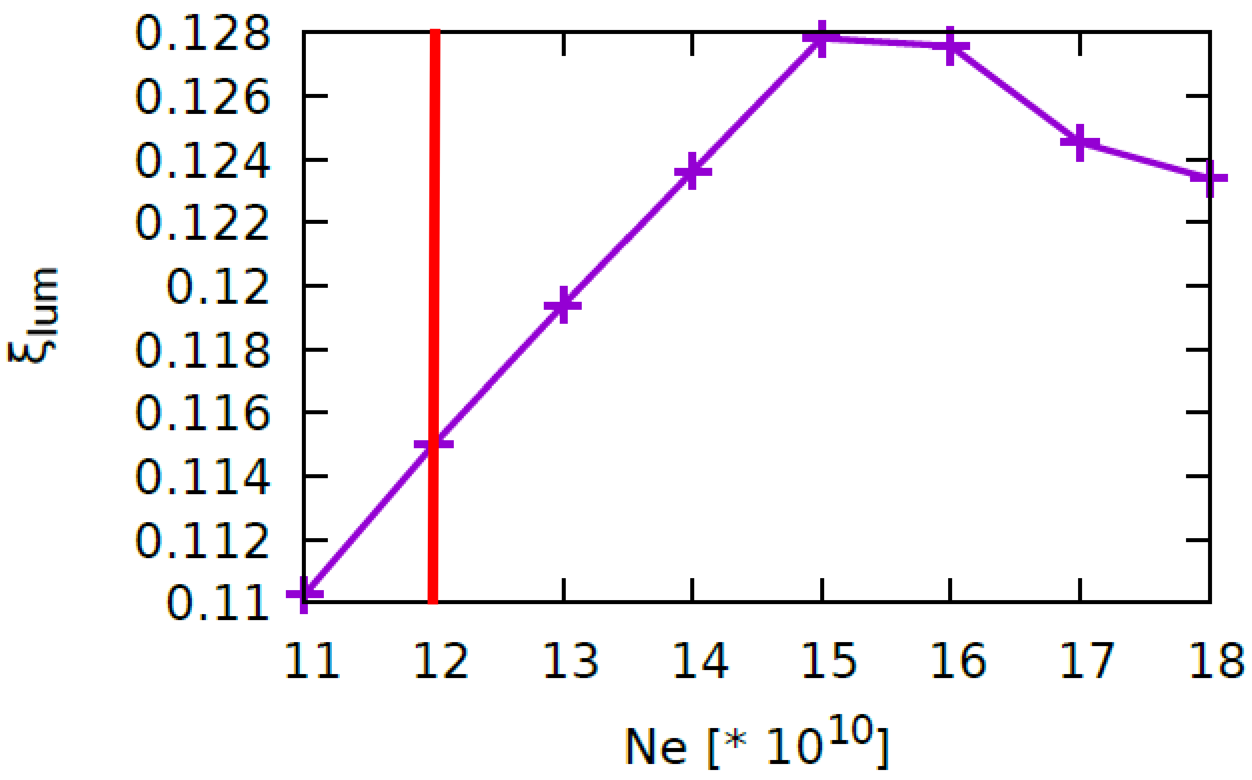
$$\xi_y = \frac{2r_e\beta_y^0}{N\gamma}\frac{L}{f_0}$$

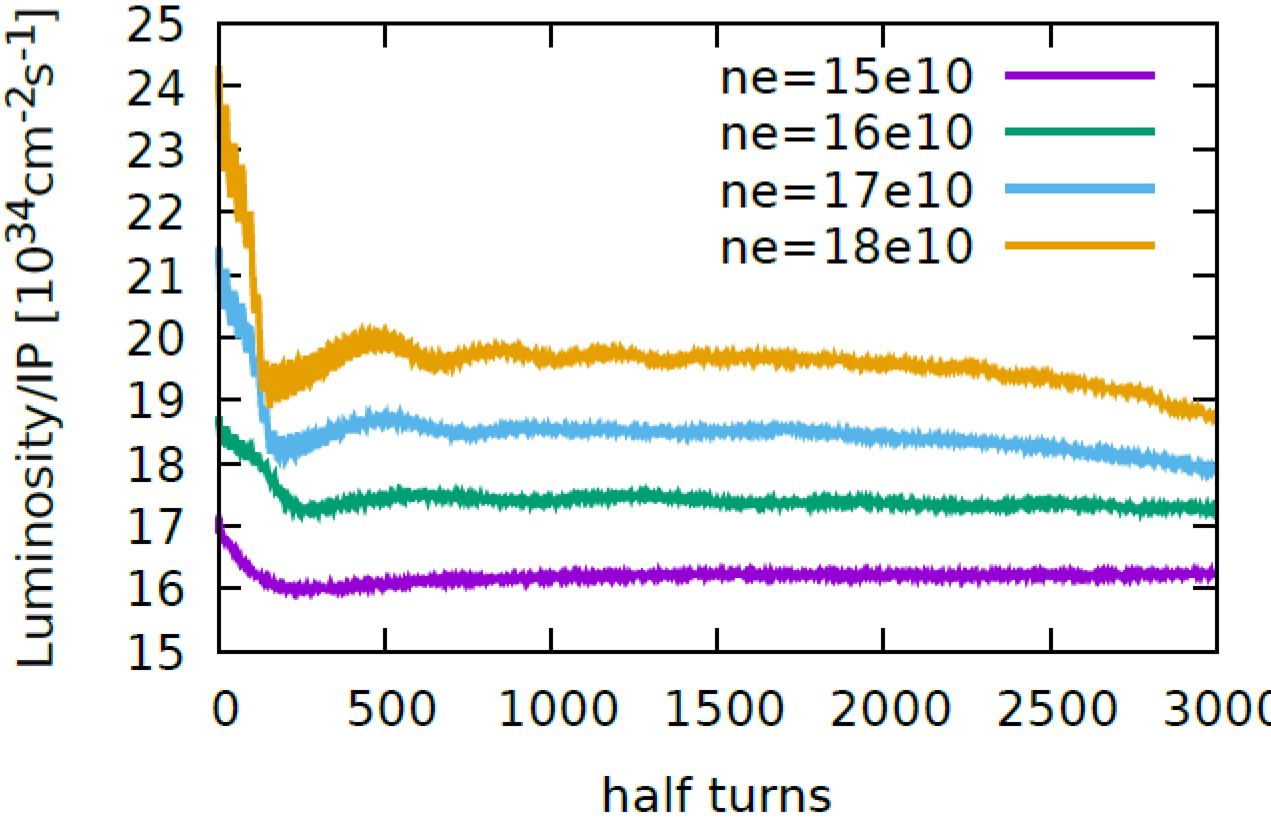




W

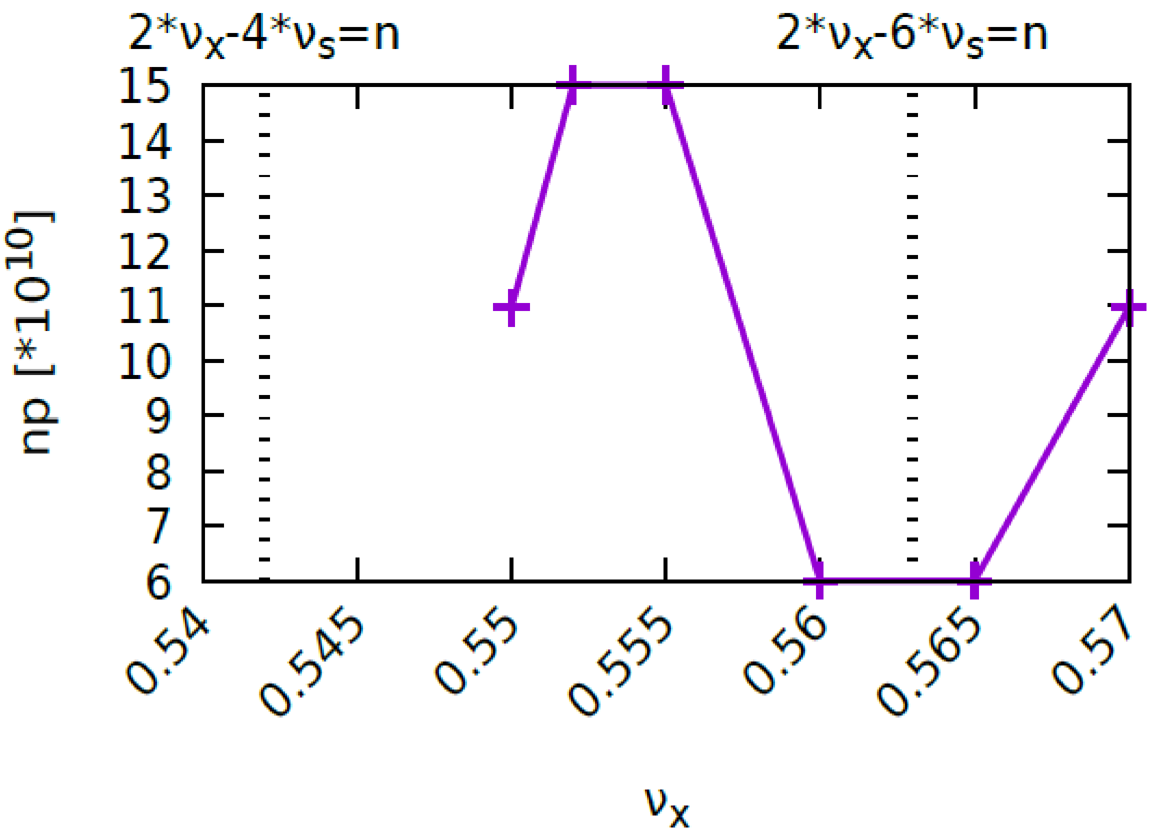
Bunch Current Limit @ W

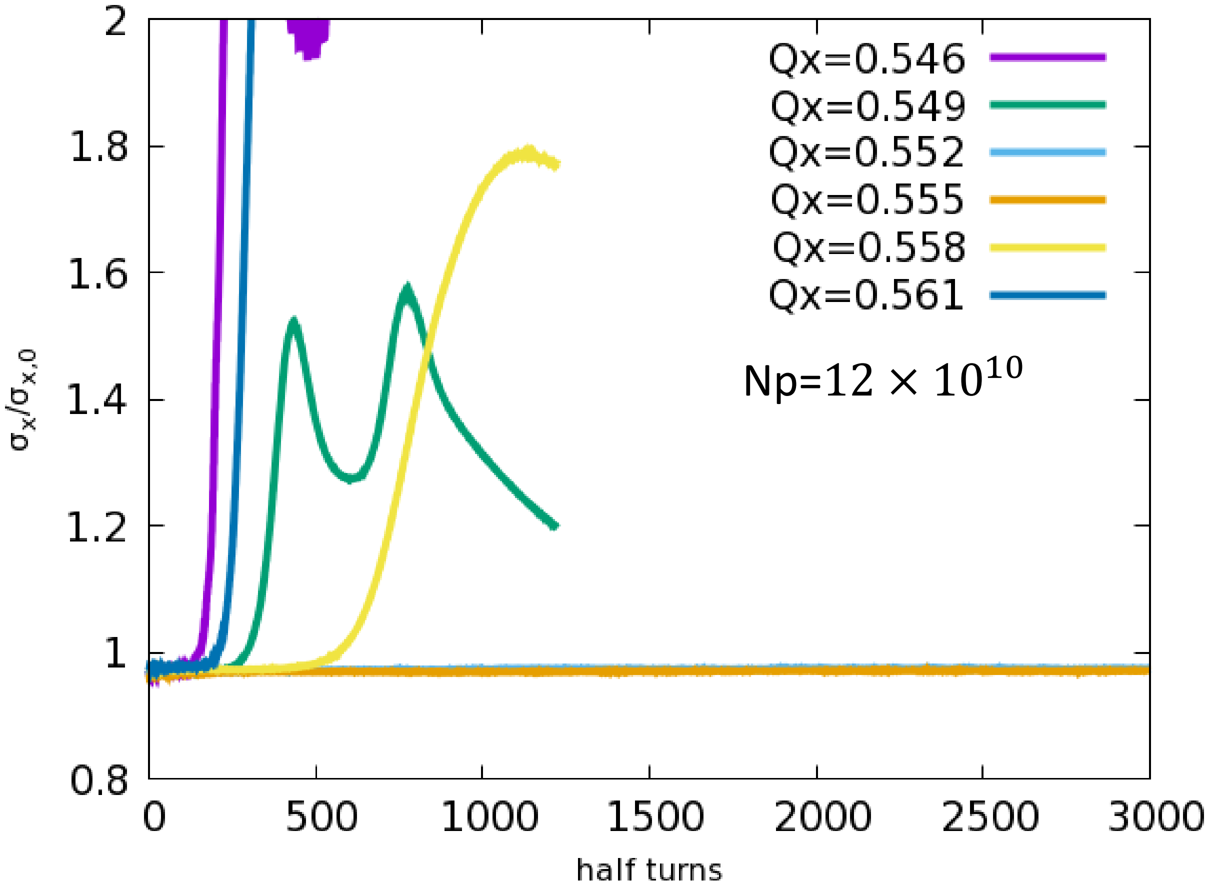




Bunch Current Limit versus Horizontal Tune

Collision is stable in the range of [0.552, 0.555]

