

# Off-momentum Optics at SuperKEKB

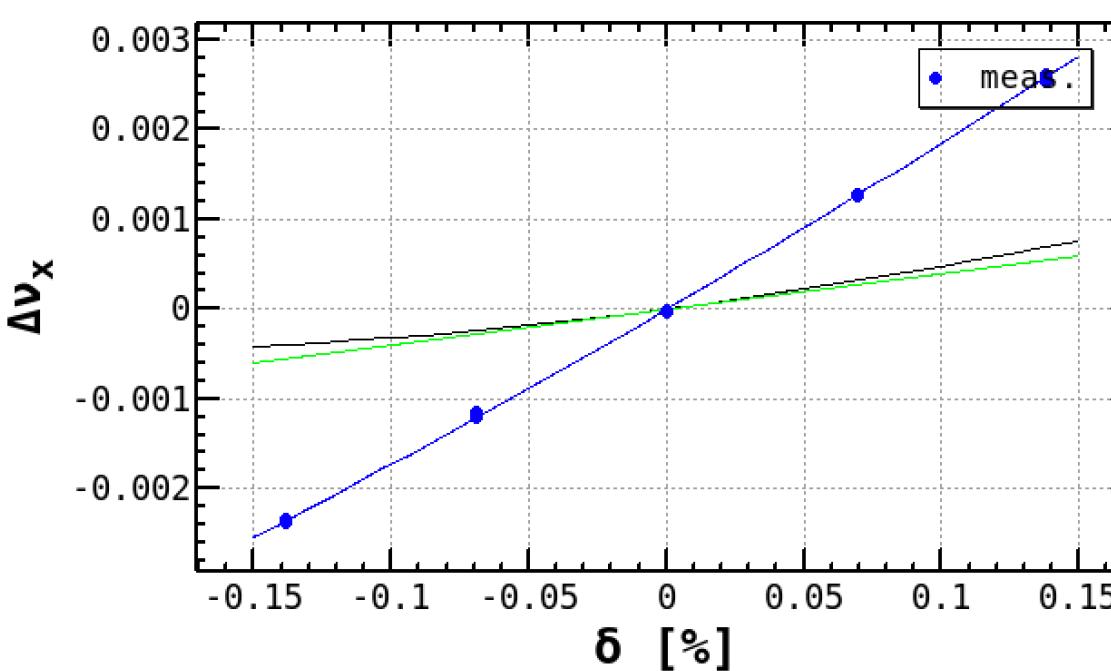
1. Chromatic phase-advance
2. Chromatic XY coupling

Y. Ohnishi

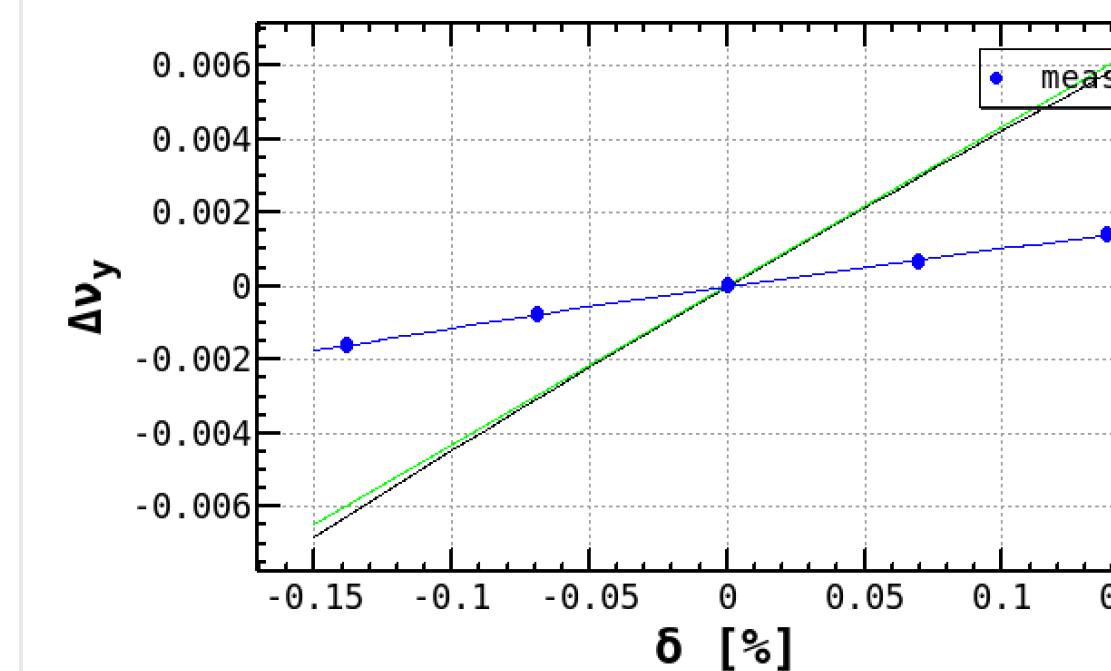
This work was supported by JSPS KAKENHI Grant Number 17K05475.

**LER**  $\beta_x^* = 100 \text{ mm}$   $\beta_y^* = 4 \text{ mm}$

$\xi_x: 1.787(\text{meas})/ 0.401(\text{model})$

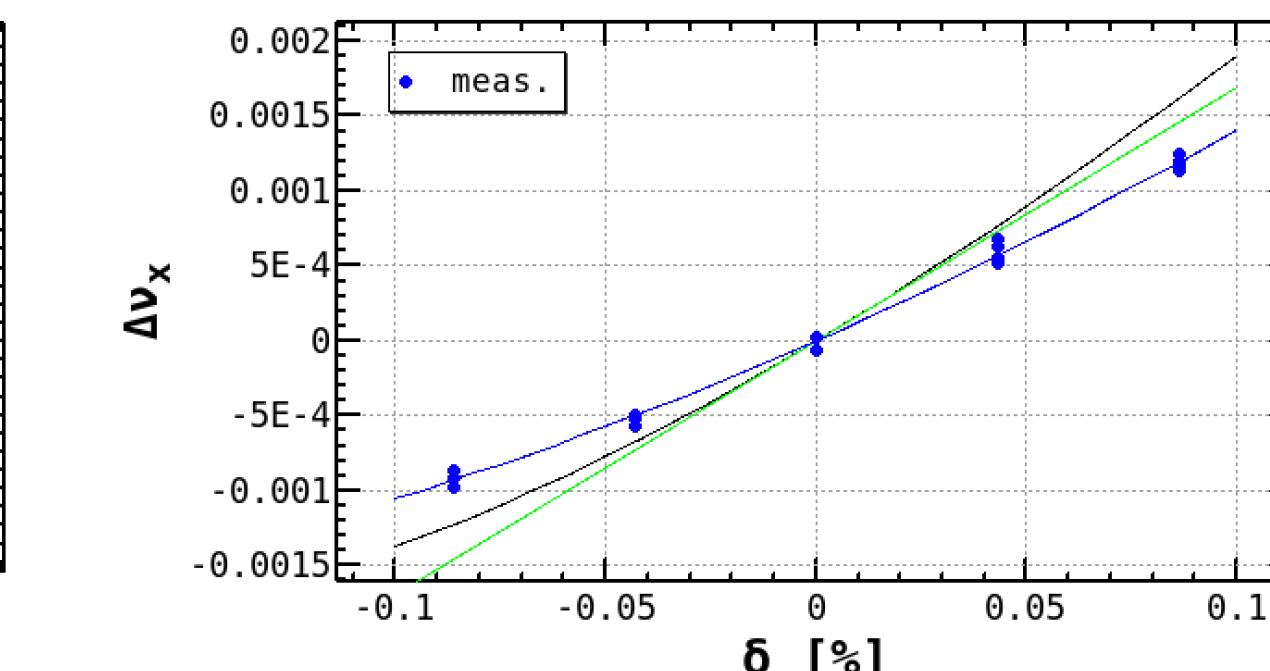


$\xi_y: 1.080(\text{meas})/ 4.327(\text{model})$

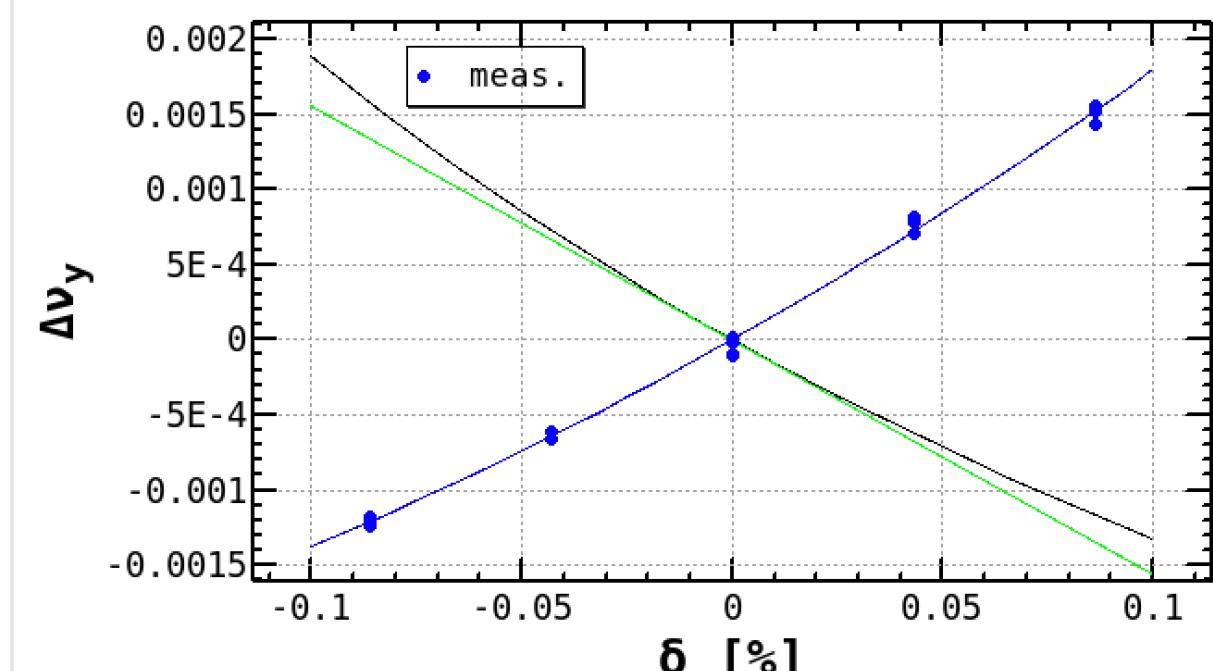


**HER**  $\beta_x^* = 100 \text{ mm}$   $\beta_y^* = 4 \text{ mm}$

$\xi_x: 1.234(\text{meas})/ 1.691(\text{model})$



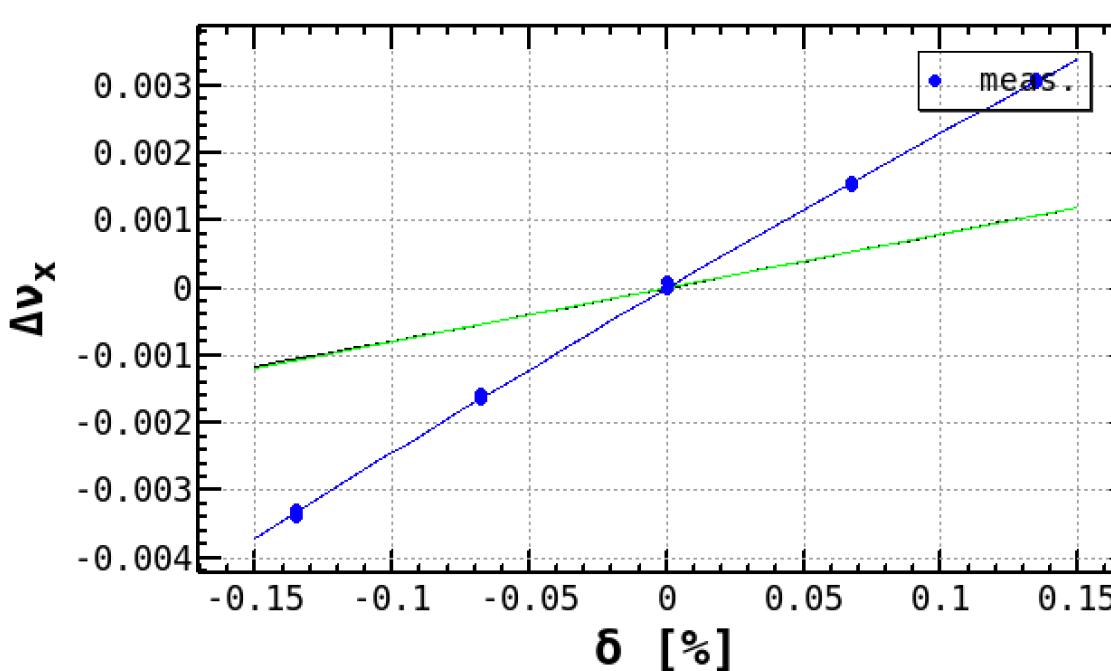
$\xi_y: 1.587(\text{meas})/ -1.560(\text{model})$



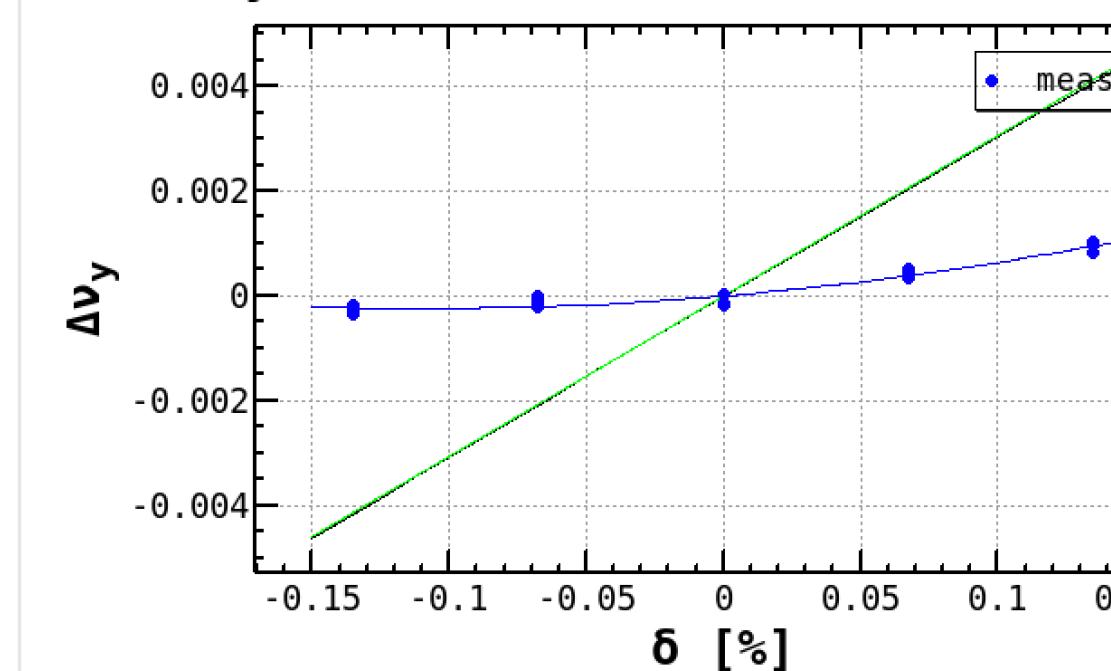
**blue: measurement / black(green): model**

**LER**  $\beta_x^* = 200 \text{ mm}$   $\beta_y^* = 4 \text{ mm}$

$\xi_x: 2.368(\text{meas})/ 0.793(\text{model})$

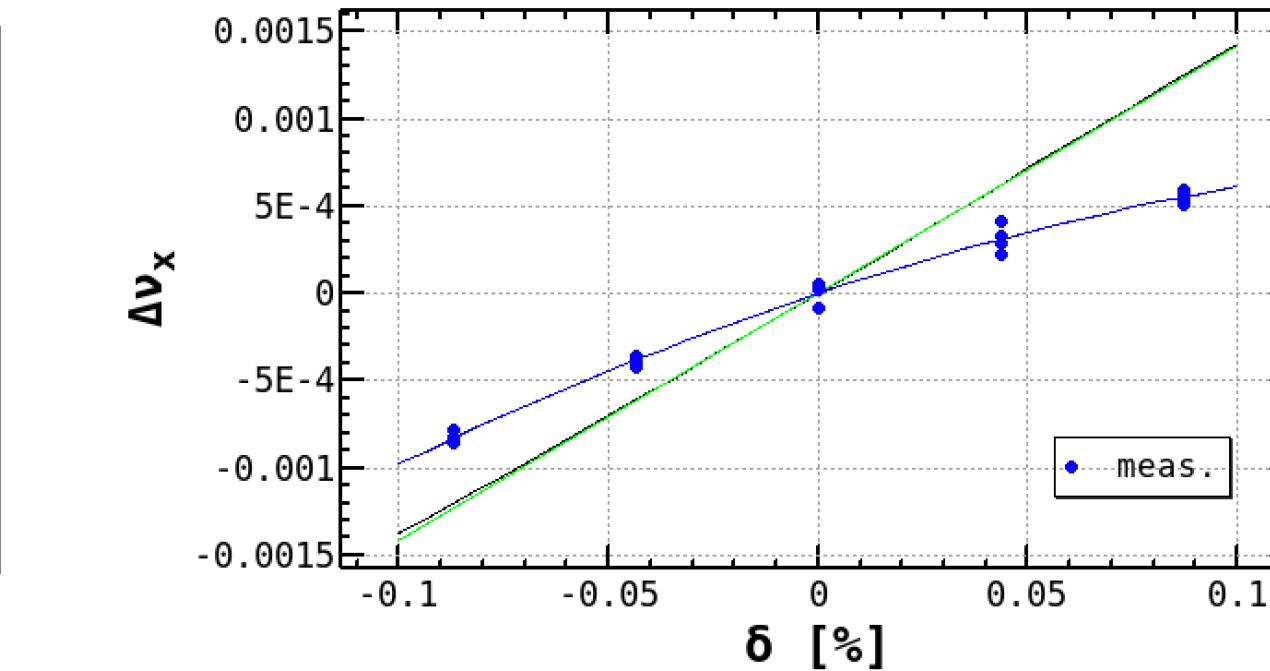


$\xi_y: 0.438(\text{meas})/ 3.071(\text{model})$

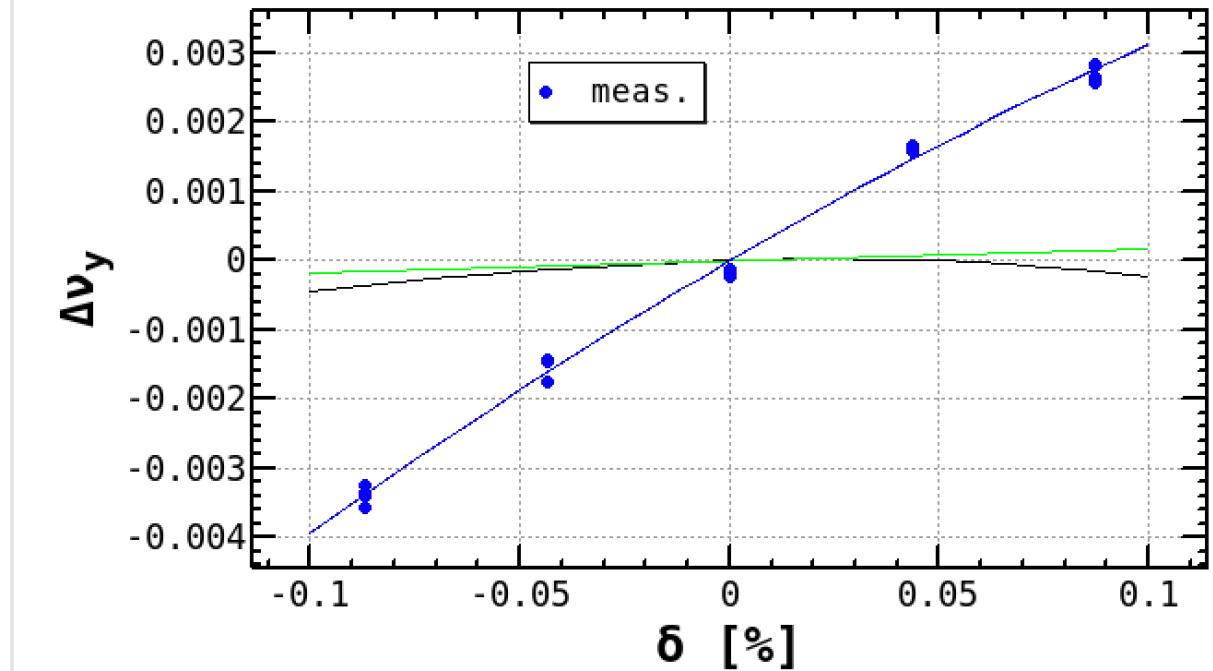


**HER**  $\beta_x^* = 200 \text{ mm}$   $\beta_y^* = 2 \text{ mm}$

$\xi_x: 0.794(\text{meas})/ 1.421(\text{model})$



$\xi_y: 3.533(\text{meas})/ 0.171(\text{model})$



# Why there is a discrepancy between model and measurement ?

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## Natural chromaticity in Phase 2

$$\begin{aligned} \text{LER : } \xi_x &= -69 / \xi_y = -146 \\ \text{HER : } \xi_x &= -97 / \xi_y = -168 \end{aligned}$$

$$\begin{aligned} \text{QC2: } \xi_x &= -8 / \text{QC1: } \xi_y = -67 \\ \text{QC2: } \xi_x &= -25 / \text{QC1: } \xi_y = -100 \end{aligned}$$

$$\begin{aligned} \text{LER : } \Delta\xi_x &= 1\sim1.5 / \Delta\xi_y = 2.5\sim3.3 \\ \text{HER : } \Delta\xi_x &= 0.5\sim0.7 / \Delta\xi_y = 3\sim3.3 \end{aligned}$$