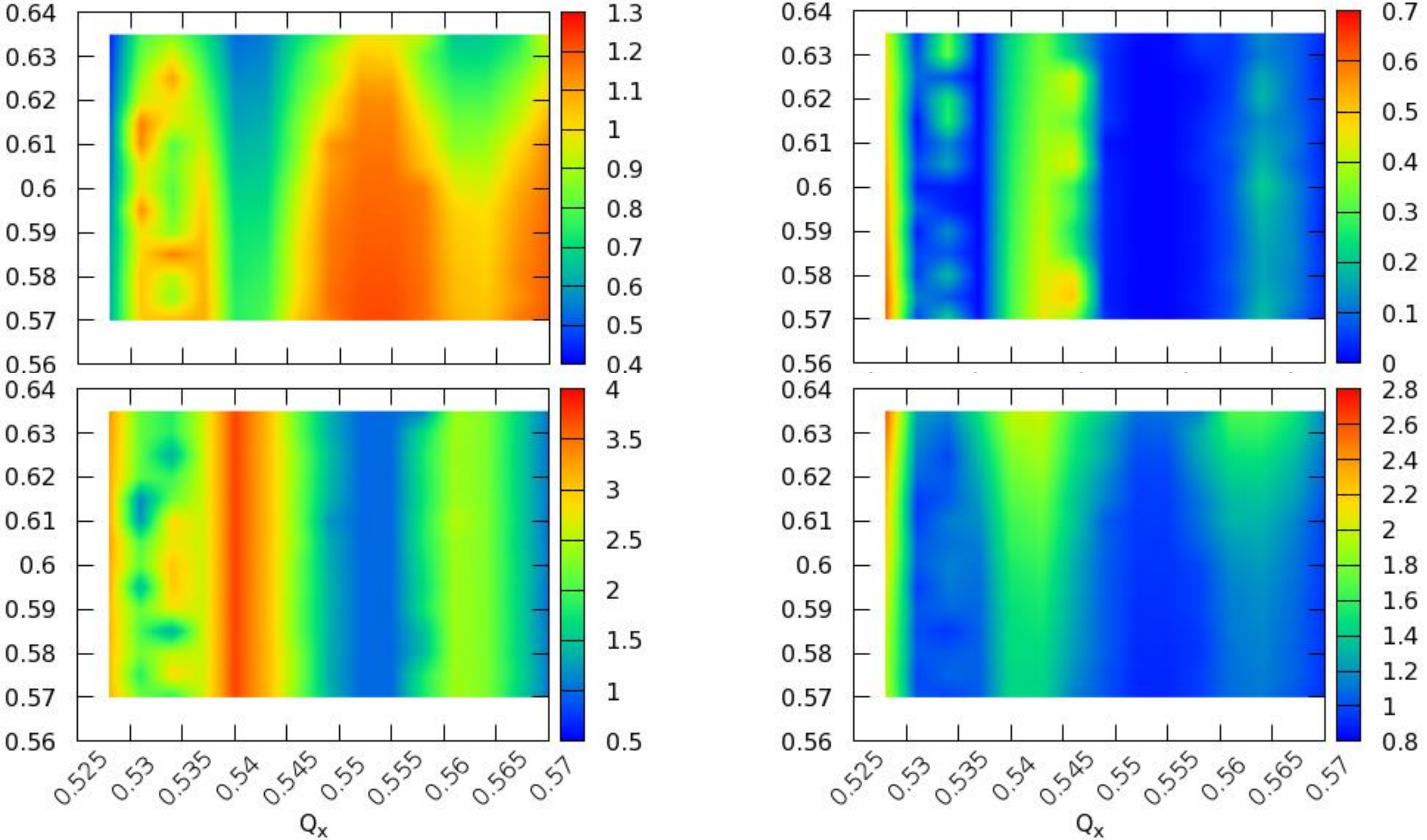




Tune Scan @ W

Luminosity

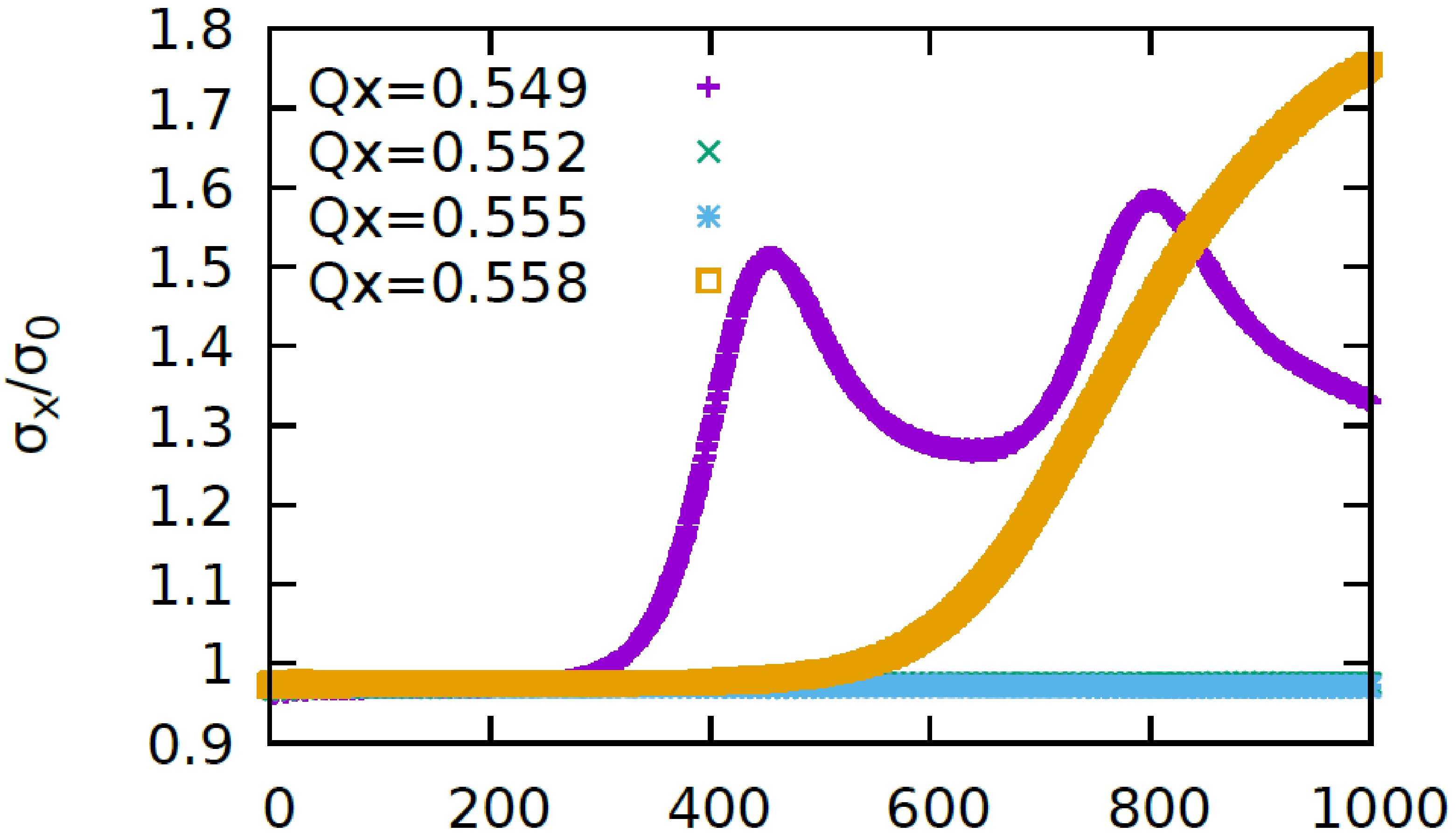
Turn-by-Turn Luminosity Distortion



σ_x

σ_y

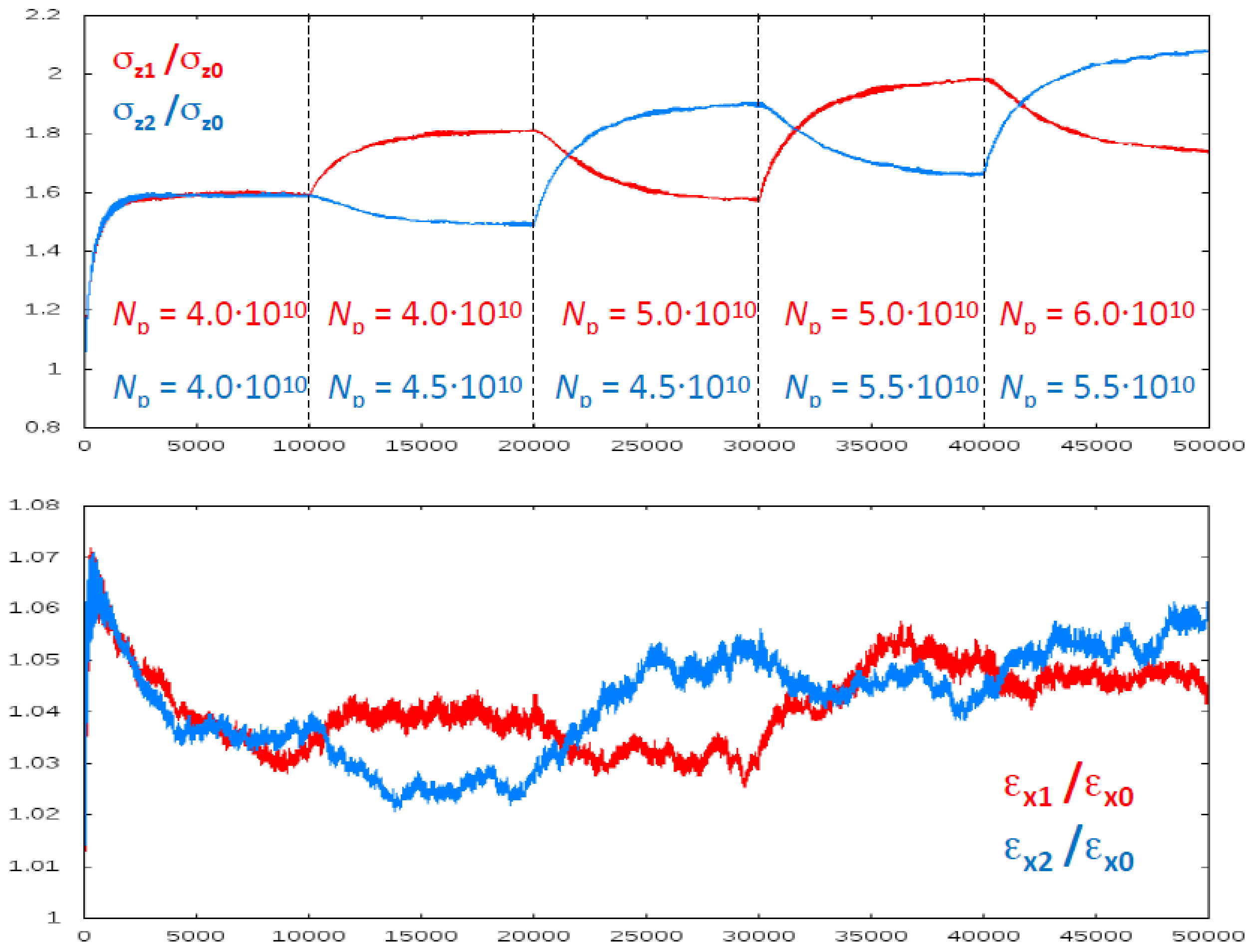
Tune Scan @ W (Qy=0.590)



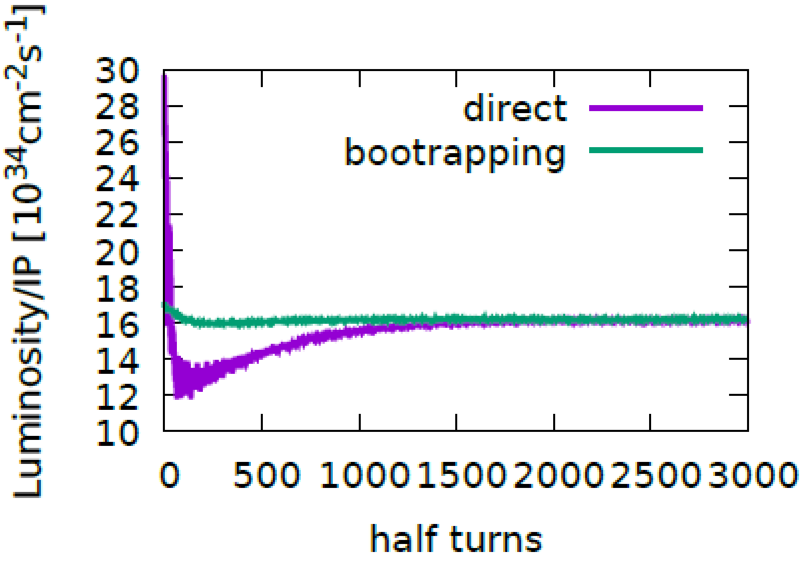
Bootstrapping Injection

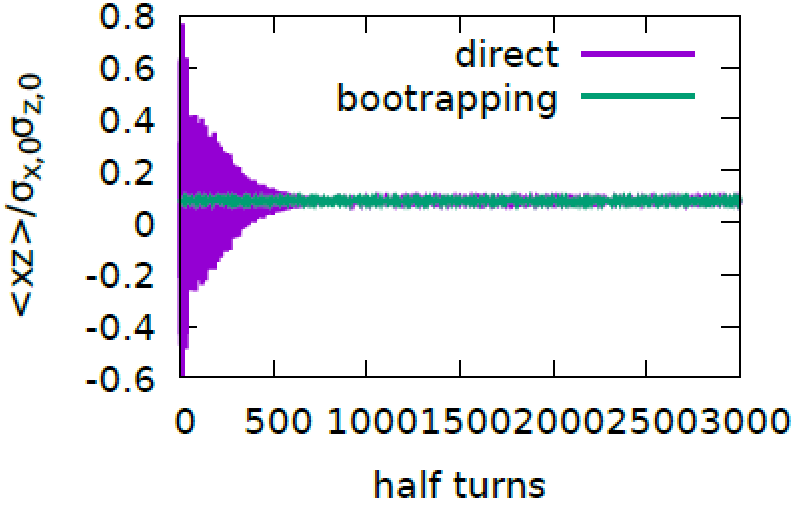
D. Shatilov

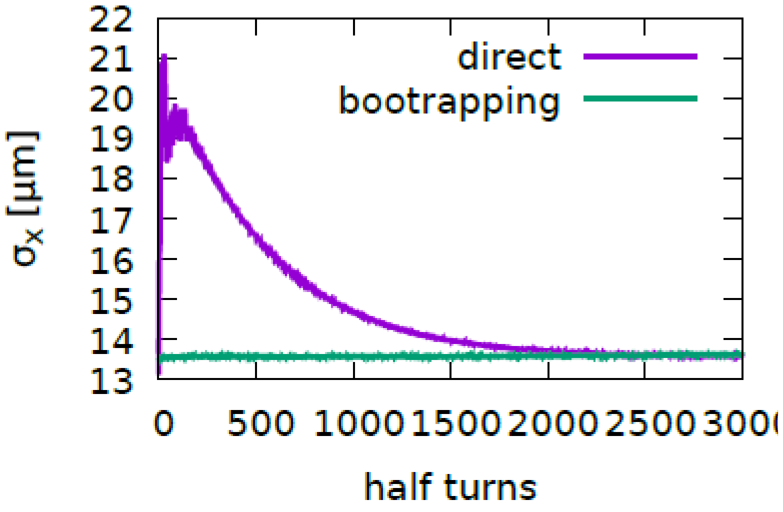
$$\mathrm{Np} \propto \frac{\alpha_p \sigma_\delta \sigma_z}{\beta_x^*} \text{ (K. Oide)}$$