between model and real machine

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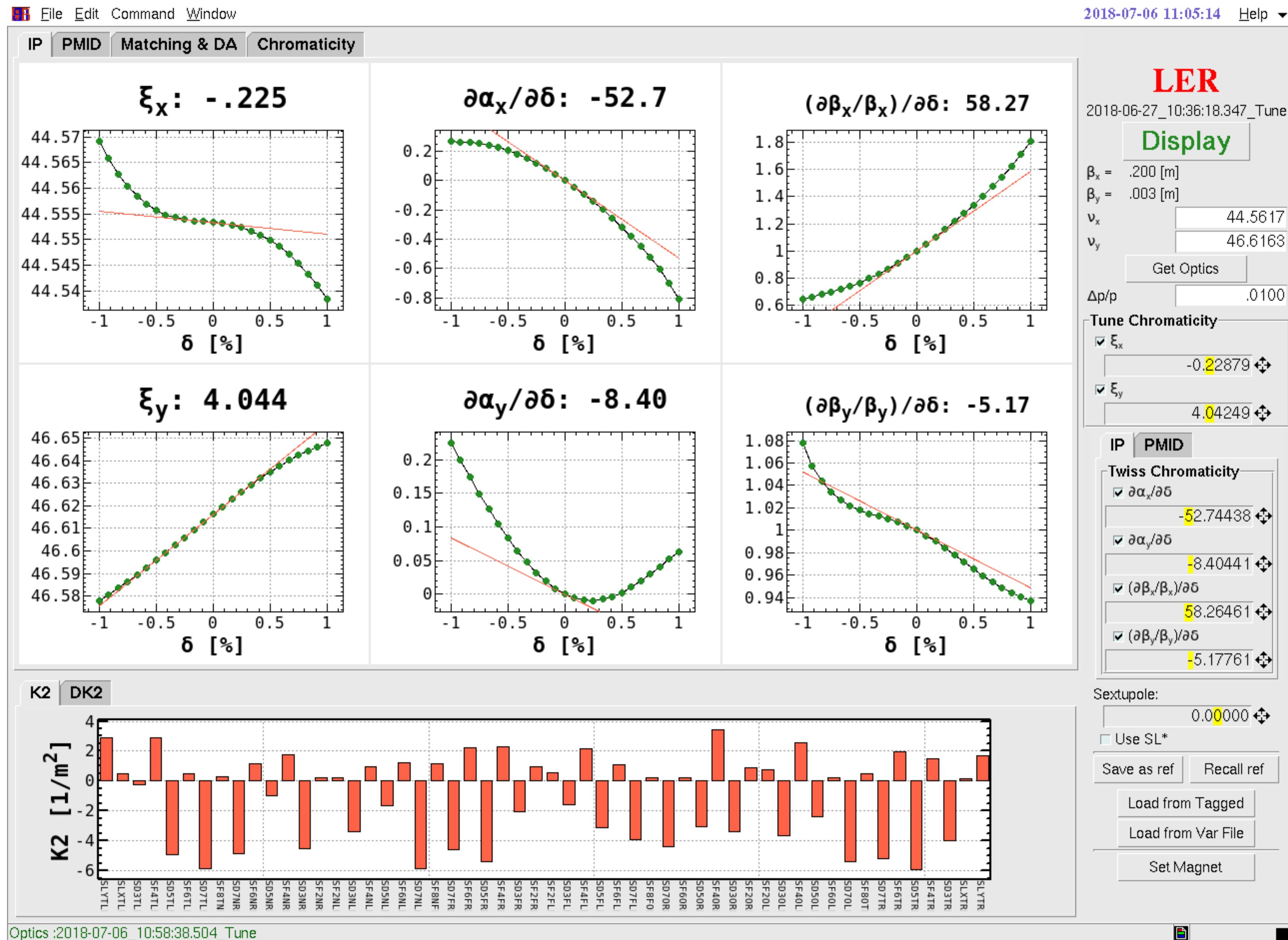
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between model and real machine

1 ~ 2 % discrepancy is not so large ?



Control chromaticity and Twiss chromaticity

Relative change is OK.

# Measurement of Optical Functions based on COD

$$\begin{aligned}\Delta x_i^j &= \frac{\Delta\theta_j}{2\sin(\pi\nu)} \sqrt{\beta_i} \sqrt{\beta_i} \cos(|\psi_i - \psi_j| - \pi\nu) & i = 1 \sim N \quad j = 1 \sim M \\ &= A_j \{C_j (\cos \pi\nu + a_{ij} S_i \sin \pi\nu) + S_j (\cos \pi\nu - a_{ij} C_i \sin \pi\nu)\}\end{aligned}$$

**Fitting parameters:**

$$\begin{aligned}A_j &= \frac{\Delta\theta_j}{2\sin \pi\nu} \\ C_{i,j} &= \sqrt{\beta_{i,j}} \cos \psi_{i,j} \\ S_{i,j} &= \sqrt{\beta_{i,j}} \sin \psi_{i,j} \\ a_{ij} &= sign(\psi_i - \psi_j)\end{aligned}$$

$$N \times M > 2(M + N)$$

$$N = \sim 450 \text{ BPMs}, \quad M = 6 \text{ steerings}$$

**Least-square fitting:**  $\chi^2 = \sum_{i,j}^{N,M} (\Delta x_i^j - \Delta x_{i,meas}^j)^2$  **with**

$$\sum_{i=1}^N \frac{1}{\beta_i} = \sum_{i=1}^N \frac{1}{\beta_{i,design}}$$

## Definition of chromatic phase-advance:

$$\Delta\psi_{x,i} = \psi_{x,i} - \psi_{x,i-1}$$

$$\chi_i(x) = \frac{1}{2\pi} \frac{\partial \Delta\psi_{x,i}}{\partial \delta}$$

$$\xi_x = \frac{\partial \nu_x}{\partial \delta}$$

$$\Delta\psi_{y,i} = \psi_{y,i} - \psi_{y,i-1} \quad i = 2, 3, 4, \dots, N$$

$$\chi_i(y) = \frac{1}{2\pi} \frac{\partial \Delta\psi_{y,i}}{\partial \delta} \quad N = 440 - 460 \text{ (#BPMs)}$$

$$\xi_y = \frac{\partial \nu_y}{\partial \delta} \quad \longleftarrow \text{Chromaticity}$$

$$\begin{pmatrix} \chi_{1,m}(x) - \chi_{1,d}(x) \\ \chi_{2,m}(x) - \chi_{2,d}(x) \\ \vdots \\ \chi_{N,m}(x) - \chi_{N,d}(x) \\ \chi_{1,m}(y) - \chi_{1,d}(y) \\ \chi_{2,m}(y) - \chi_{2,d}(y) \\ \vdots \\ \chi_{N,m}(y) - \chi_{N,d}(y) \\ \xi_{x,m} - \xi_{x,d} \\ \xi_{y,m} - \xi_{y,d} \end{pmatrix}$$

$$= M_{resp} \begin{pmatrix} \Delta K_{2,1}/K_{2,1} \\ \Delta K_{2,2}/K_{2,2} \\ \vdots \\ \Delta K_{2,M}/K_{2,M} \end{pmatrix}$$

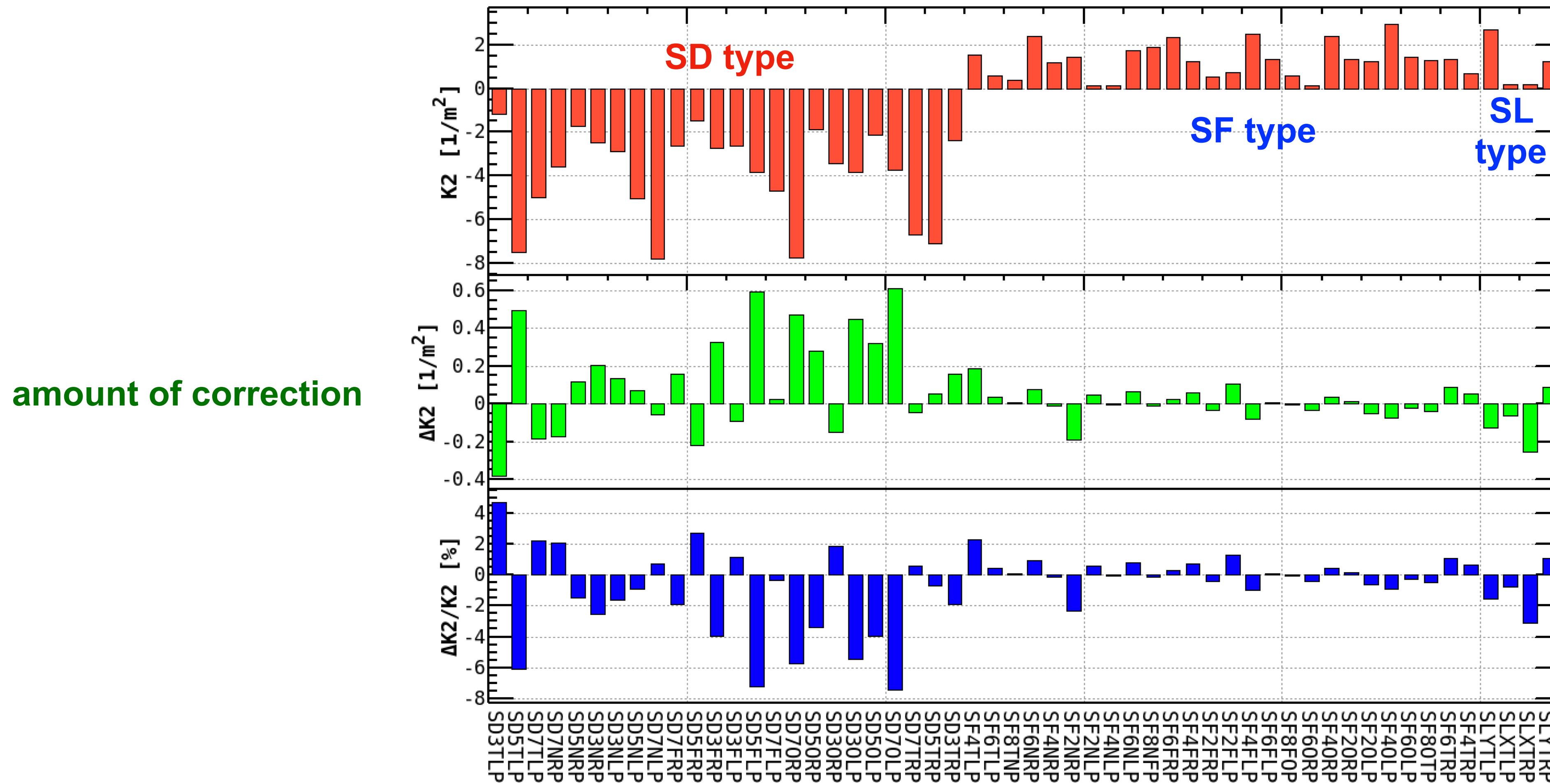
SD + SF + SL : M = 54 (#families)

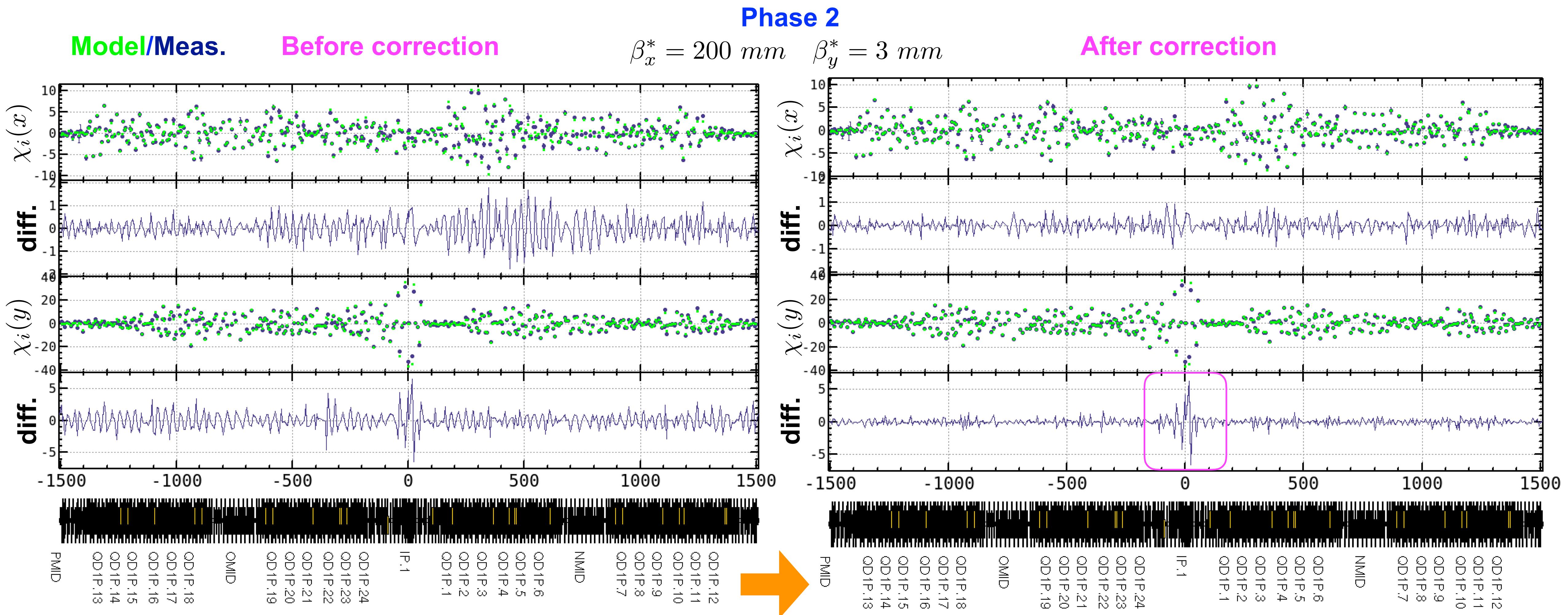
We solve these equations to obtain  $\Delta K_2$ .

$$a_f = \frac{K_2 + \Delta K_2}{K_2}$$

# We have 54 families (pairs

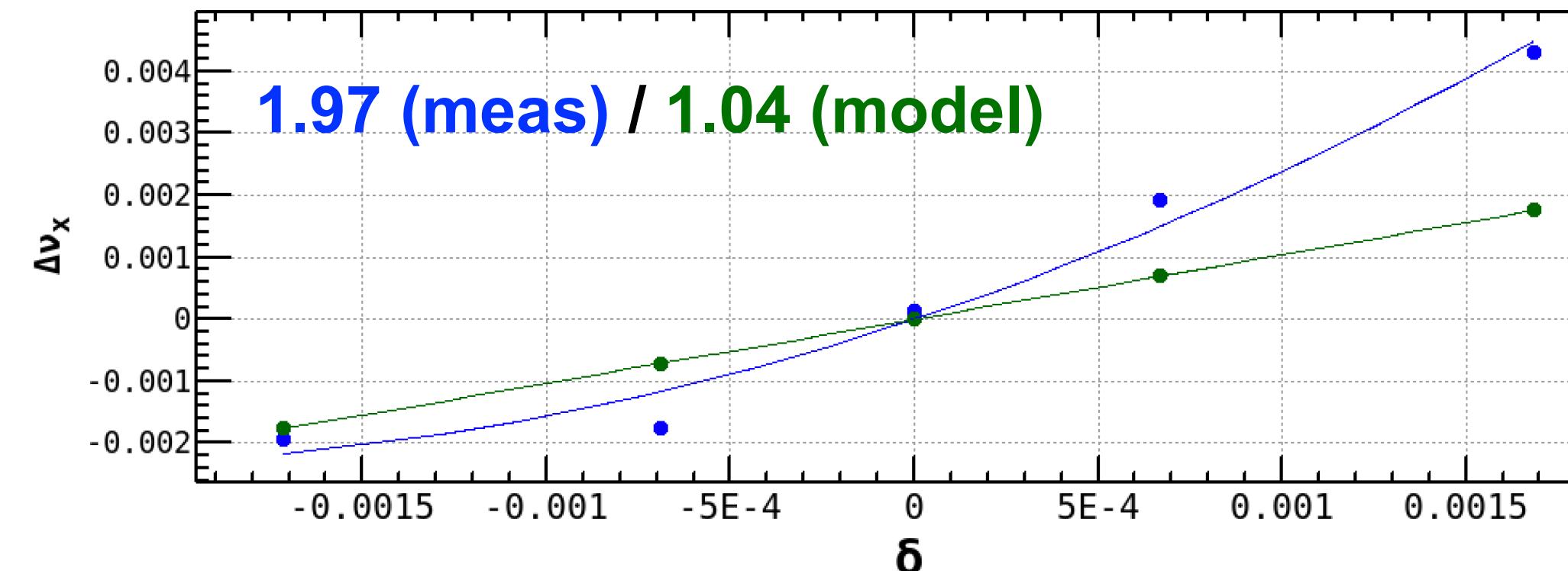
# LER



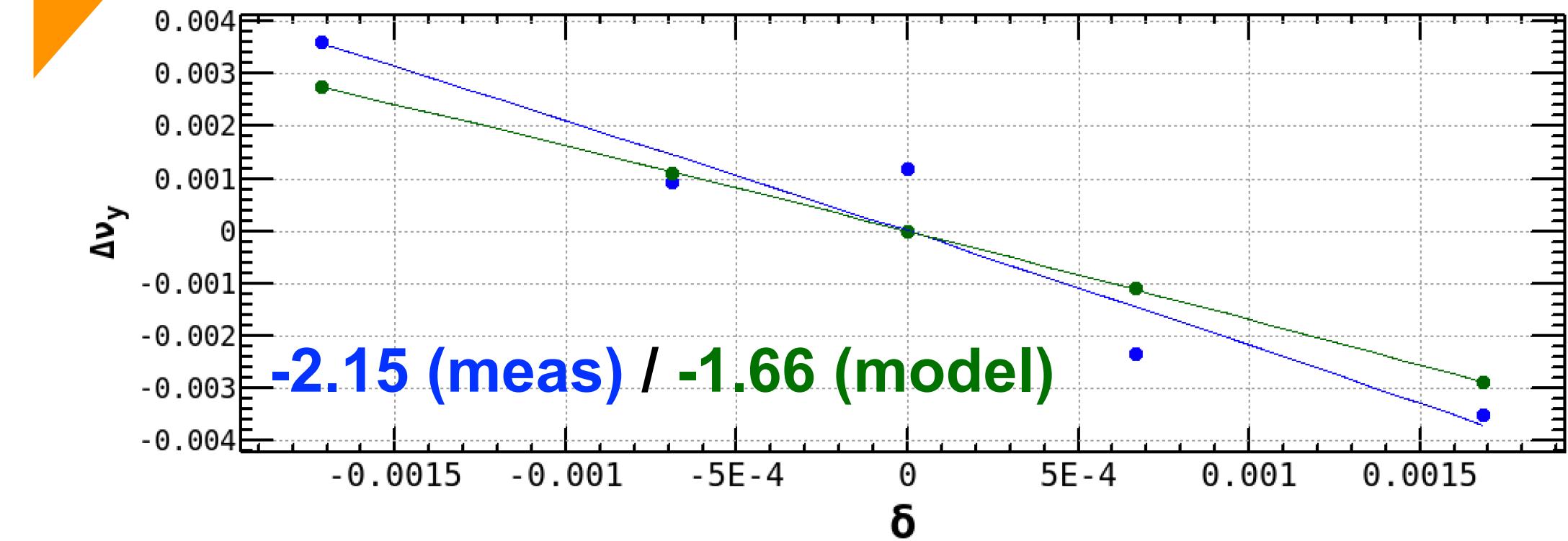
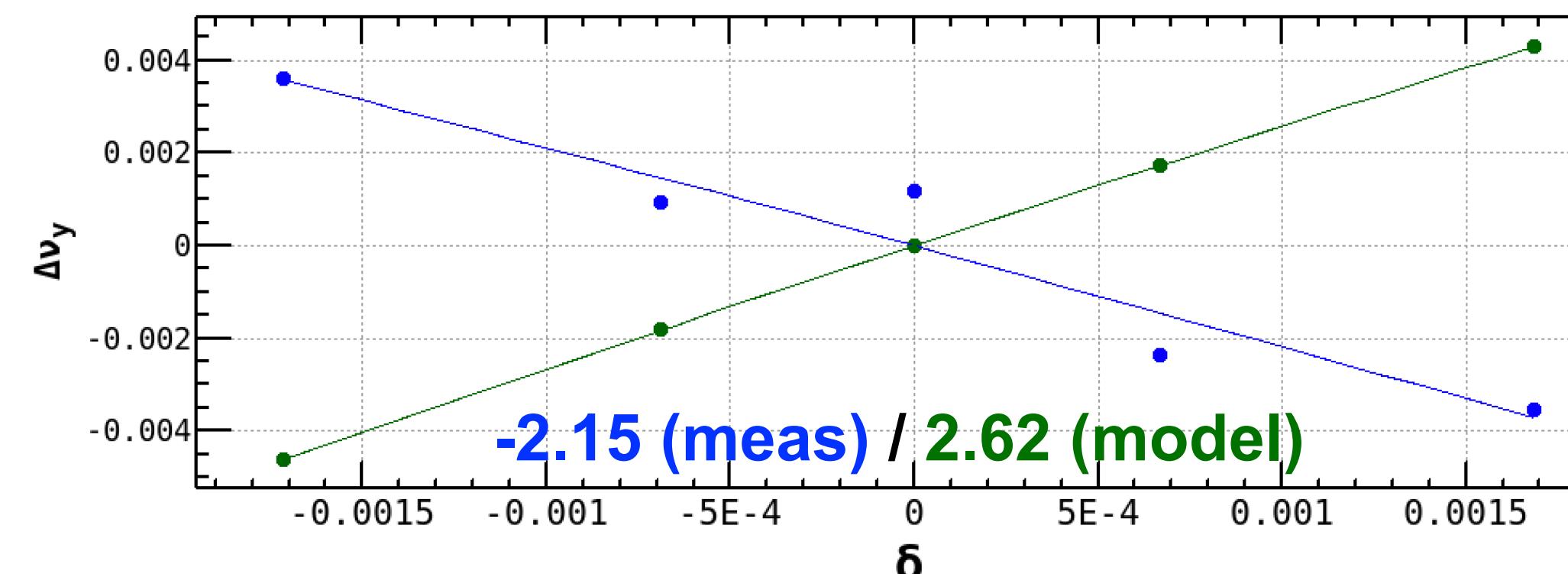
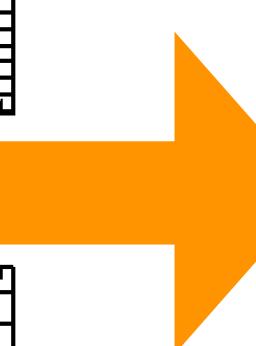
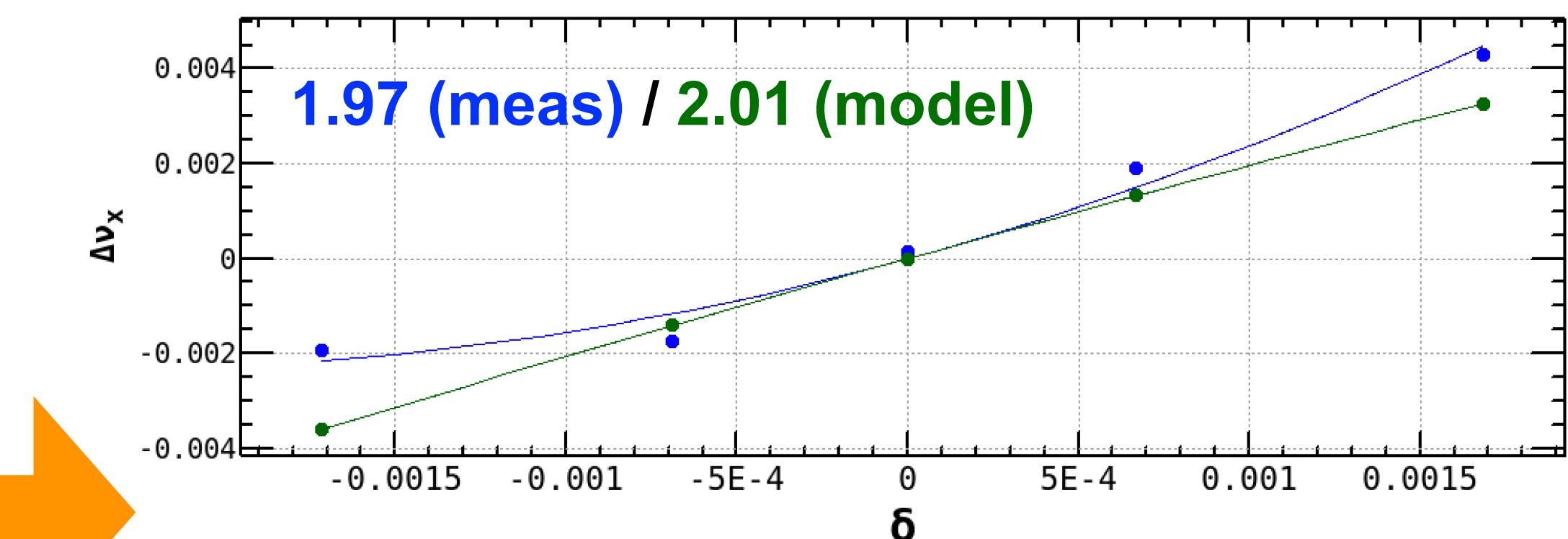