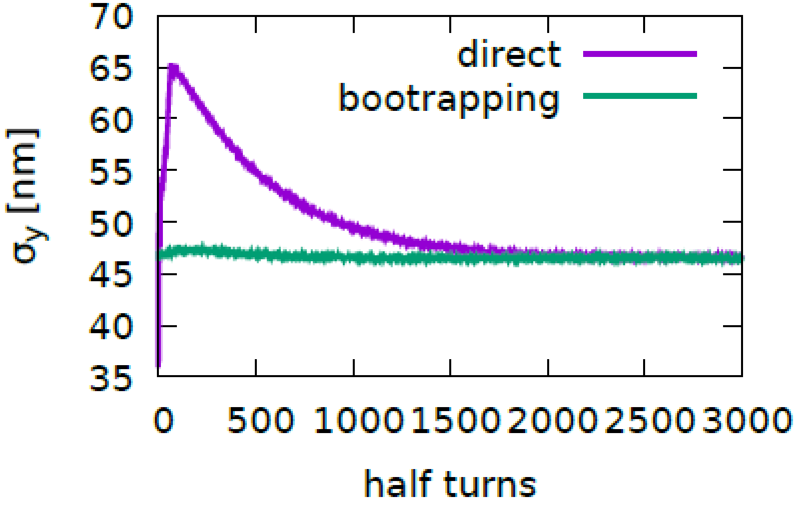


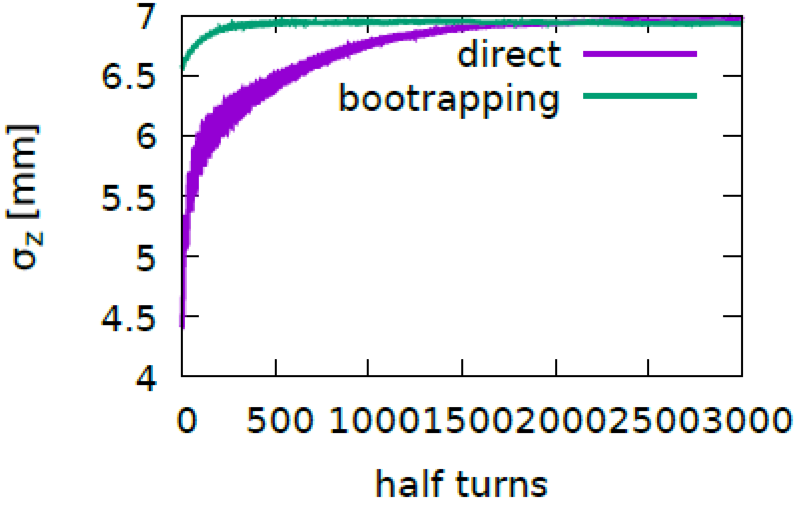
Bootstrapping is necessary? (15e10*15e10)







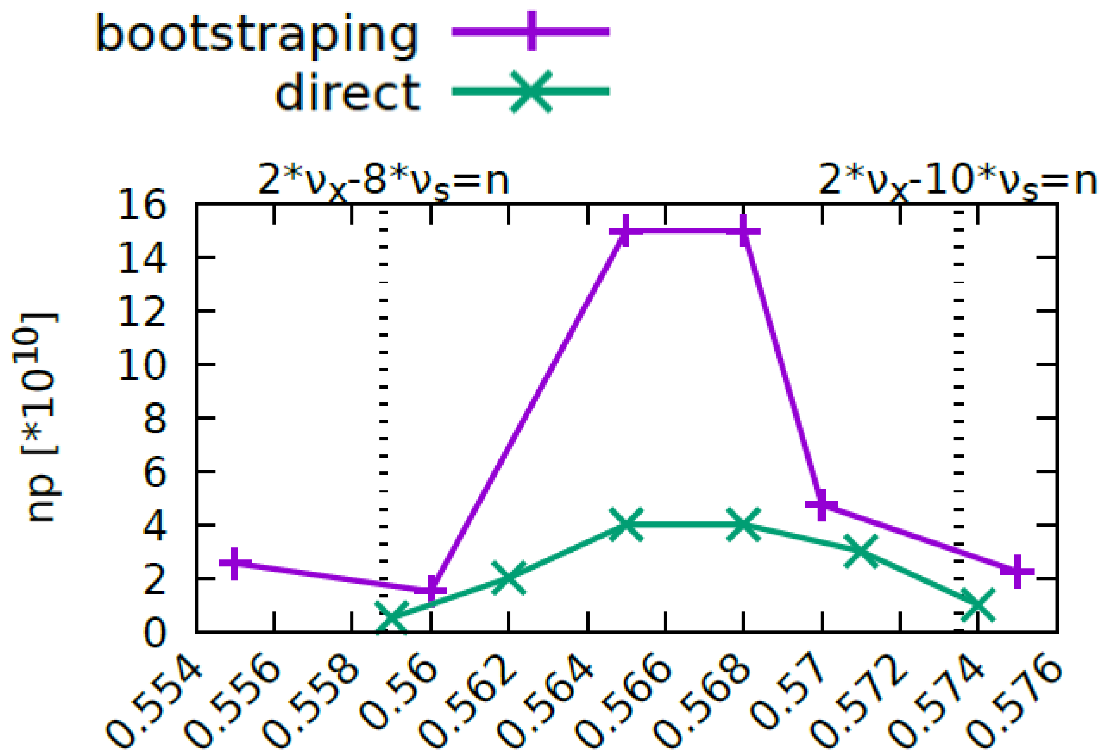




Bunch Current Limit versus Horizontal Tune

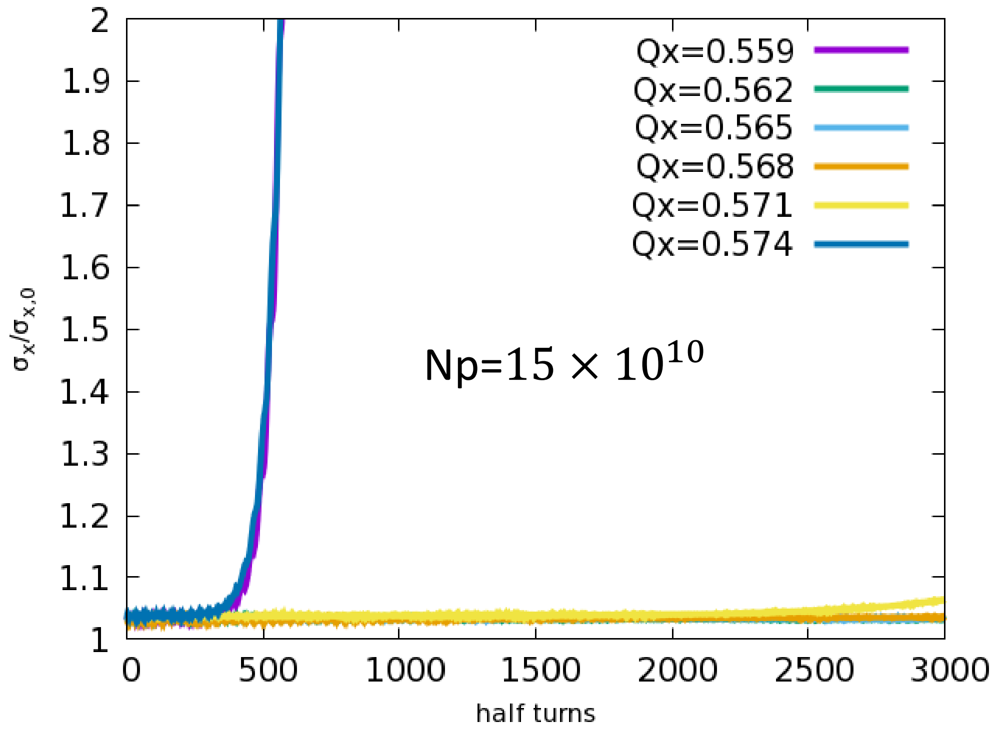
Criteria: $\frac{\sigma_x}{\sigma_0}$<1.1

Width of safe Qx ~ 0.006



- Collision is stable in the range of [0.562, 0.568]

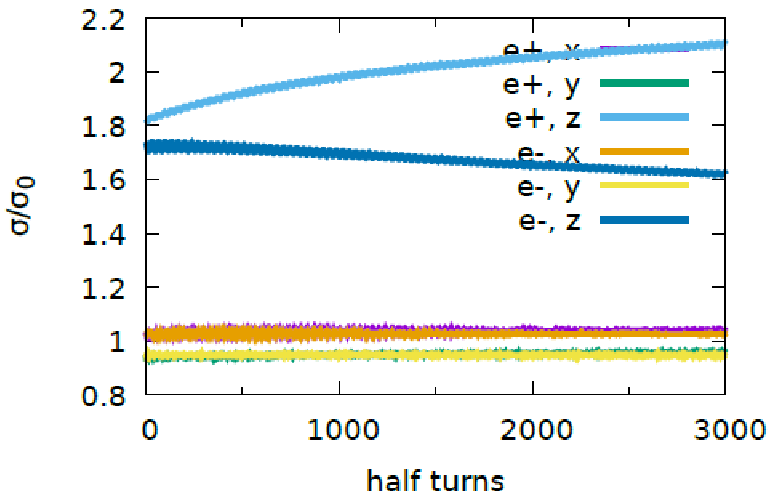
It is similar in case of Z-2T, with np=12$\times 10^{10}$