**The correction is successfully performed except for the IR.**

**Before correction**

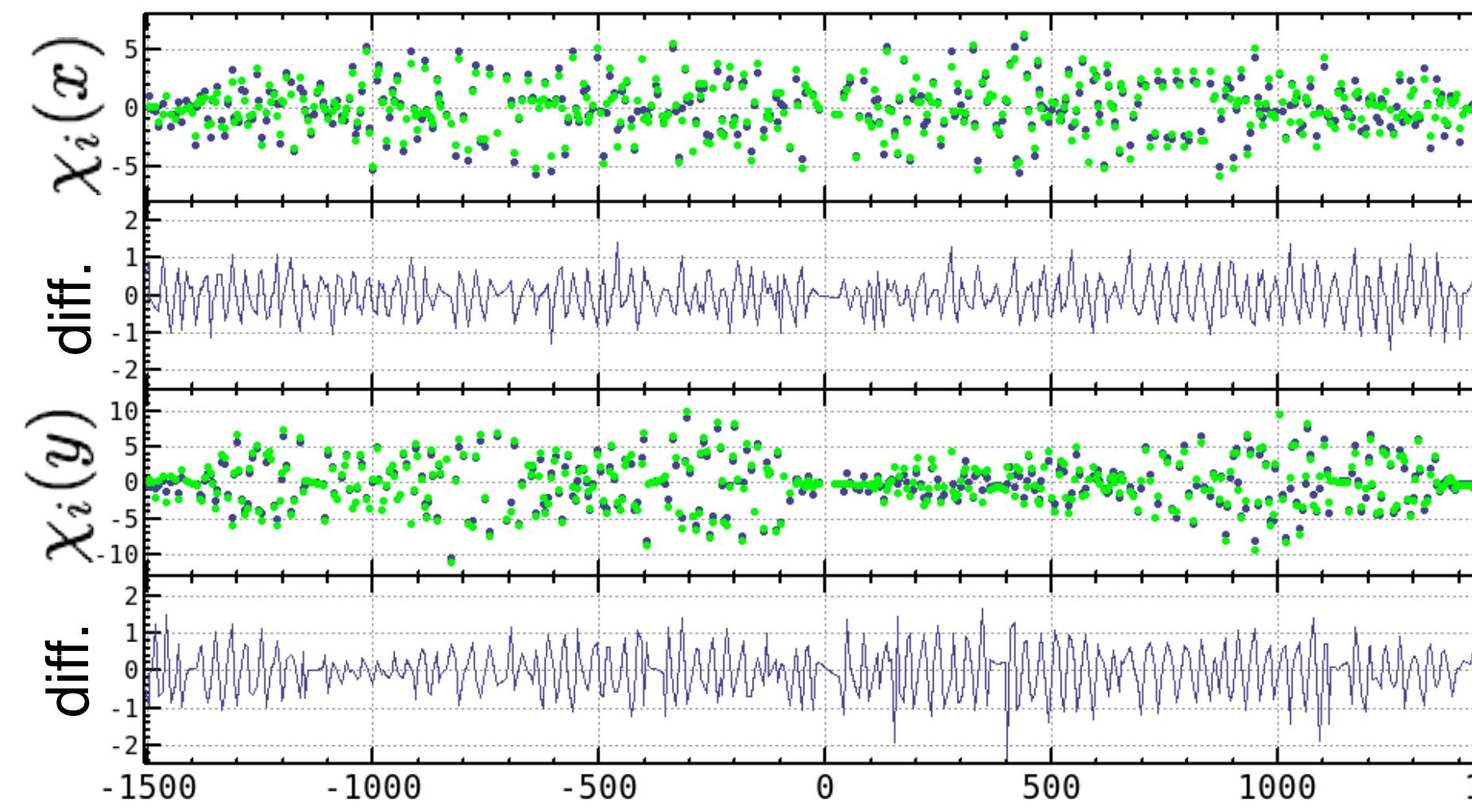


**After correction**

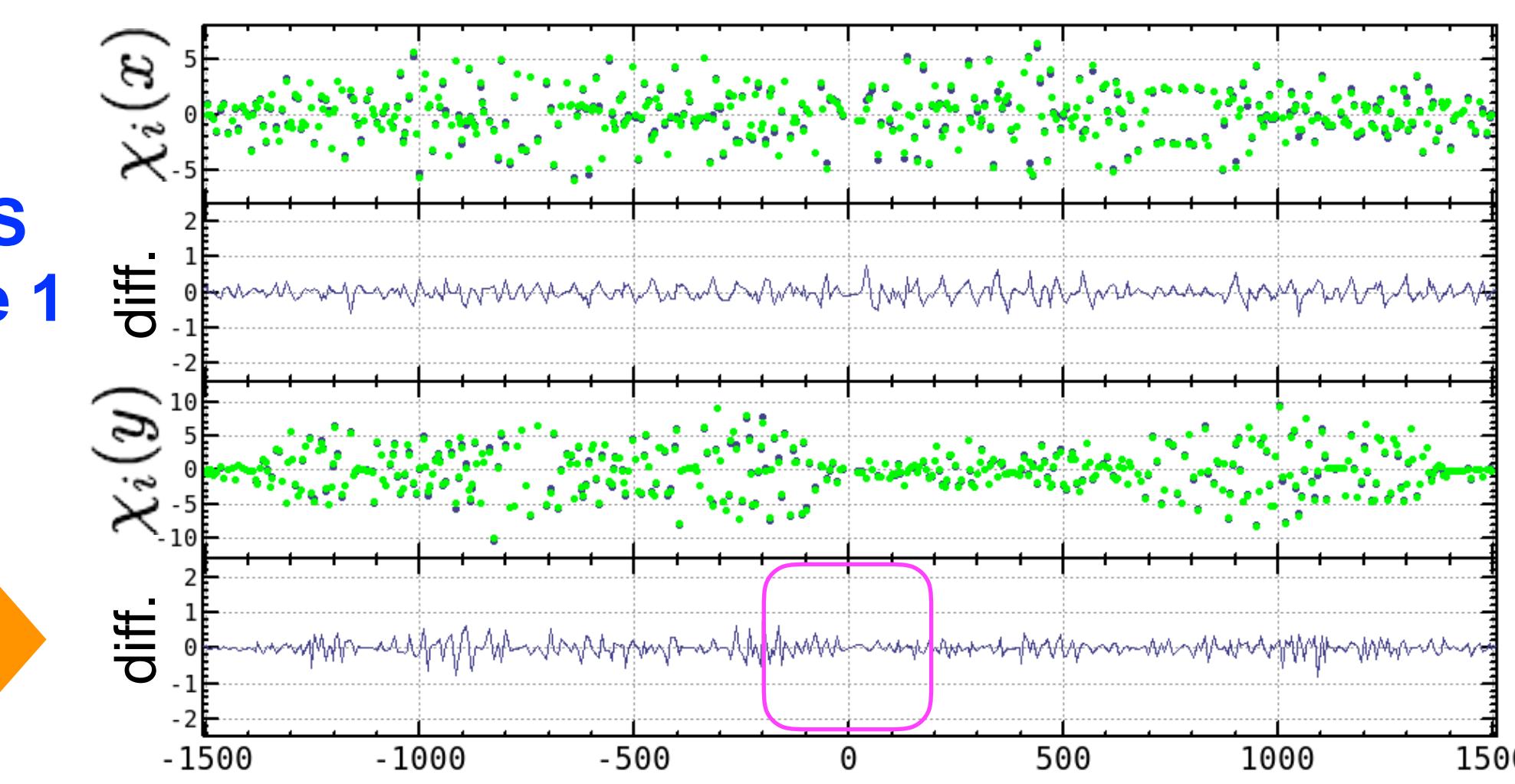
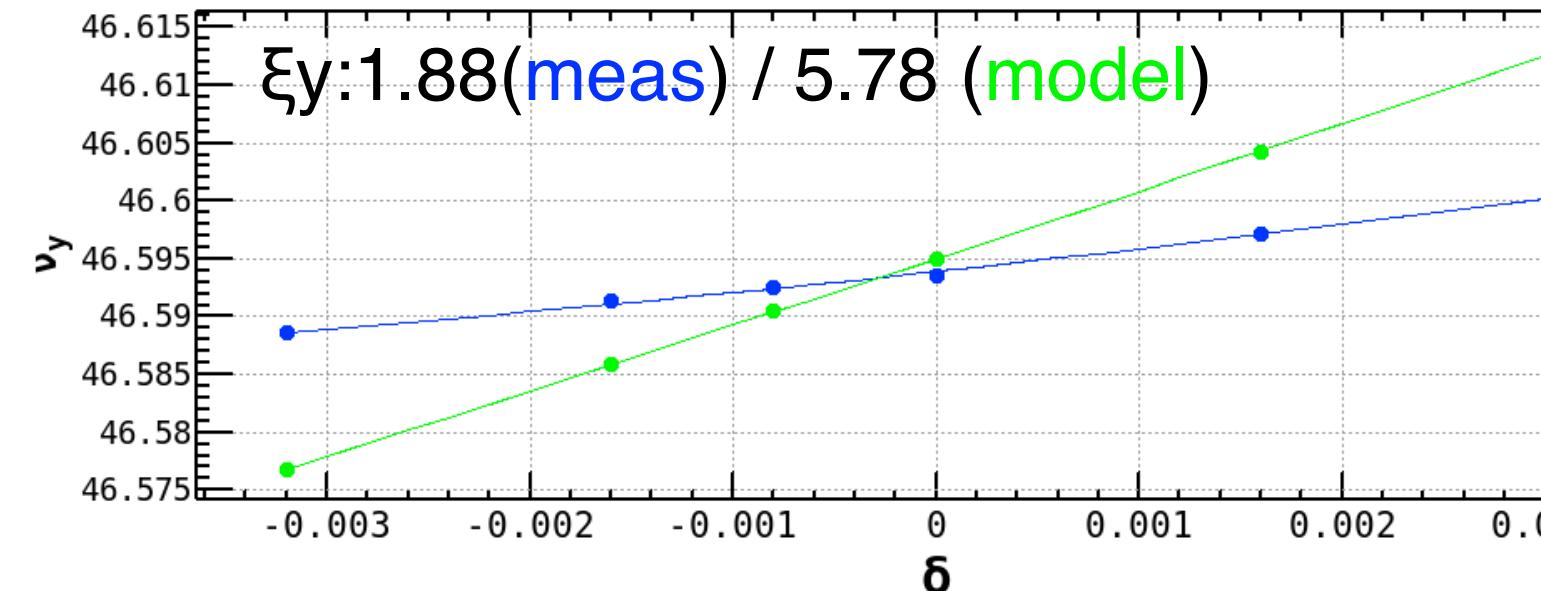
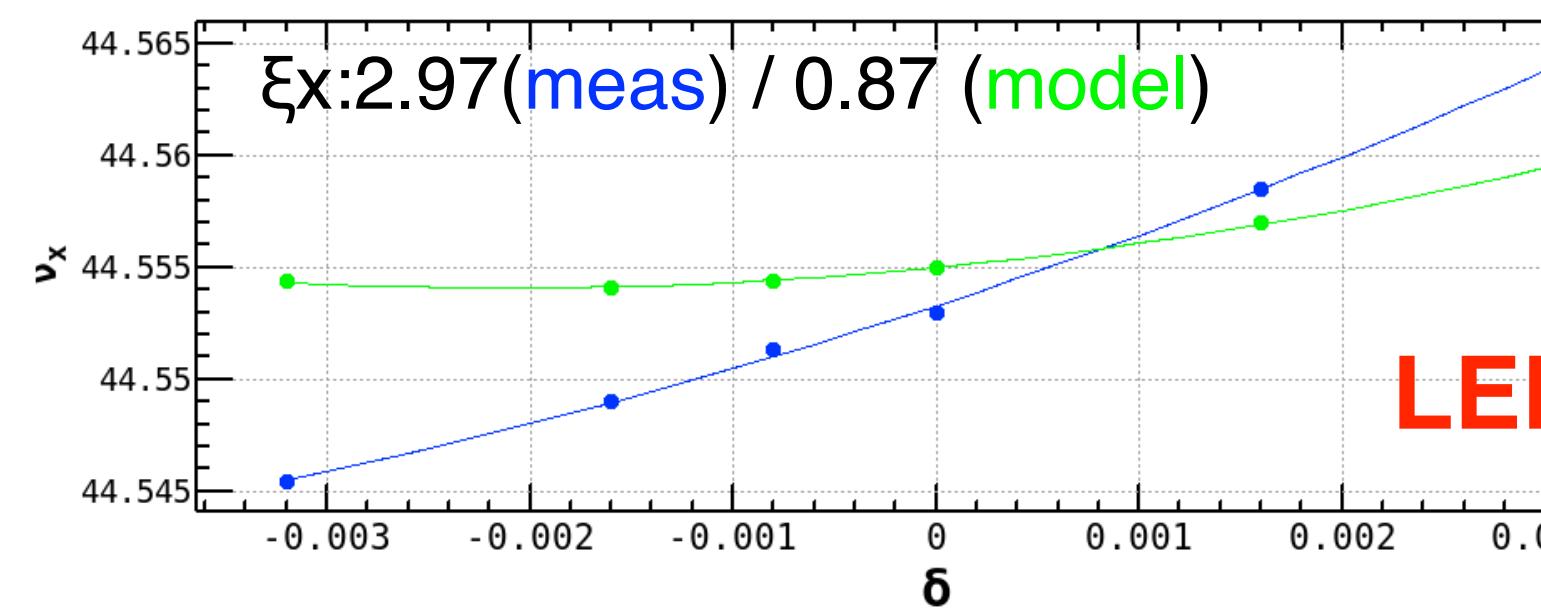
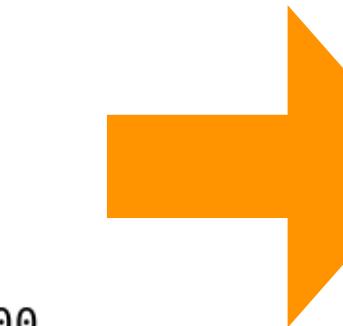


The chromaticity is well reproduced by the model after the correction.

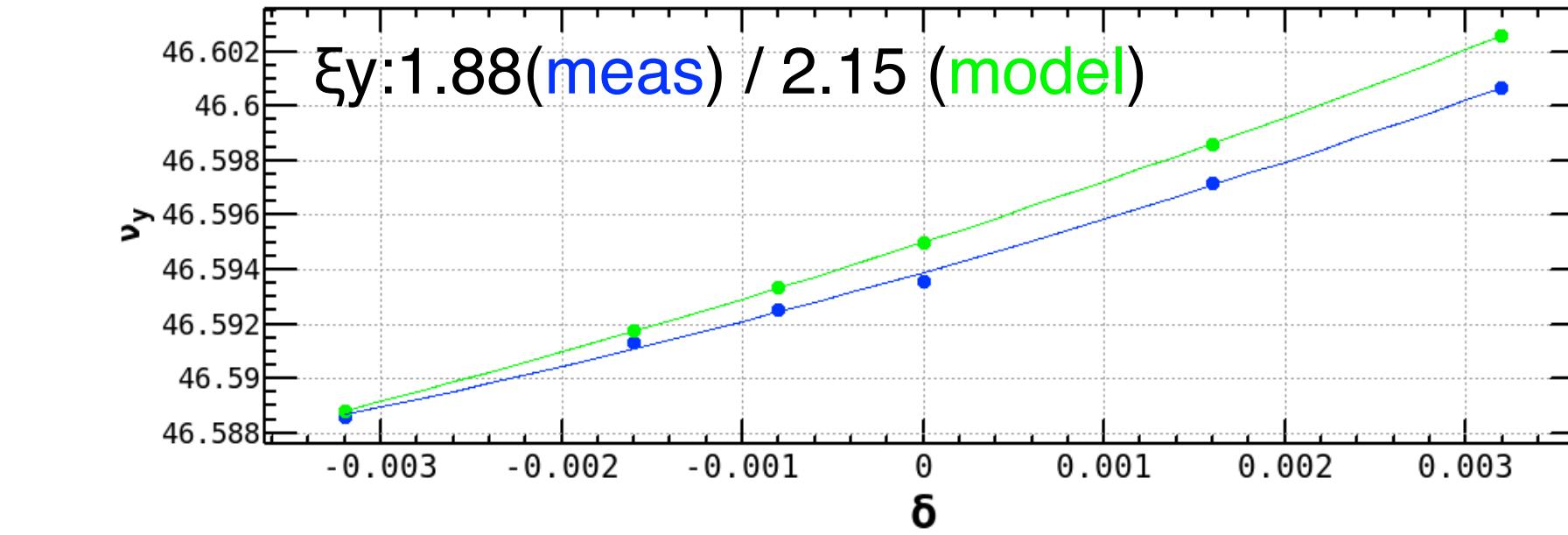
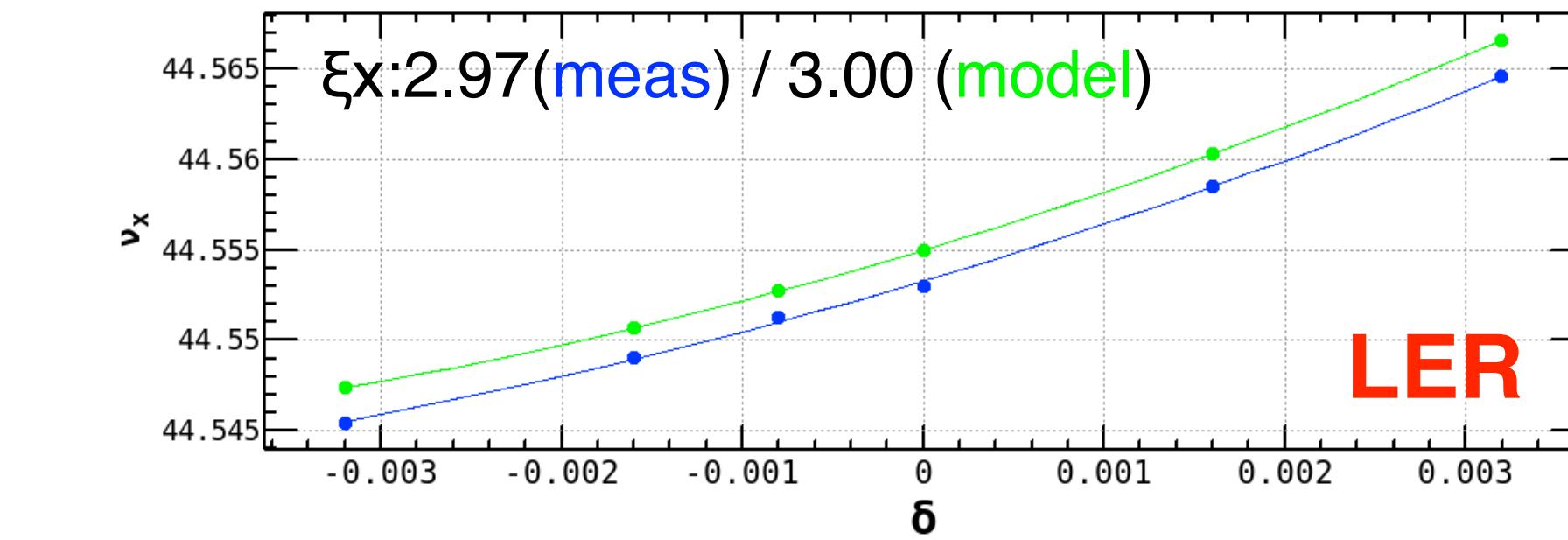
# Chromatic Phase-Advance in LER (Phase I)