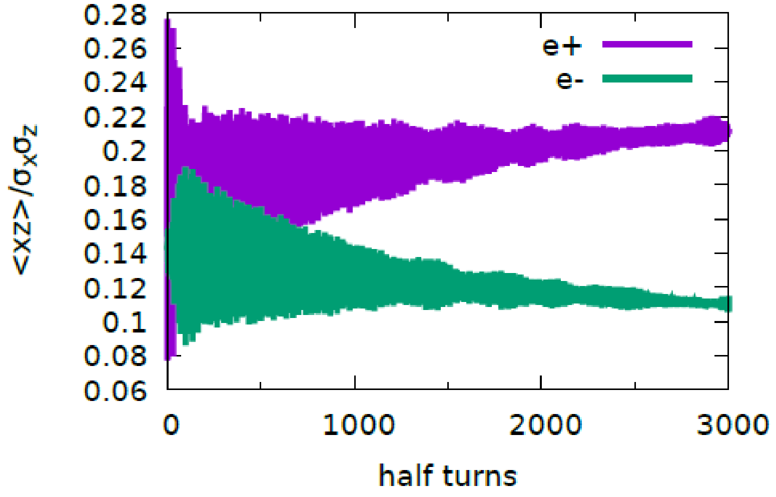


Bunch Current

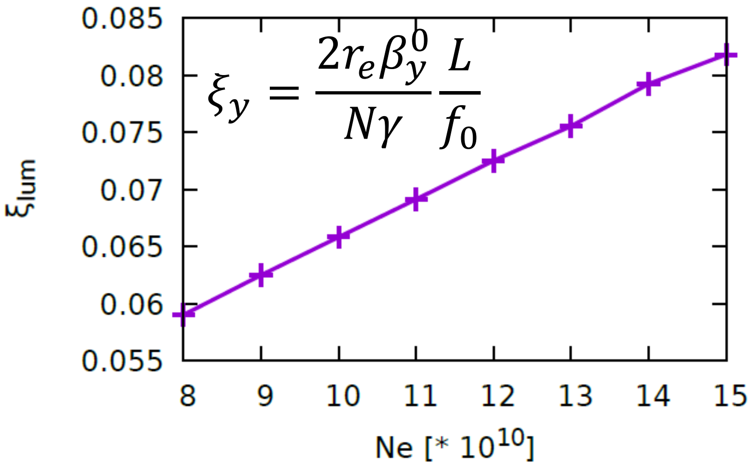
Asymmetrical Bunch Current

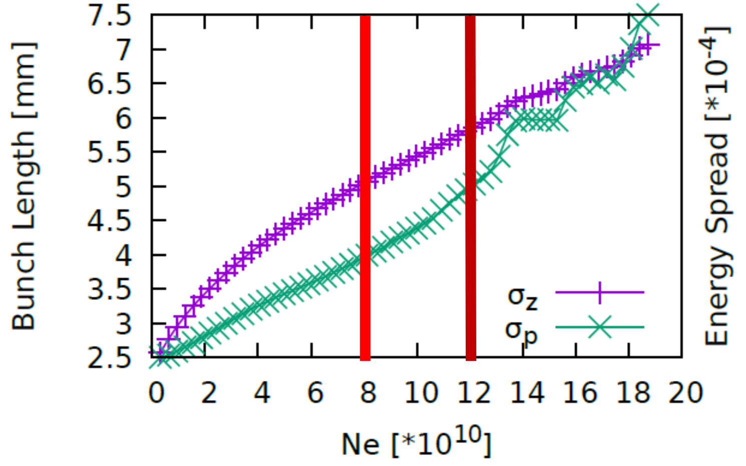
Np=13e10 vs 15e10





Bunch Current Limit



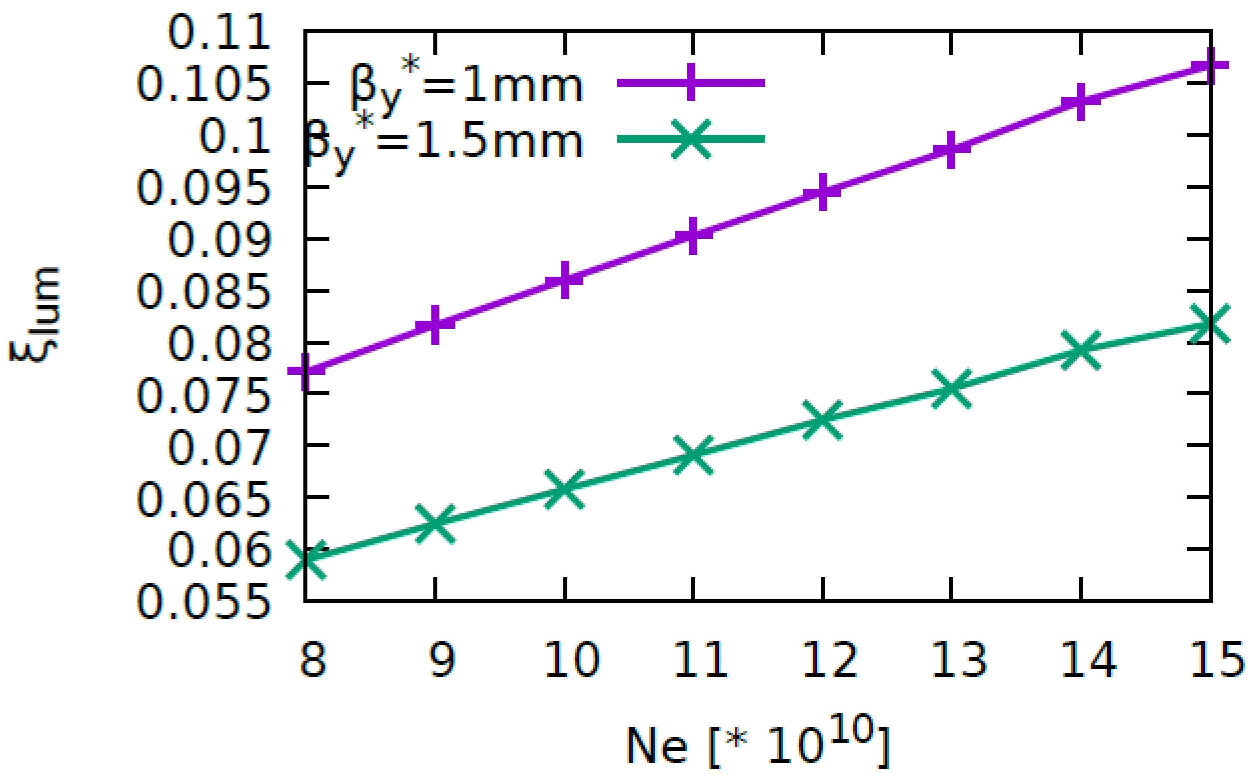


Bunch Lengthening and Energy Spread **w/o Collision**

Z

β_y^*=1.5mm->1mm (Solenoid: 3T -> 2T)

$$\xi_y = \frac{2r_e\beta_y^0}{N\gamma}\frac{L}{f_0}$$



With same beam current,

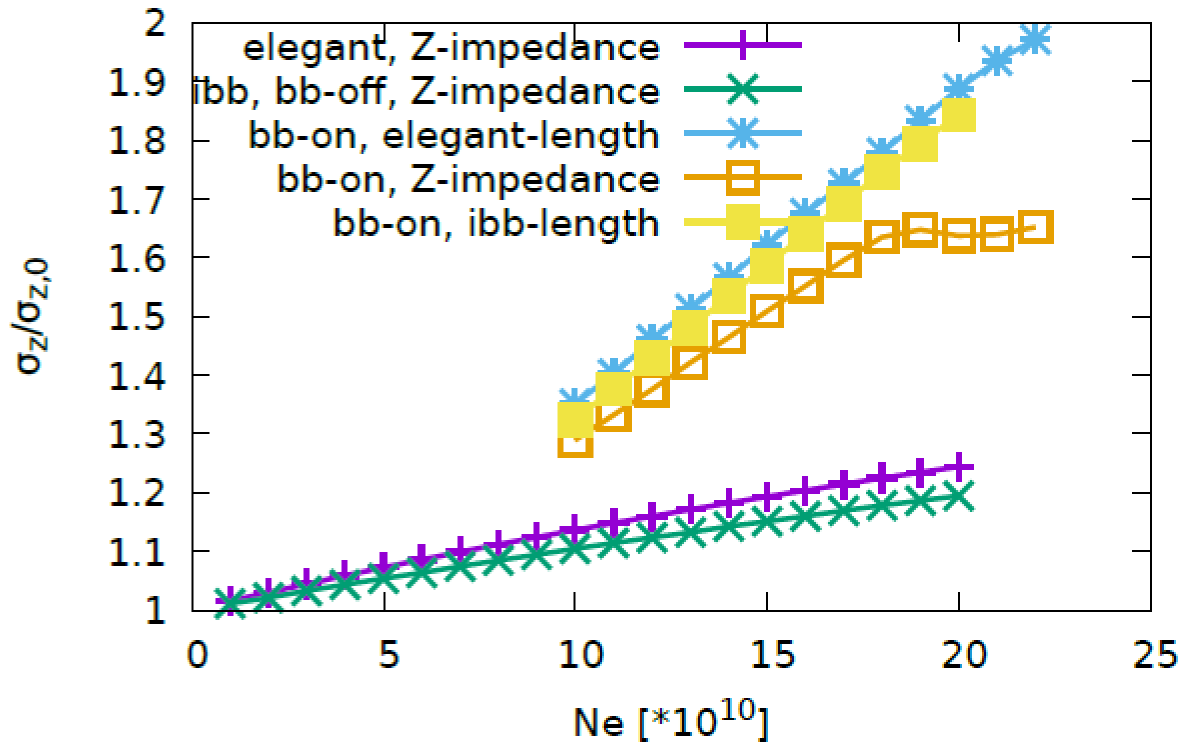
- smaller β_y^*+weaker solenoid, luminosity increase by a factor of two.
- bunch population increase from 8×10^{10} to 12×10^{10}, luminosity increase about 20%.

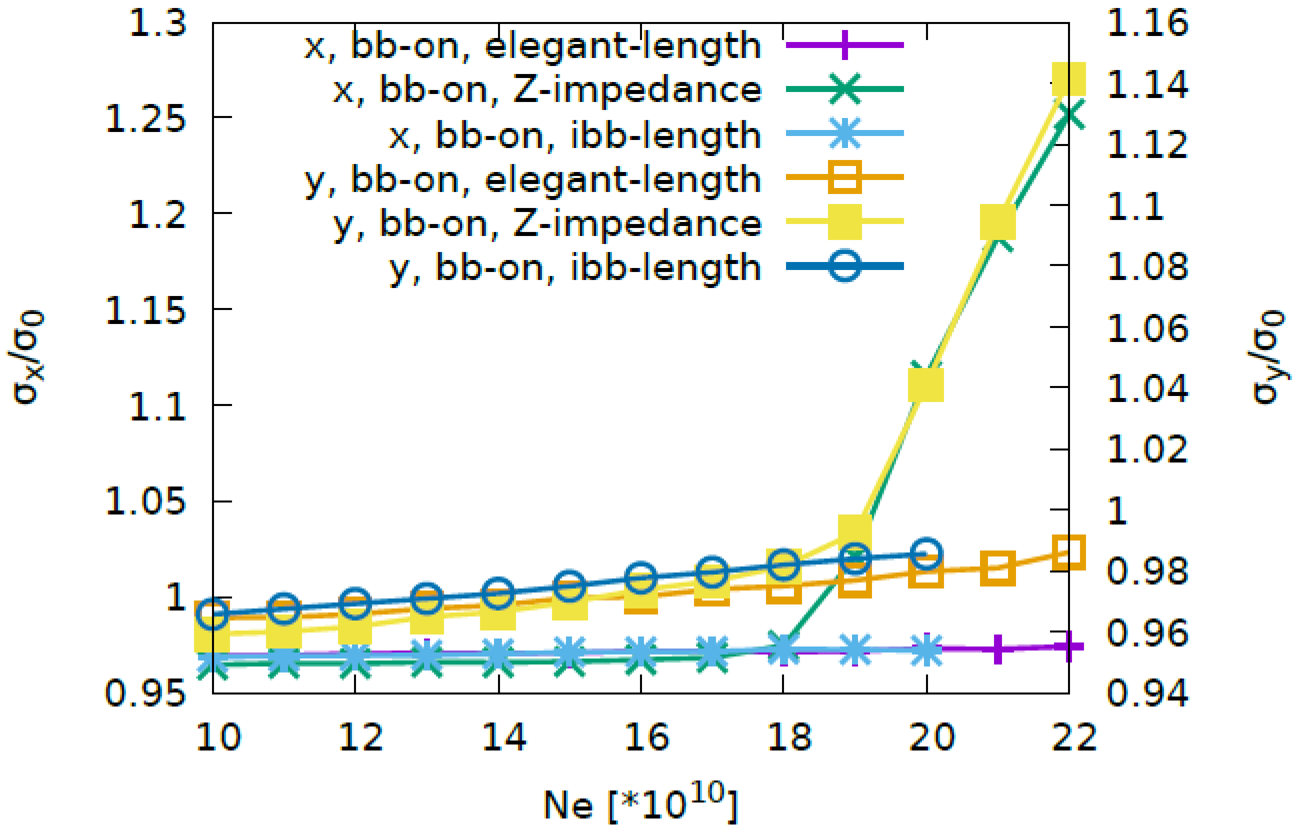
Crosstalk between Beam-Beam Interaction & Collective Effect

- Bunch length caused by impedance => $\sigma_z, \beta_z, \epsilon_z$
- In the conventional case, it is fine since the longitudinal dynamics is not sensitive to the beam-beam interaction
- In CEPC/FCC, the beamstrahlung effect will also lengthen the bunch
- It is self-consistent to consider the longitudinal wake field and beamstrahlung

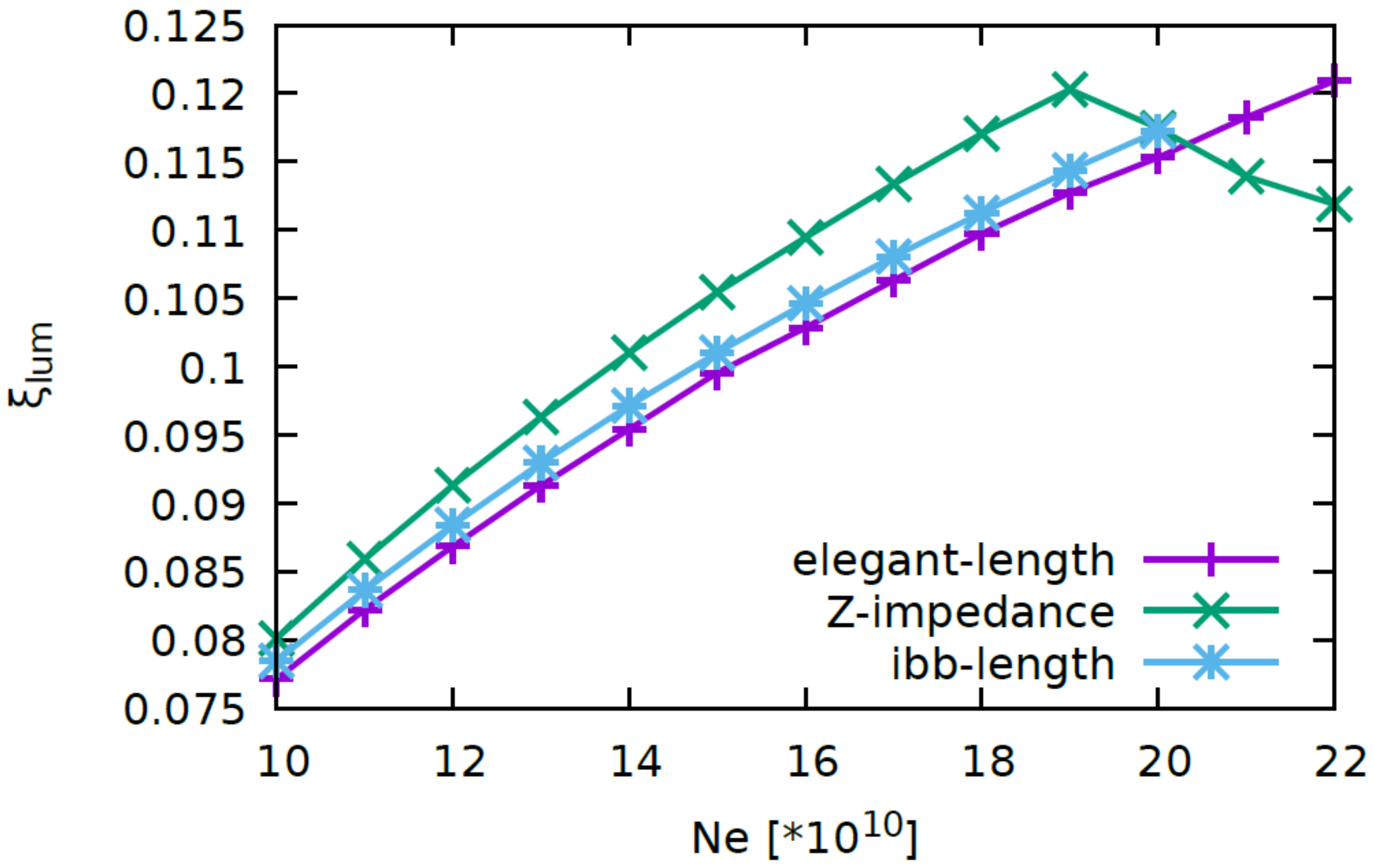
$$I(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{I}(\omega) \qquad \tilde{V}(\omega) = -\tilde{I}(\omega) Z_{\|}(\omega)$$