No QCS  
in Phase 1

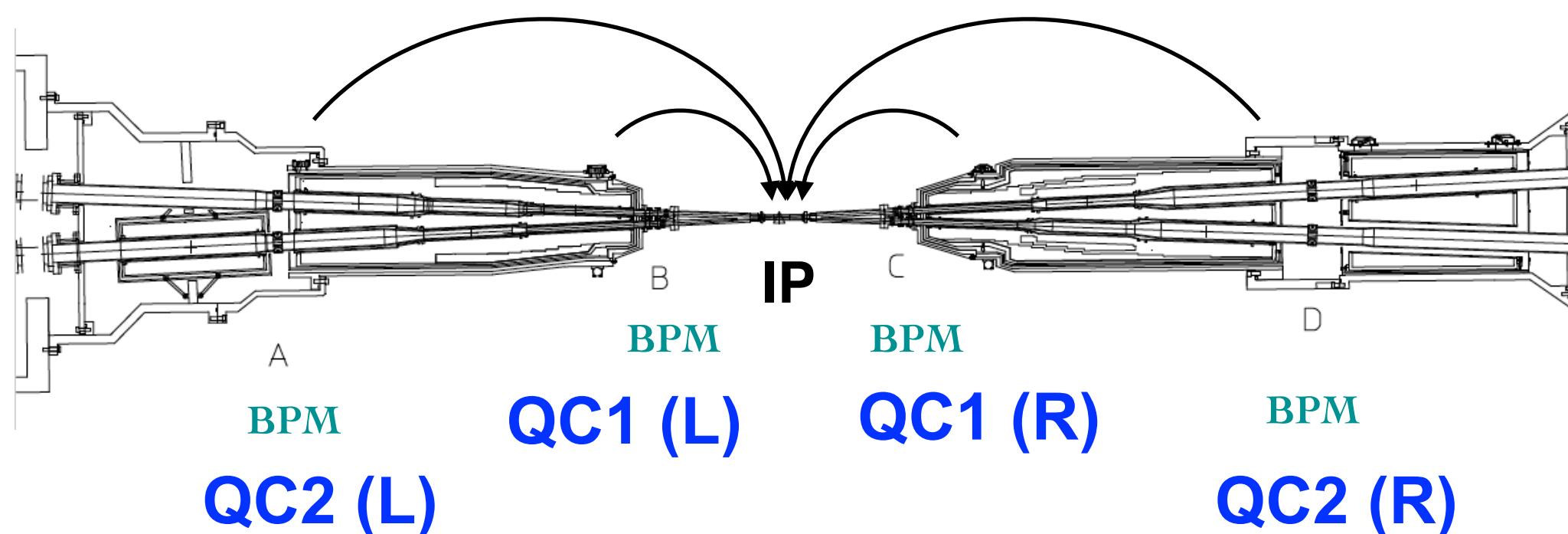


after



# **Measurement of Chromatic XY Coupling based on TbT Analysis**

## Reconstruct phase space at IP by using 2 BPMs



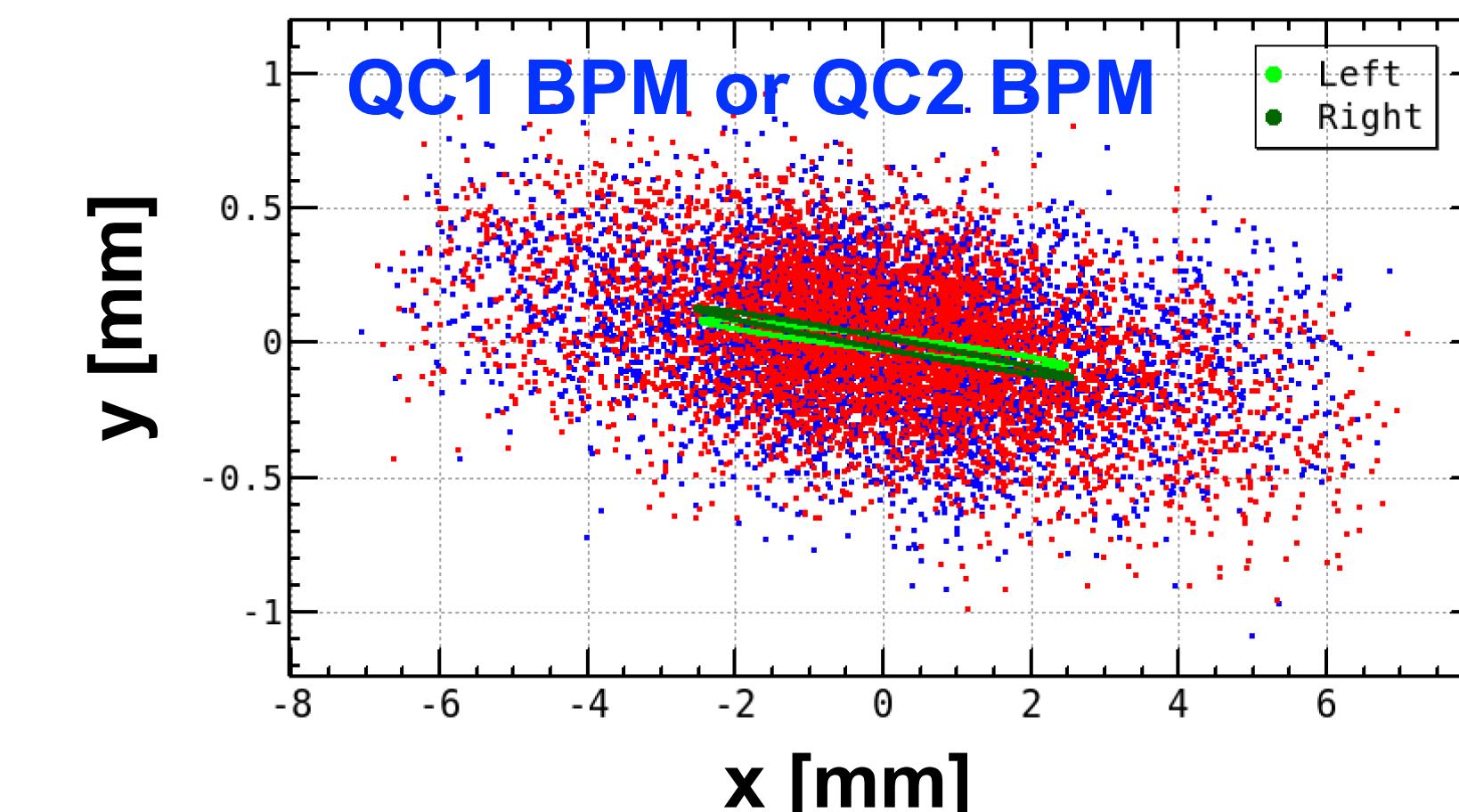
reconstruct  $(x, p_x, y, p_y)$  at IP from two BPMs

$$\begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix} = \begin{pmatrix} \mu & 0 & r_4 & -r_2 \\ 0 & \mu & -r_3 & r_1 \\ -r_1 & -r_2 & \mu & 0 \\ -r_3 & -r_4 & 0 & \mu \end{pmatrix} \begin{pmatrix} u \\ p_u \\ v \\ p_v \end{pmatrix}$$