RMS size with longitudinal impedance

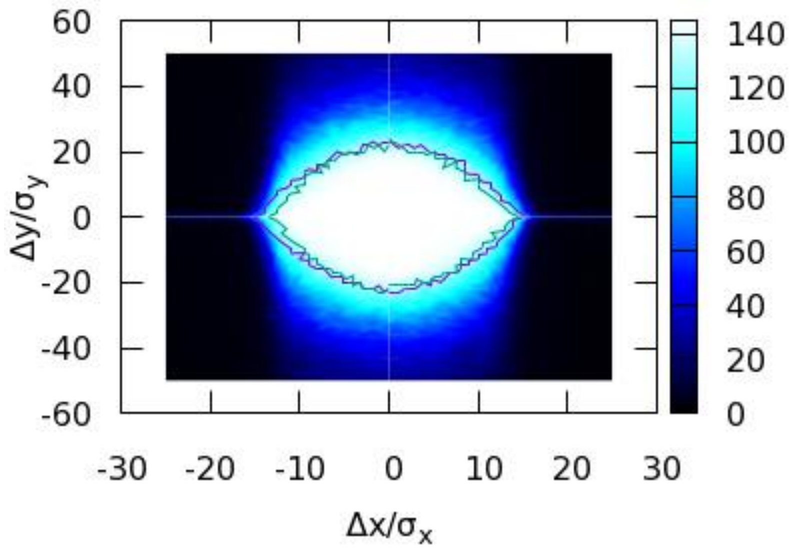


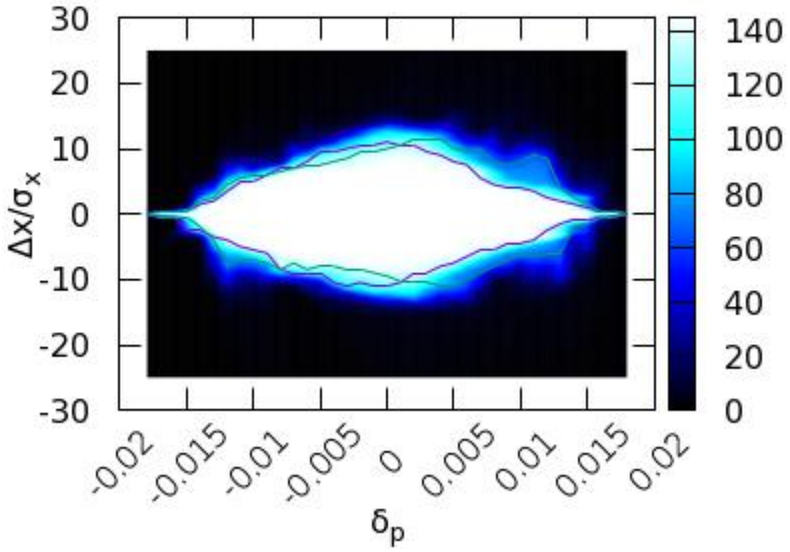


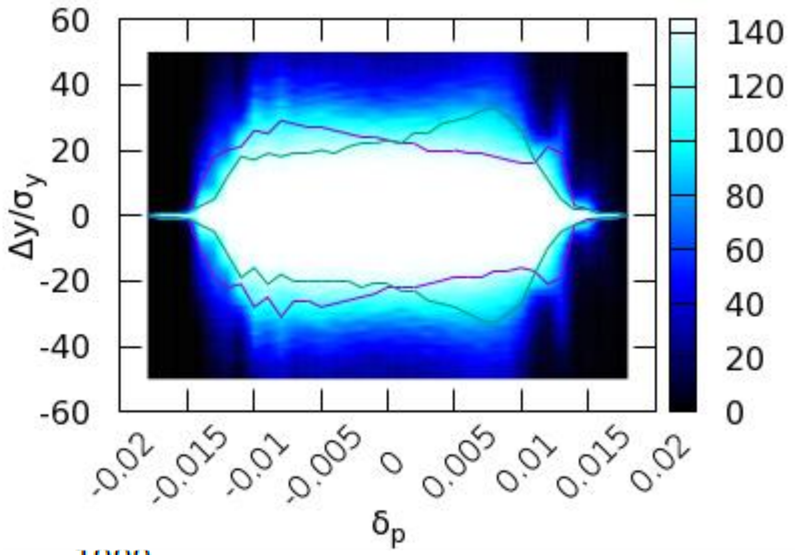
Beam-Beam Performance with longitudinal impedance

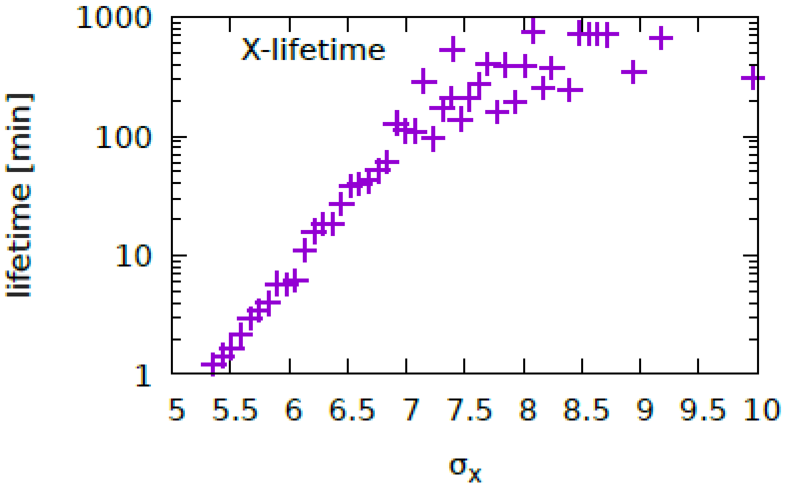


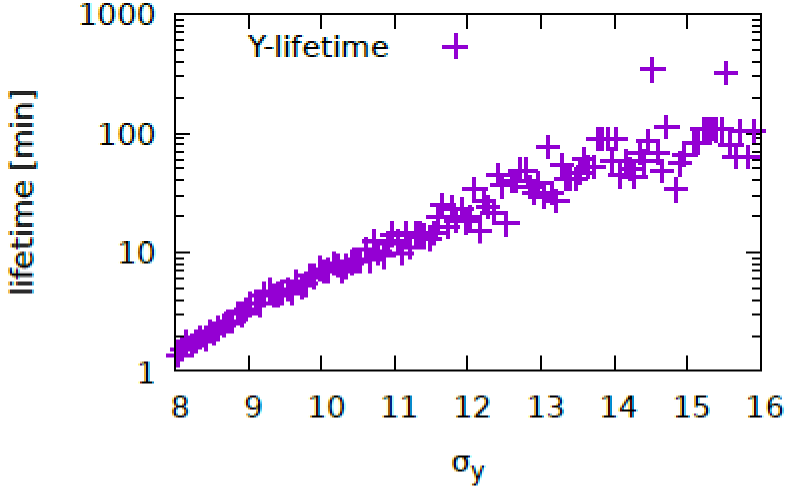
Dynamic Aperture & Lifetime: Case 0

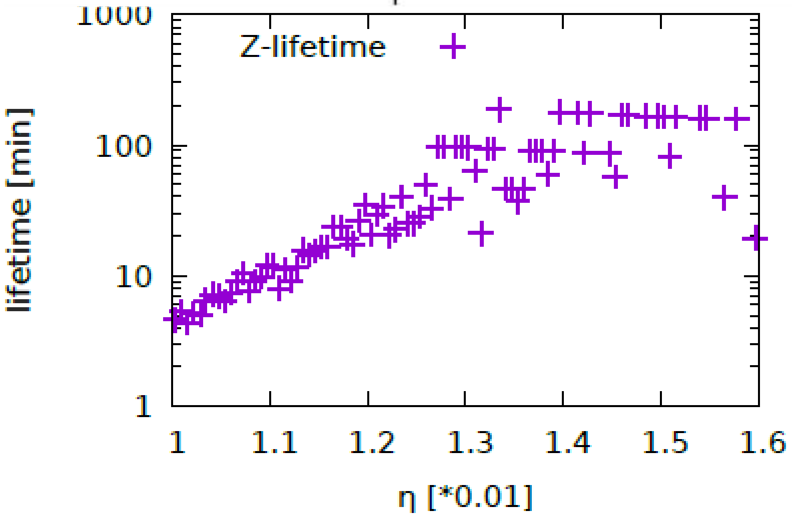




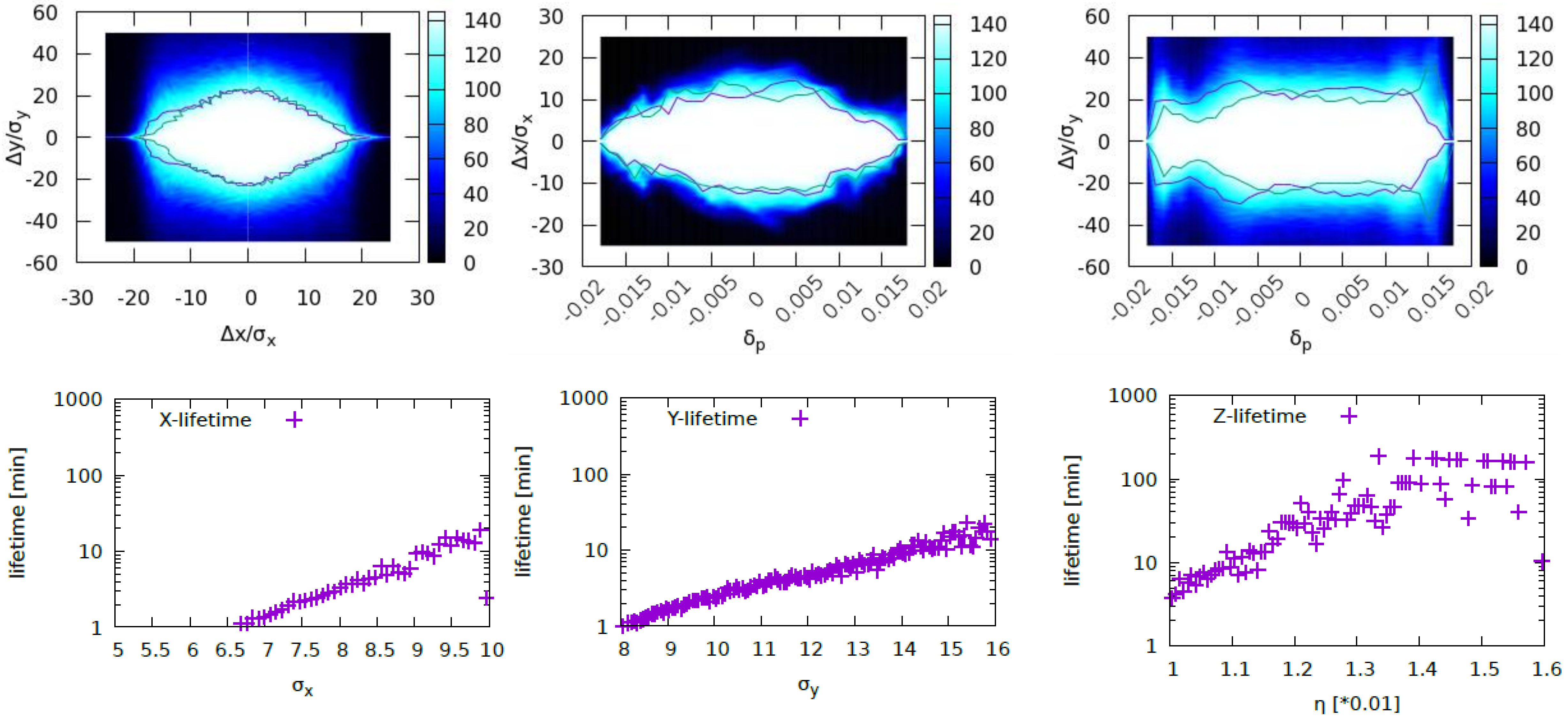




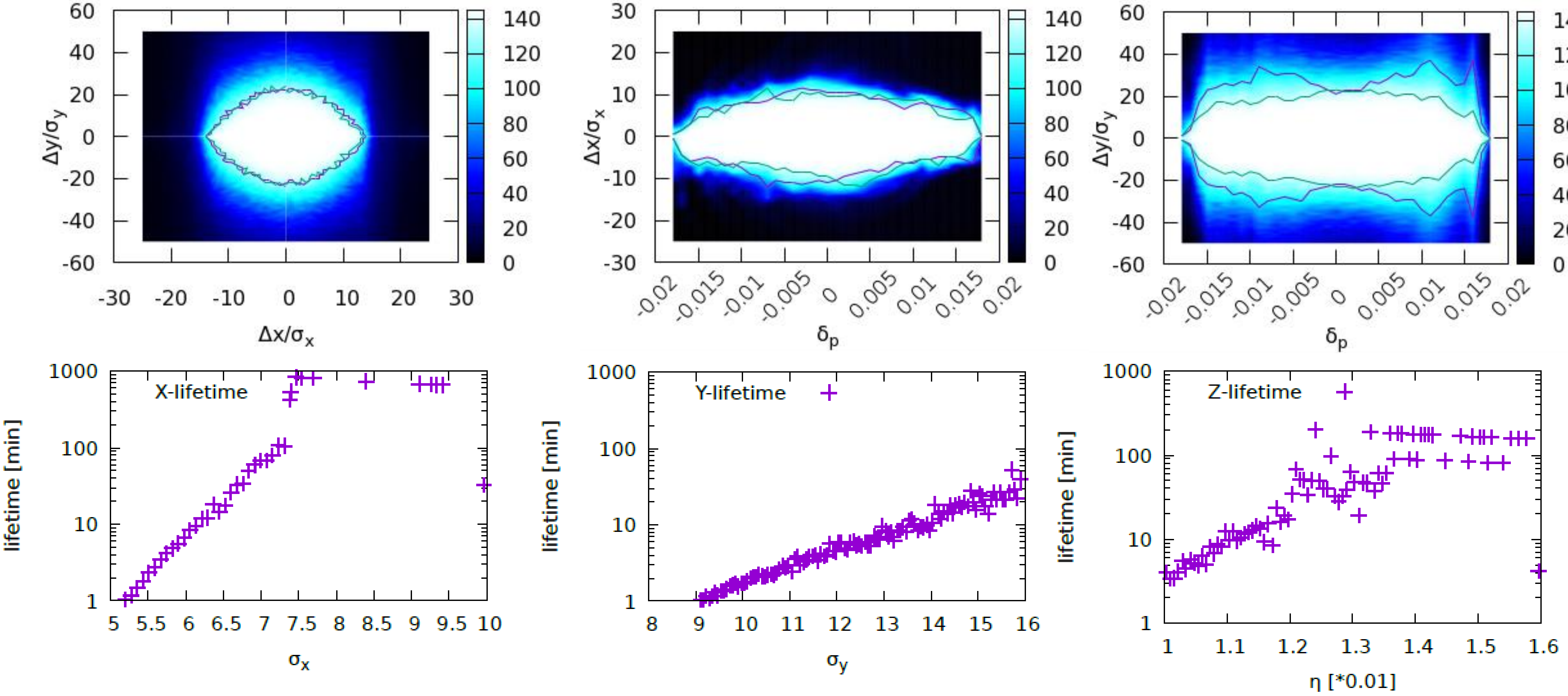




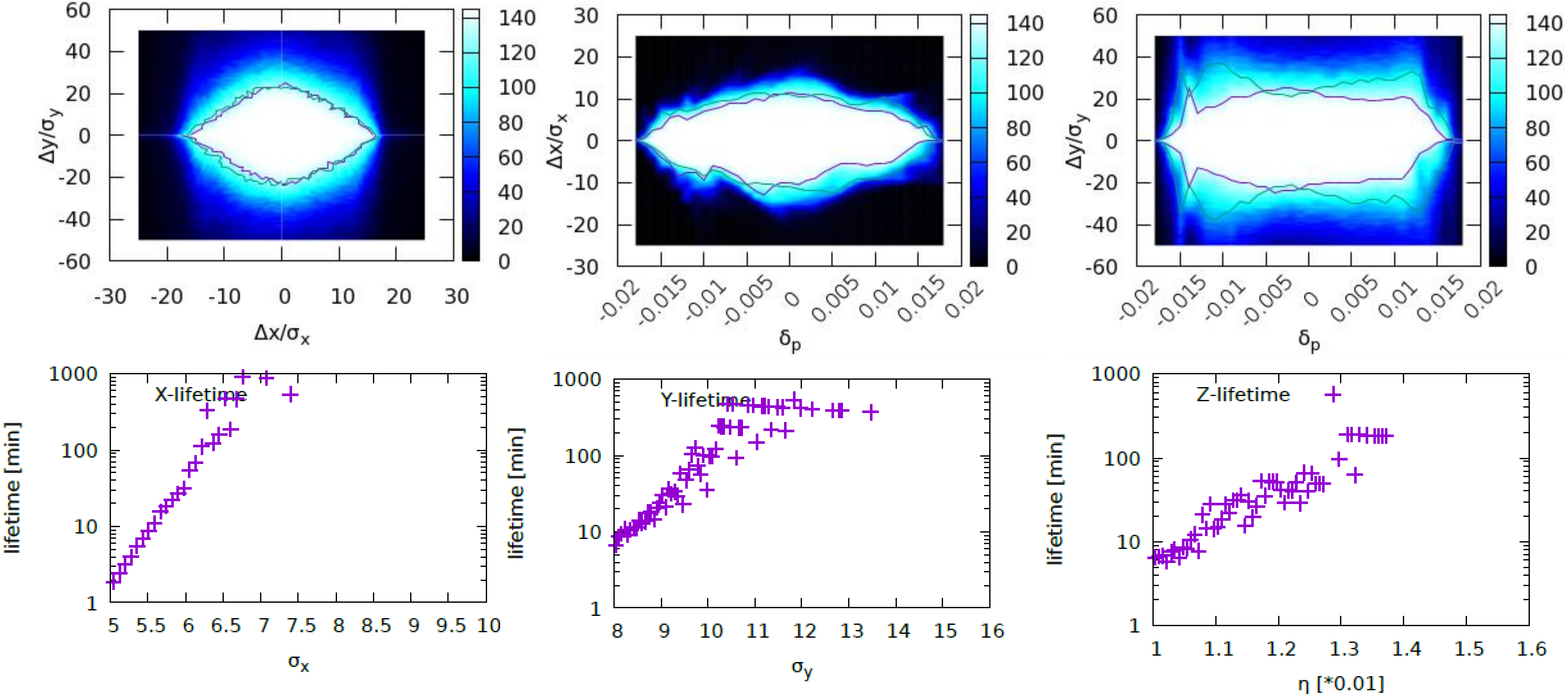
Dynamic Aperture & Lifetime: Case 1



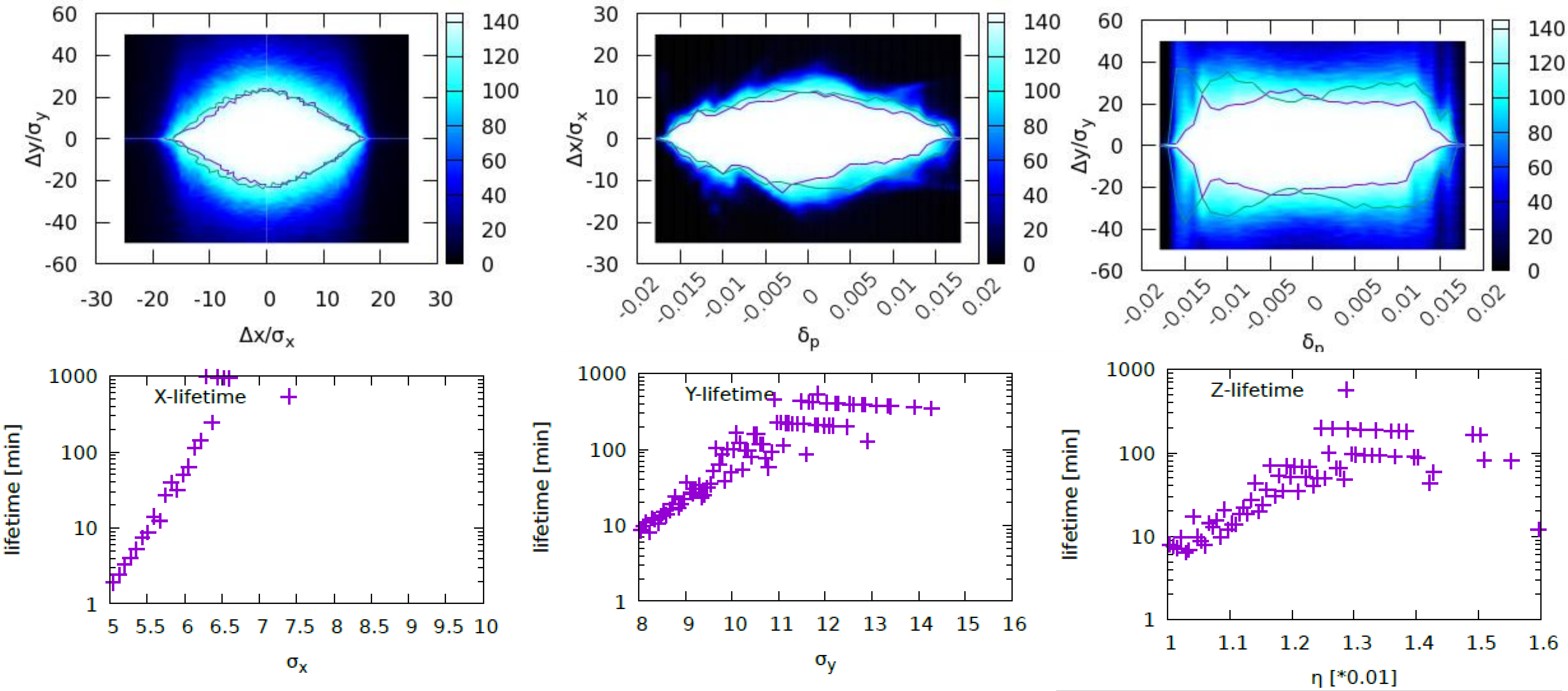
Dynamic Aperture & Lifetime: Case 2



Dynamic Aperture & Lifetime: Case 3

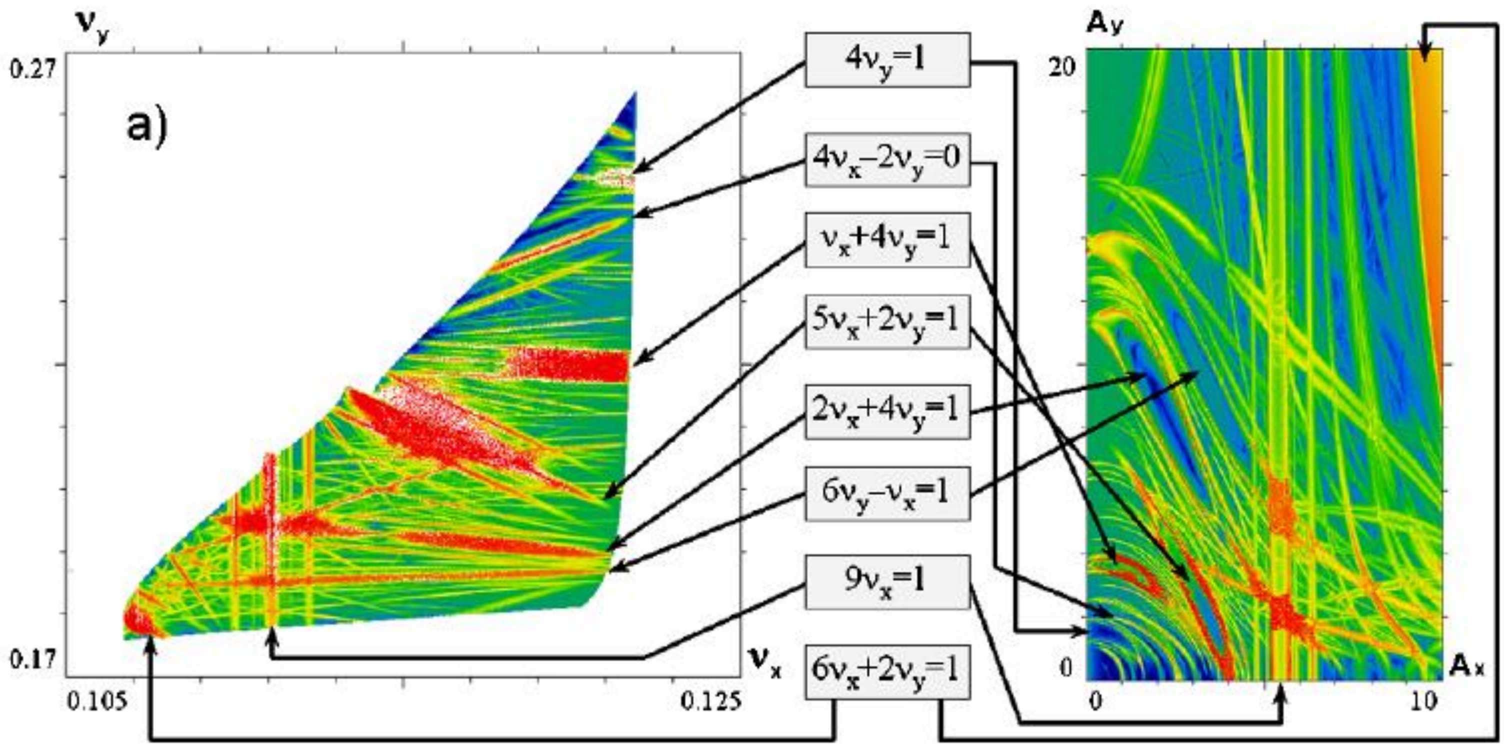


Dynamic Aperture & Lifetime: Case 4



J. Laskar, Icarus 88, 266 (1990).
D. Shatilov, E. Levichev, E. Simonov and M. Zobov, PRST-AB, 14, 014001 (2011)

Application of FMA to Beam-Beam Effects



Beam-beam resonances in the tune and amplitude planes for DAFNE, crab~0.4.

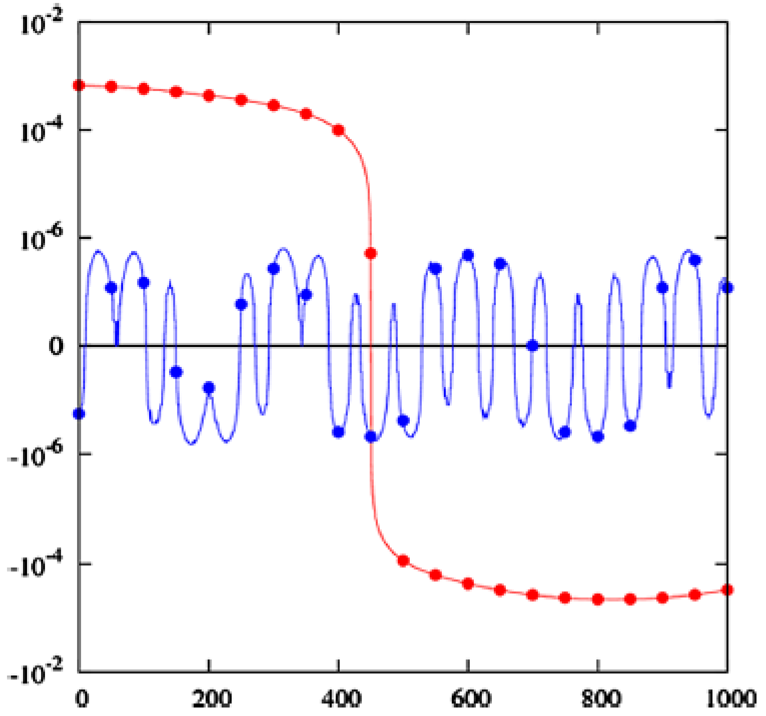


FIG. 3. Betatron tune deviation versus the window shift (in turns) for two trajectories: chaotic (red) and regular (blue).

The diffusion index is calculated as $log10(\sigma_\nu)$, where σ_ν is the rms spread of tunes.

Could a So-Called Diffusion Map help us?

- FMA does not work well with chromaticity
- FMA will fail with strong synchrotron radiation and beamstrahlung effect
- In a lattice, with strong non-linearity, beam-beam interaction, strong SR fluctuation, could we construct a Diffusion Map to do some analysis?

K. Ohmi, M. Tawada, K. Oide and S. Kamada,
"Study of the diffusion processes caused by the beam-beam interactions", APAC 2004

Diffusion Process caused by Beam-Beam Interaction

- Diffusion Equation with a stochastic kick:

$$\frac{\partial}{\partial s}\Psi(x,s) = B\frac{\partial^2}{\partial x^2}\Psi(x,s)$$

- For initial condition, $\Psi(x,0) = \delta(x - x_0)$, the solution is given as

$$\Psi = \frac{1}{\sqrt{4\pi Bs}}\exp\left[-\frac{(x-x_0)^2}{4Bs}\right]$$

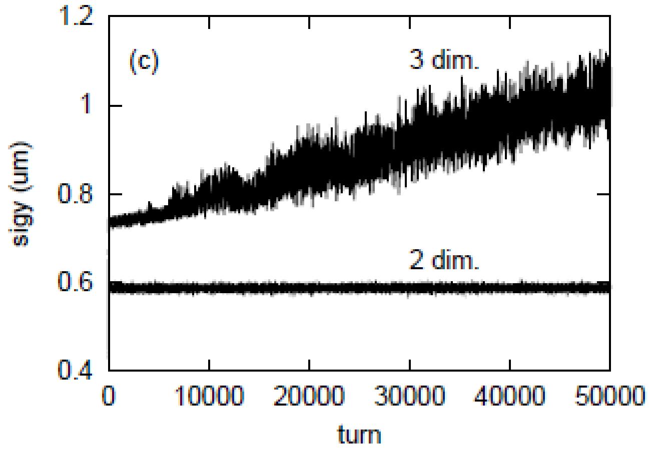
which means Ψ is Gaussian and its rms value increase as $\sigma^2 = 2Bs$

- With damping $\frac{dx}{ds} = -Dx$, the diffusion equation is replaced by Fokker-Plank equation

$$\frac{\partial}{\partial s}\Psi(x,s) = Dx\frac{\partial}{\partial x}\Psi(x,s) + B\frac{\partial^2}{\partial x^2}\Psi(x,s)$$

and the equilibrium solution is $\Psi = \exp\left[-\frac{x^2}{2B/D}\right]$

- In most of the circular accelerators, the betatron motion is much faster than the diffusion and damping, therefore the same discussion is applied by using the adiabatic invariant $\sqrt{J_x}$ instead of *x*.
- If there are some diffusion mechanisms, which are independent each other, the total diffusion coefficient is summation of each diffusion coefficient, $B = \sum B_i$



MEASUREMENTS OF TRANSVERSE BEAM HALO DIFFUSION

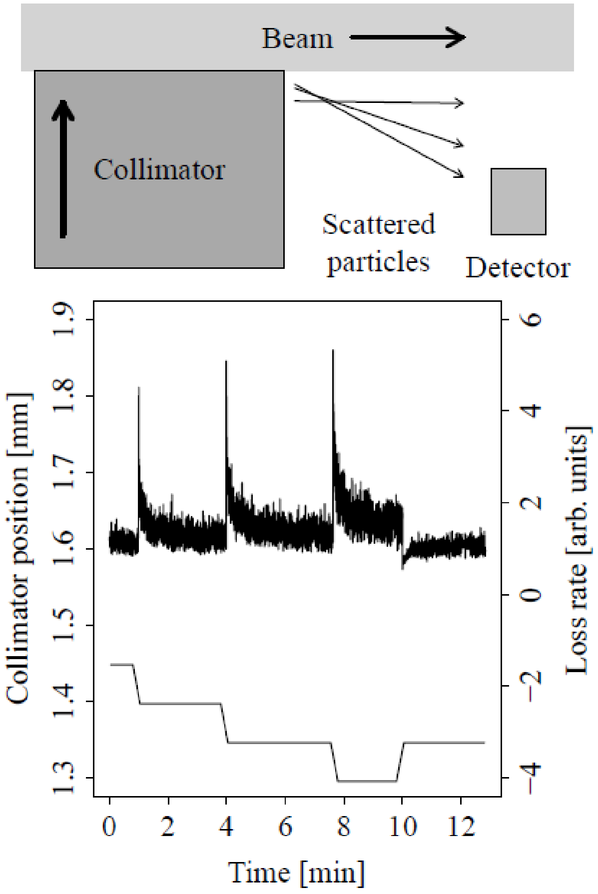
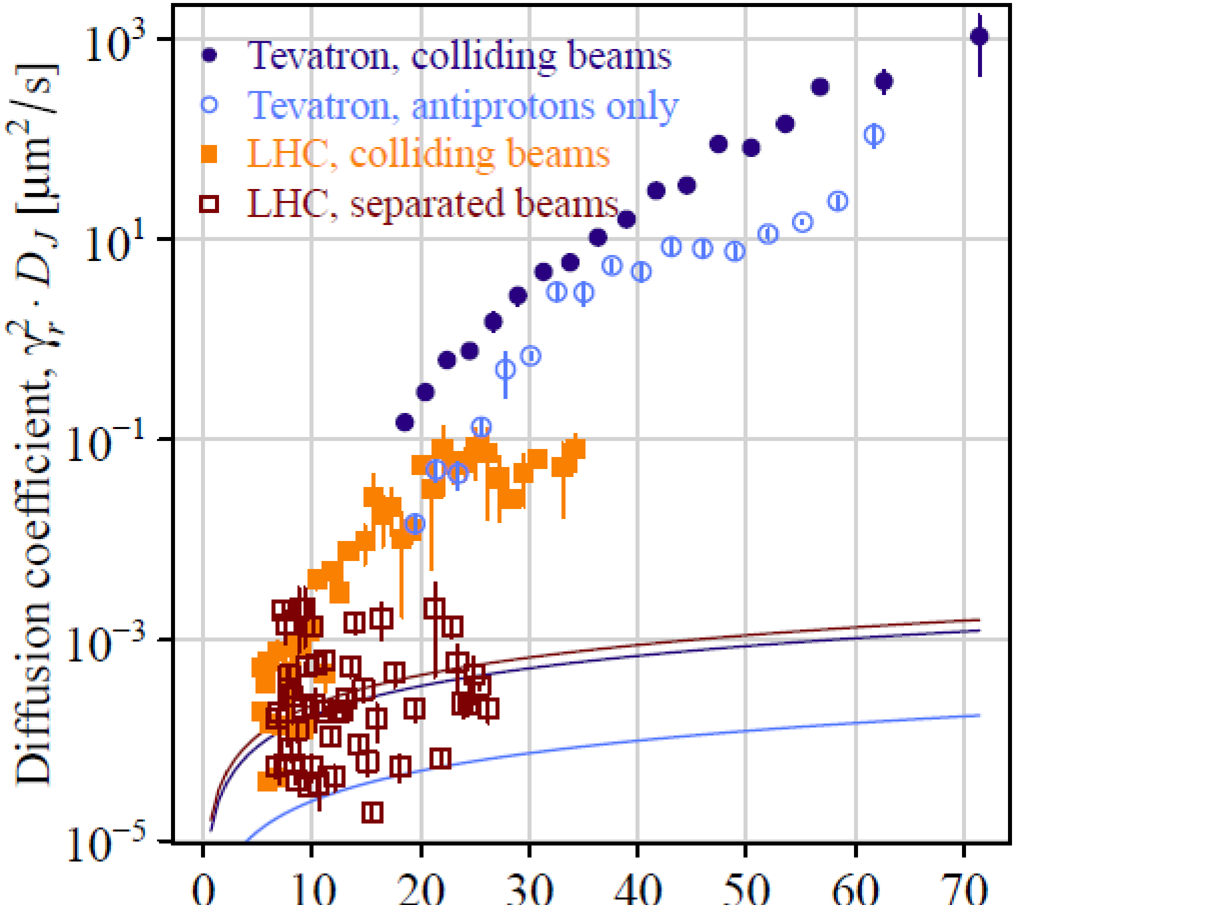


Figure 1: Schematic diagram of the apparatus (top). Example of the response of local loss rates to inward and outward collimator steps (bottom).

the particle loss rate at the collimator is equal to the flux at that location:

$$\mathrm{L} = -D\left[\partial_J f\right]_{J=J_C}$$

with phase-space density *f(J, t)* described by the diffusion equation, where *J* is the Hamiltonian action and *D* the diffu coefficient in action space.



Action near Resonances

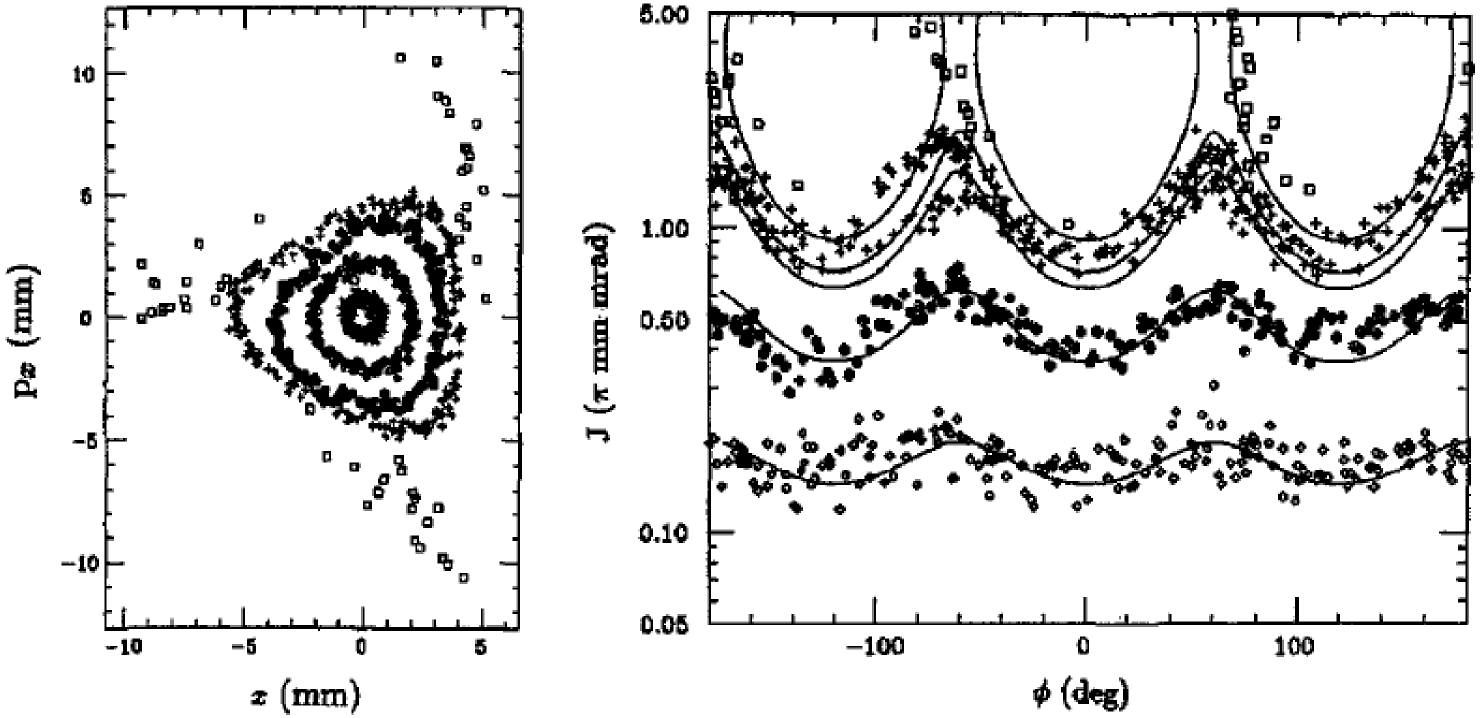


Figure 2.51: Left: The measured Poincaré map of the normalized phase-space coordinates (x, p_x) of betatron motion near a third-order resonance $3\nu_x = 11$ at the IUCF cooler ring. Note that particles outside the separatrix survive only about 100 turns. Tori for particles inside the separatrix are distorted by the third order resonance. The orientation of the Poincaré map, determined by sextupoles, rotates at a rate of betatron phase advance along the ring. The right plot shows the Poincaré map in action-angle variables (J, ϕ). The solid lines are Hamiltonian tori of Eq. (2.392).

Synchrotron Radiation

K. Hirata and F. Ruggiero, “Treatment of Radiation for Multiparticle Tracking in Electron Storage Rings”, 1989

$$\begin{pmatrix} X \\ P \end{pmatrix}_{out} = \lambda U(\mu) \begin{pmatrix} X \\ P \end{pmatrix}_{in} + \sqrt{\epsilon(1-\lambda^2)} \begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \end{pmatrix}.$$

Let us calculate the ratio of radiation losses in quadrupoles and dipoles per unit FODO cell, which contains one QF and QD each. Using Eqs. (C3)–(C5),

$$\begin{aligned} \frac{\langle P_\mathrm{Q} \rangle}{\langle P_\mathrm{D} \rangle} &= \frac{\langle P_\mathrm{QF} \rangle + \langle P_\mathrm{QD} \rangle}{\langle P_\mathrm{D} \rangle} \\ &= \frac{\ell_c}{2\theta_c^2} \left(\beta_{x\mathrm{QF}} \frac{k_\mathrm{QF}^2}{\ell_\mathrm{QF}} + \beta_{x\mathrm{QD}} \frac{k_\mathrm{QD}^2}{\ell_\mathrm{QD}} \right) n^2 \varepsilon_x \\ &\equiv R_\mathrm{Q} n^2 \varepsilon_x, \end{aligned} \qquad \text{(C6)}$$

where we have used $n \equiv \Delta x/\sigma_x = \Delta x/\sqrt{\beta_{x\mathrm{QF}}\varepsilon_x}$. Then the

K. Oide and etc., PRAB, 19, 111005, 2016

$$R_\mathrm{Q} = \frac{2\sqrt{2}}{\theta_c^2} \left(\frac{\sqrt{2}+1}{\ell_\mathrm{QF}} + \frac{\sqrt{2}-1}{\ell_\mathrm{QD}} \right)$$

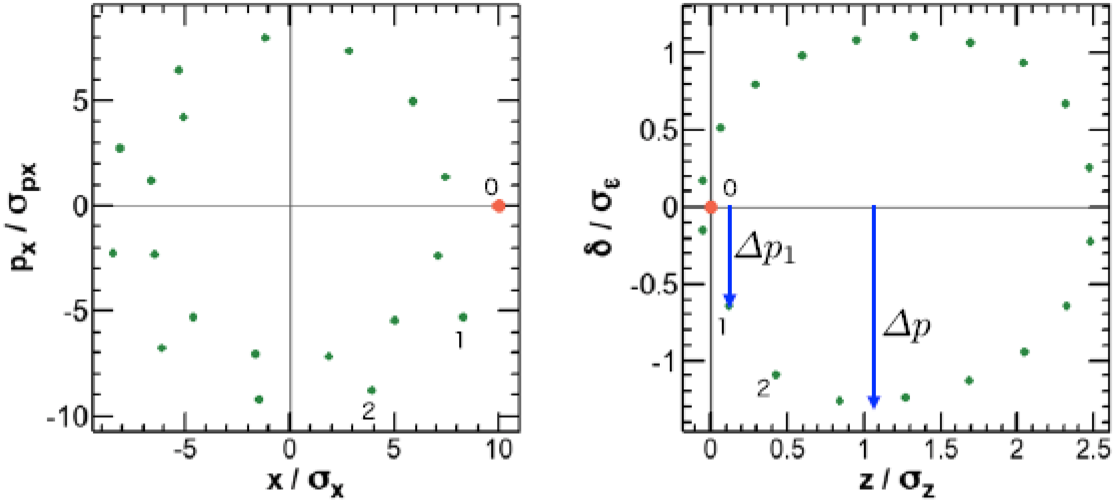
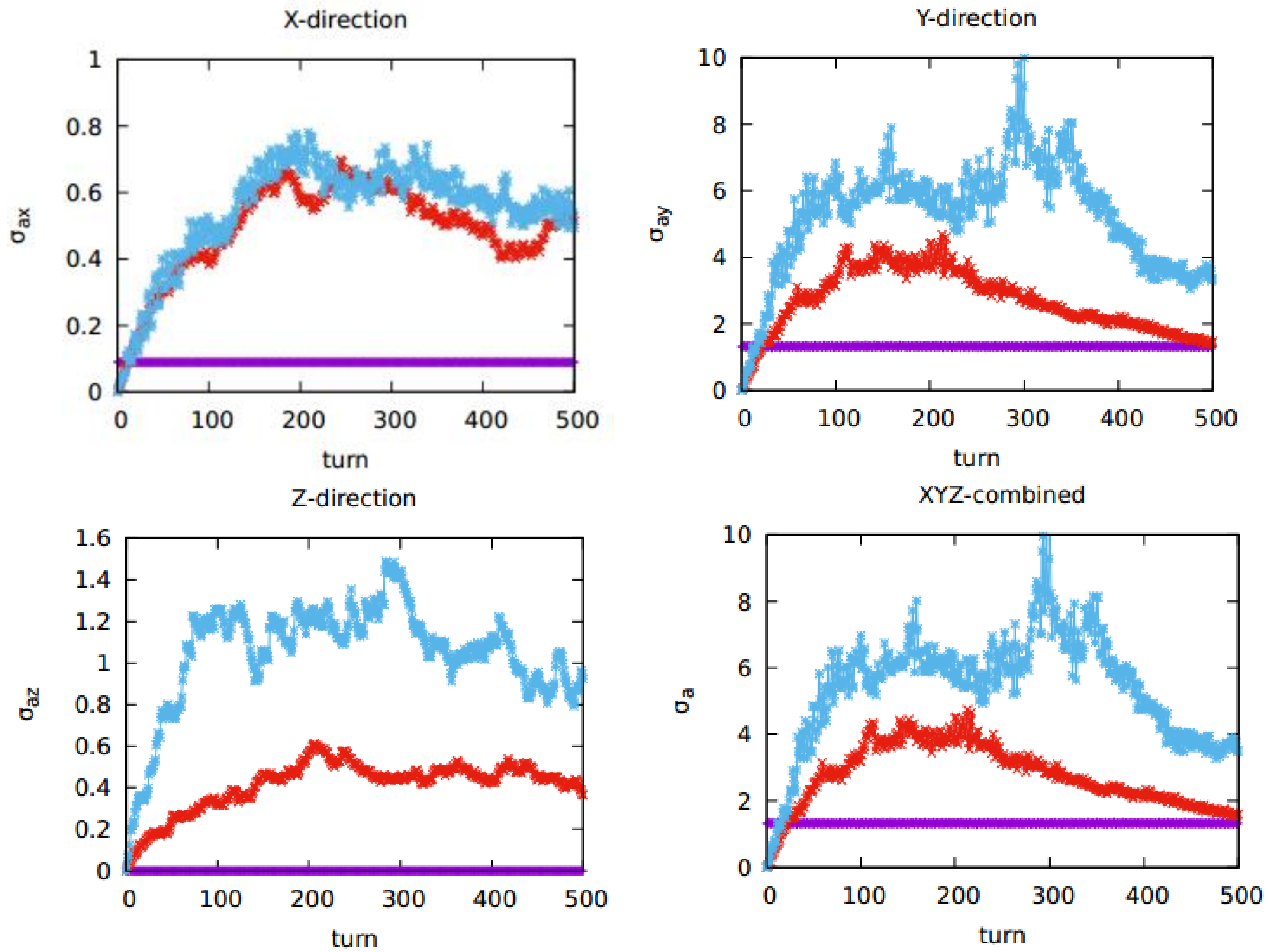
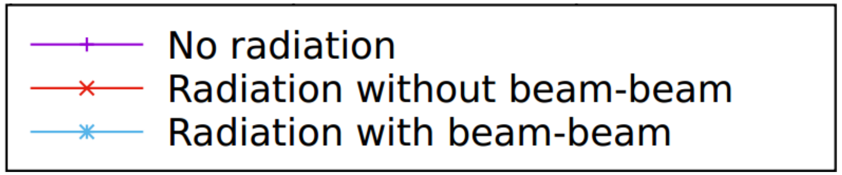


FIG. 9. Poincaré plots in $x - p_x$ (left) and $z - \delta$ (right) planes for a particle starting at $x = 10\sigma_x$, $p_x = y = p_y = z = \delta = 0$, depicted by the red dots. The numbers, 0, 1, 2 are turns. The synchrotron radiation loss in the quadrupoles excites the synchrotron motion as shown in the right plot. The amount of the energy loss in the first turn Δp_1, and the peak amplitude of the synchrotron motion Δp agrees with the estimation, Eq. (C10).

Diffusion with Different Model



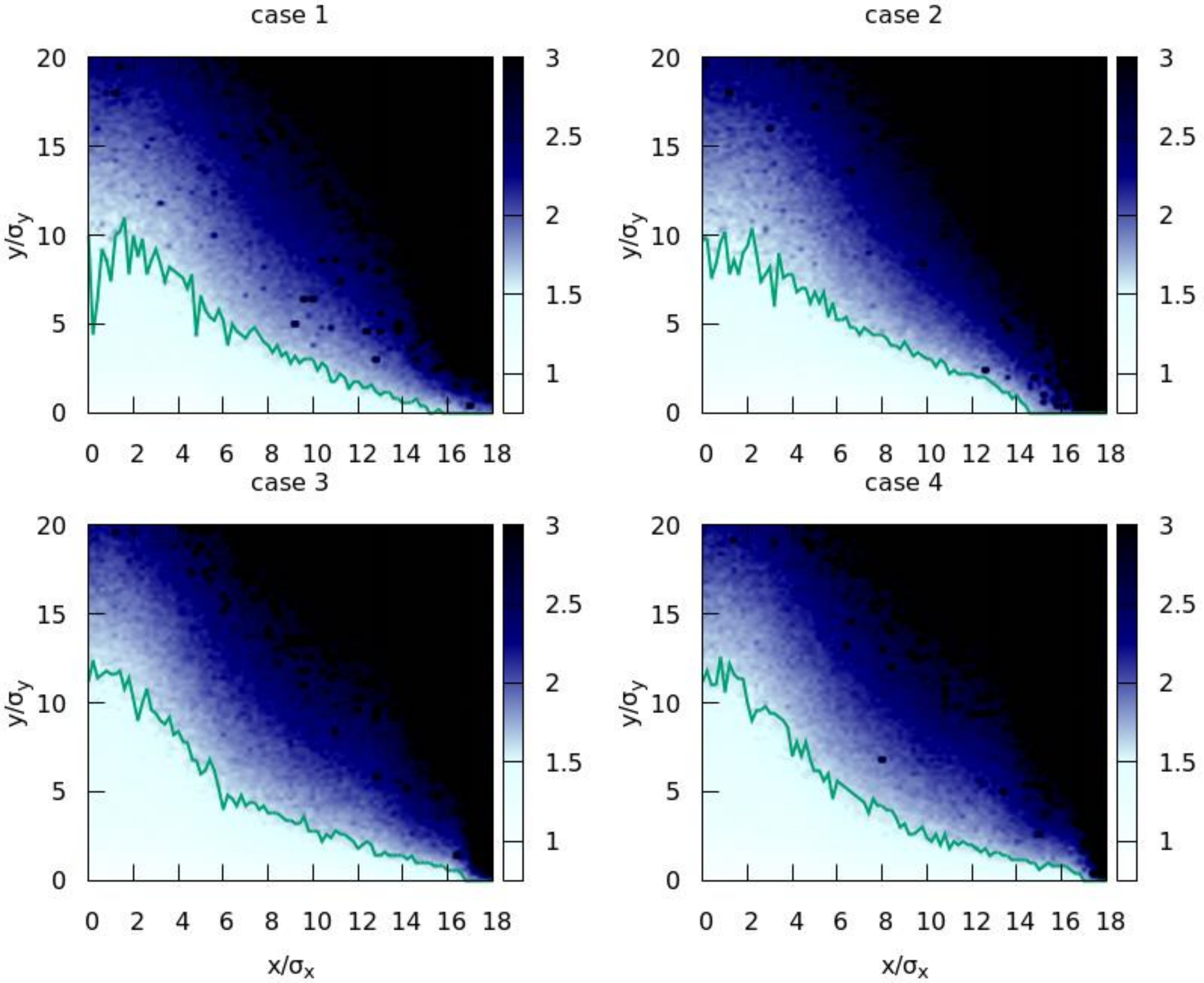


200 particles are tacked,
With same initial coordinate: $(3\sigma_x, 11\sigma_y)$

$$Ai \overset{\text{def}}{=} \sqrt{\frac{2J_i}{\varepsilon_i}} \qquad i = x, y, z$$

$$\sigma_a \overset{\text{def}}{=} \sqrt{\sigma_{ax}^2 + \sigma_{ay}^2 + \sigma_{az}^2}$$

Diffusion Map Analysis of Different Lattice

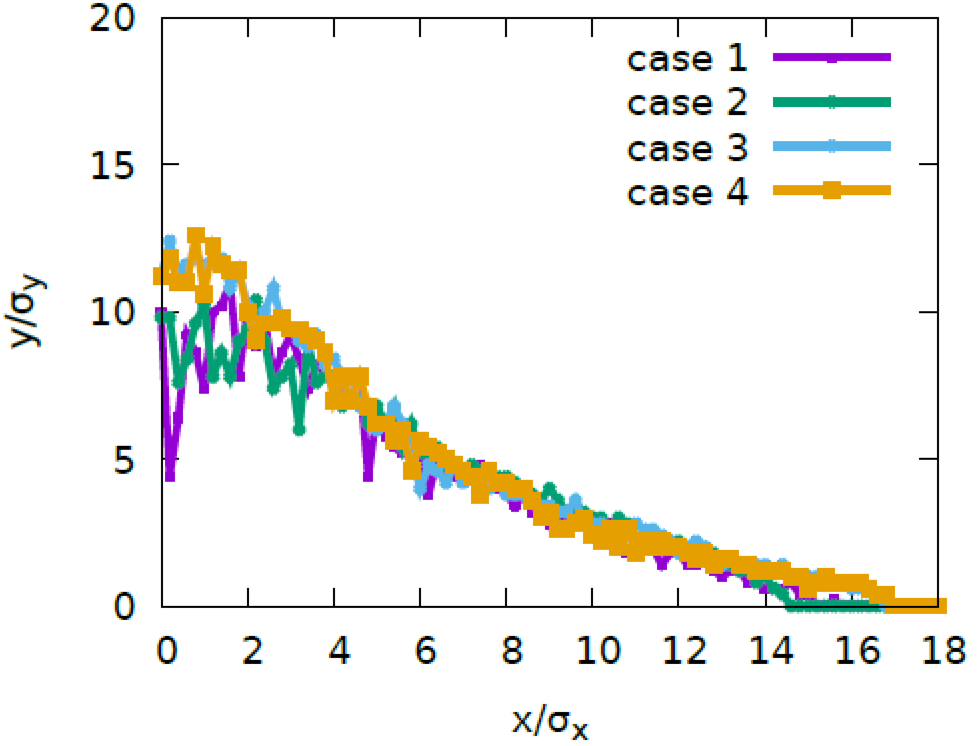


200 particles, 25 turns

$$D \equiv \log_{10}\left(\sum_{turn} \sigma_a^2\right)$$

The green lines show the boundary of D=1.6.

Slower diffusion indicates fewer halo particles.



Summary

- The beam-beam effect of CEPC CDR is briefly introduced
- The longitudinal impedance reduces the beam-beam limit according to the initial result.
- We attempt to present a diffusion map analysis method, which help to judge if one lattice is good in a non-symplectic condition. The initial result shows agreement with halo particles distribution obtained with many particle, long turns tracking.