**physical  
coordinate**

$$\mu^2 + (r_1 r_4 - r_2 r_3) = 1$$

**normal  
coordinate  
(decoupled)**



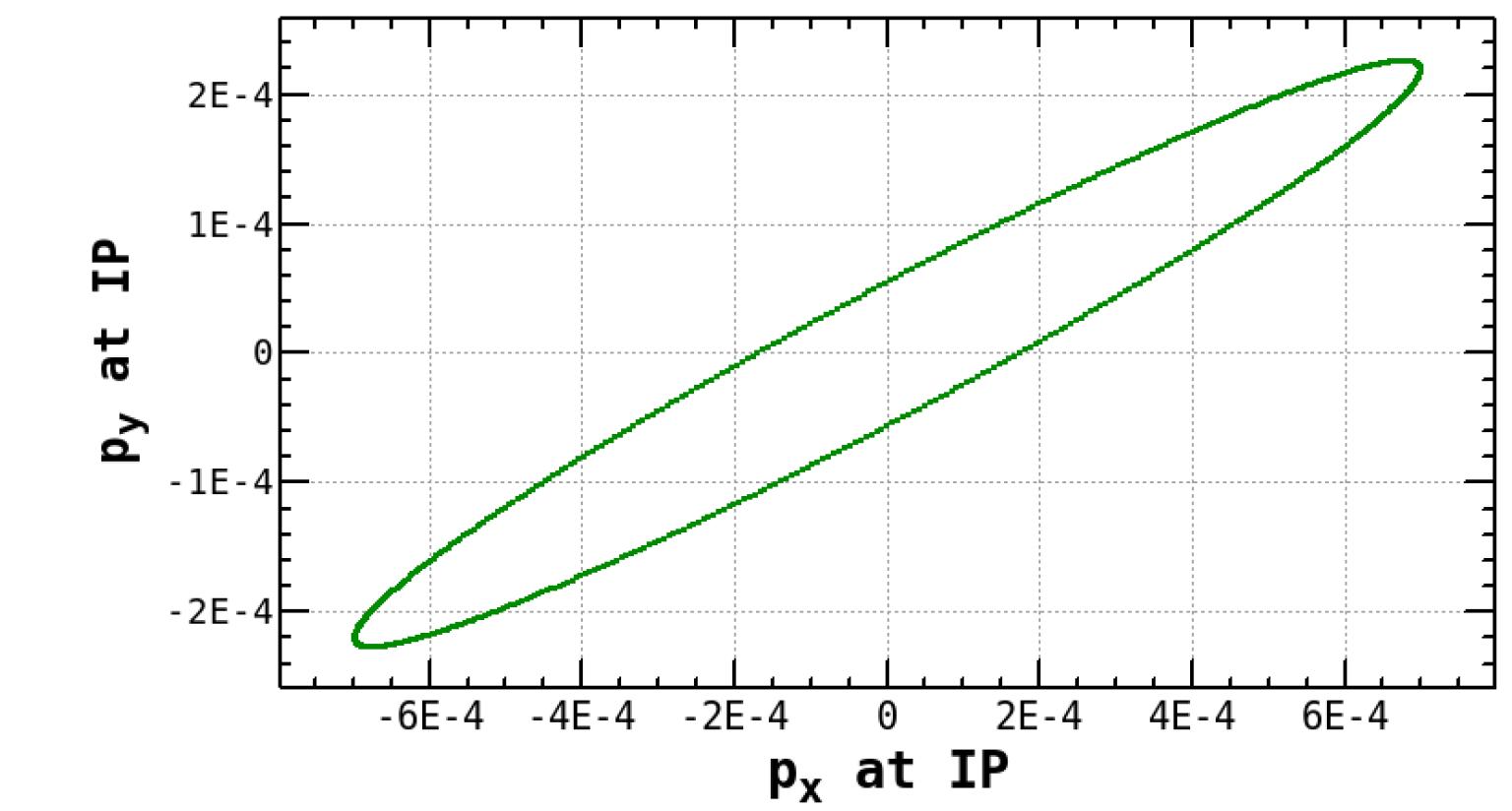
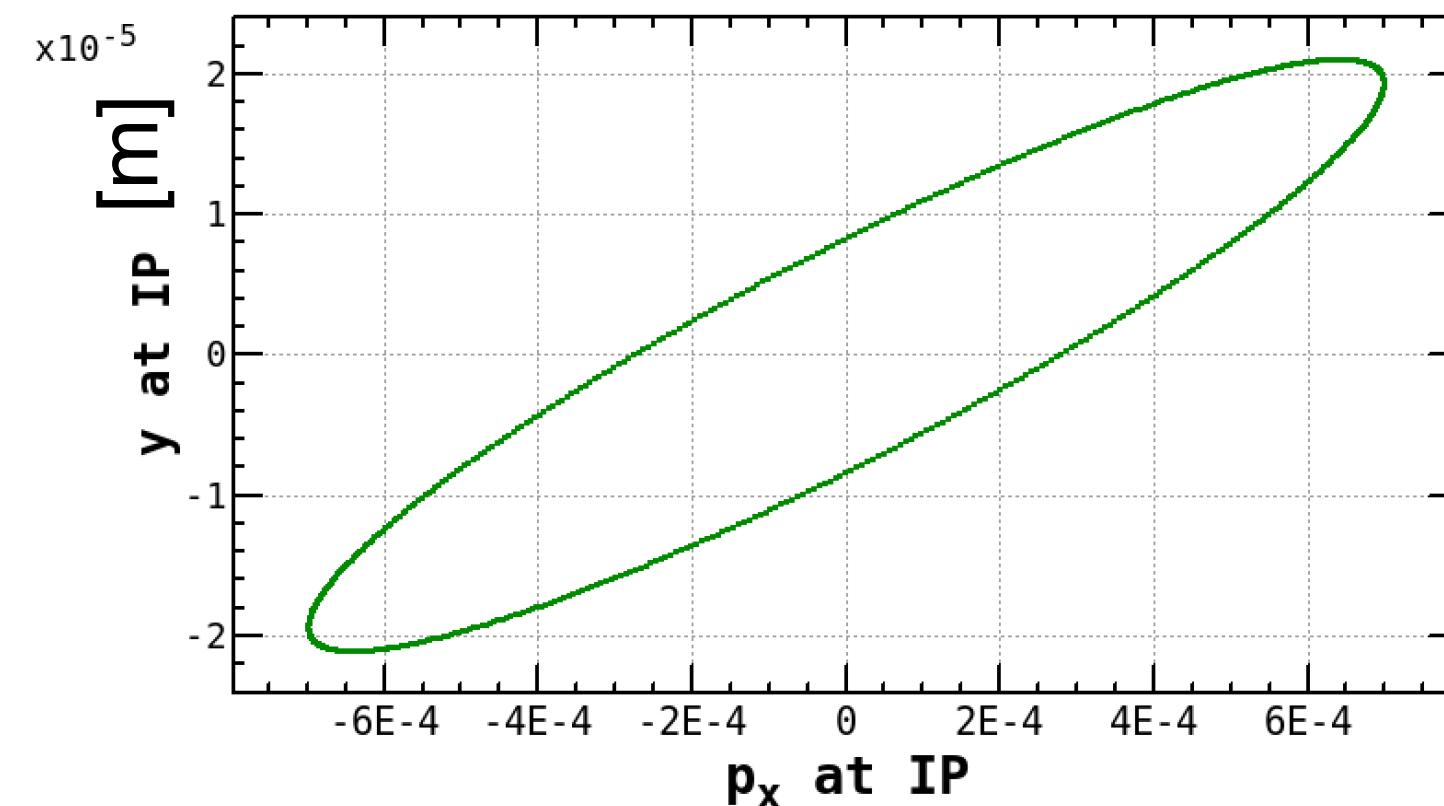
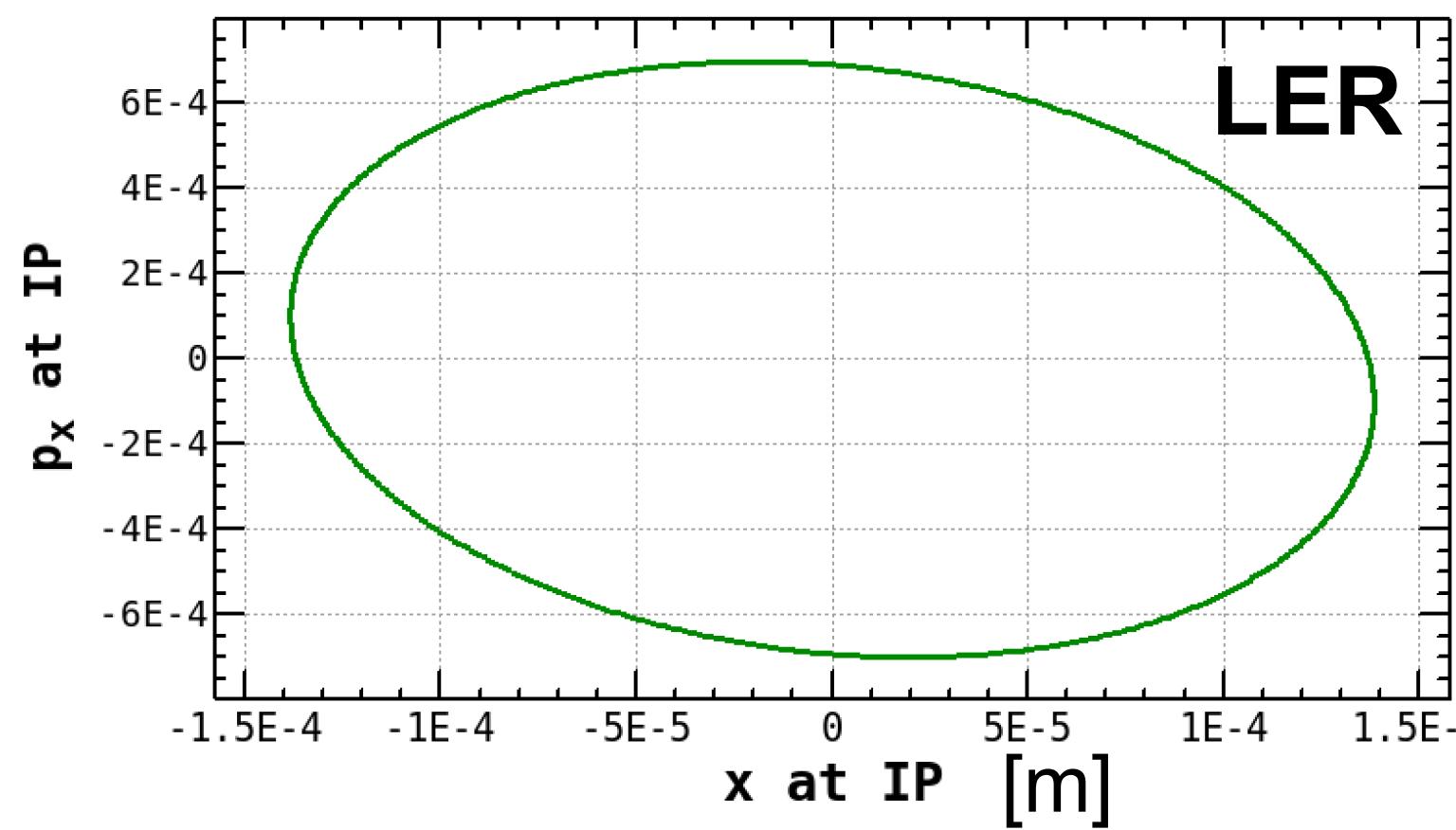
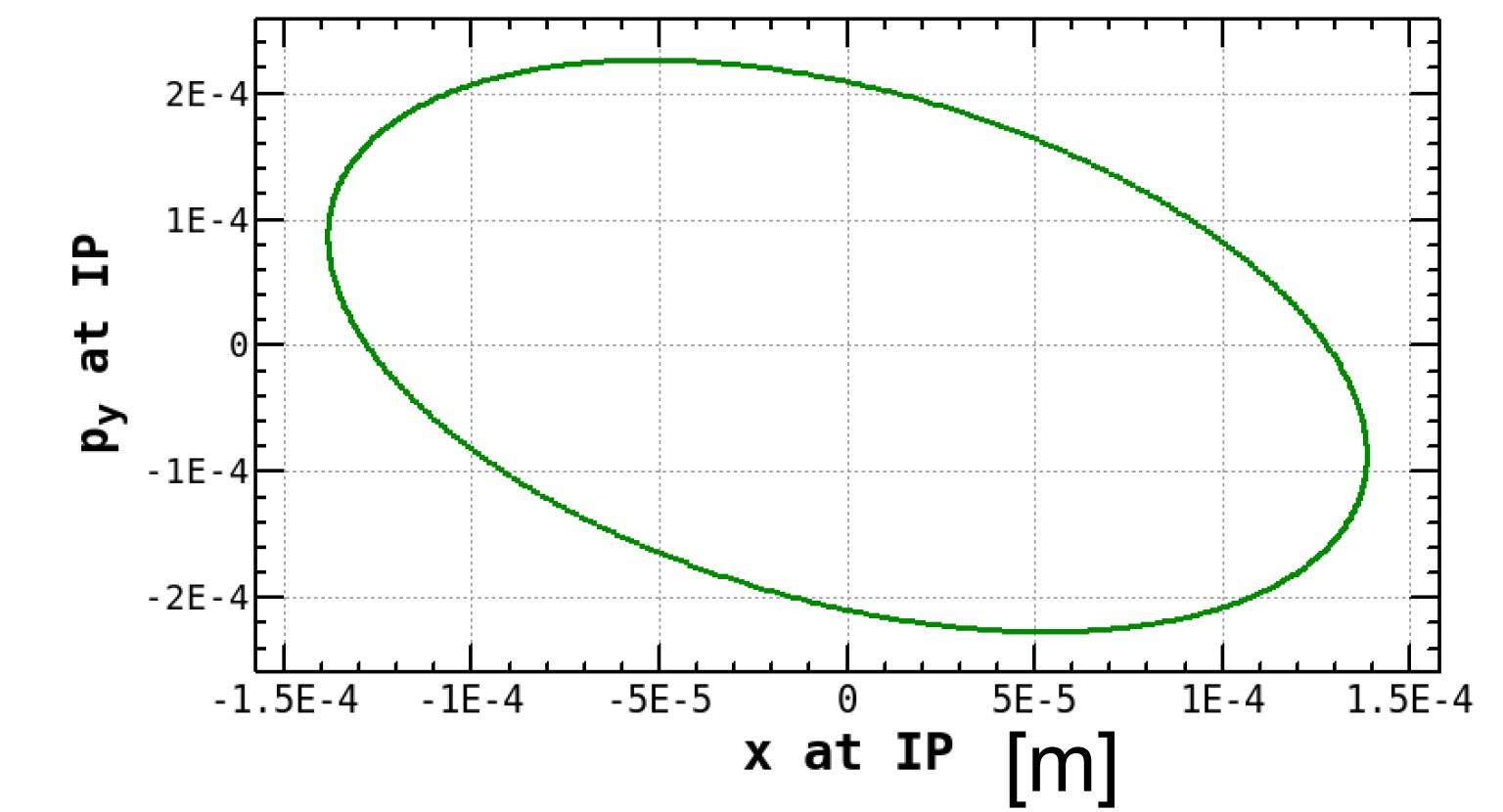
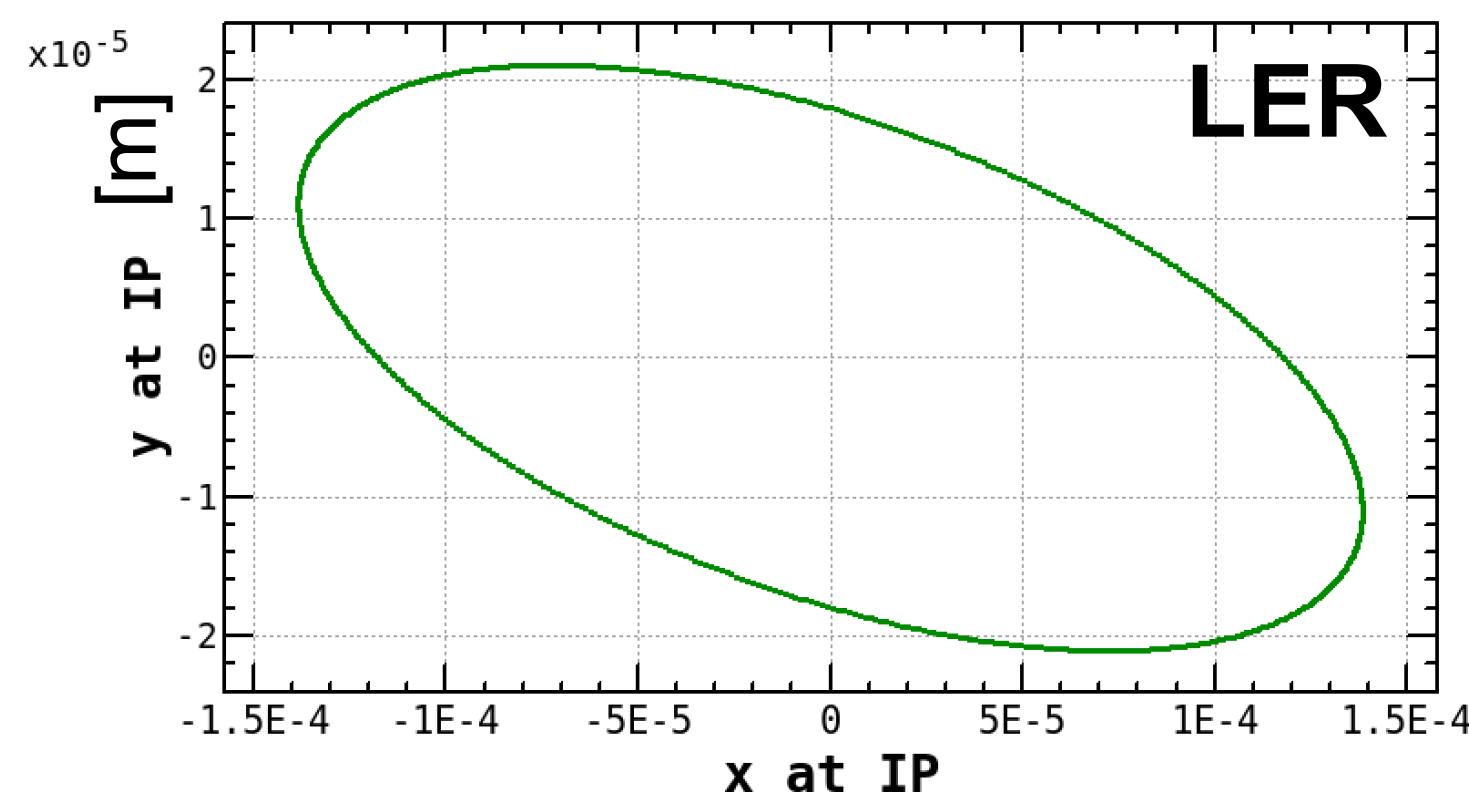
Extract normal mode of betatron oscillation

H-mode analysis

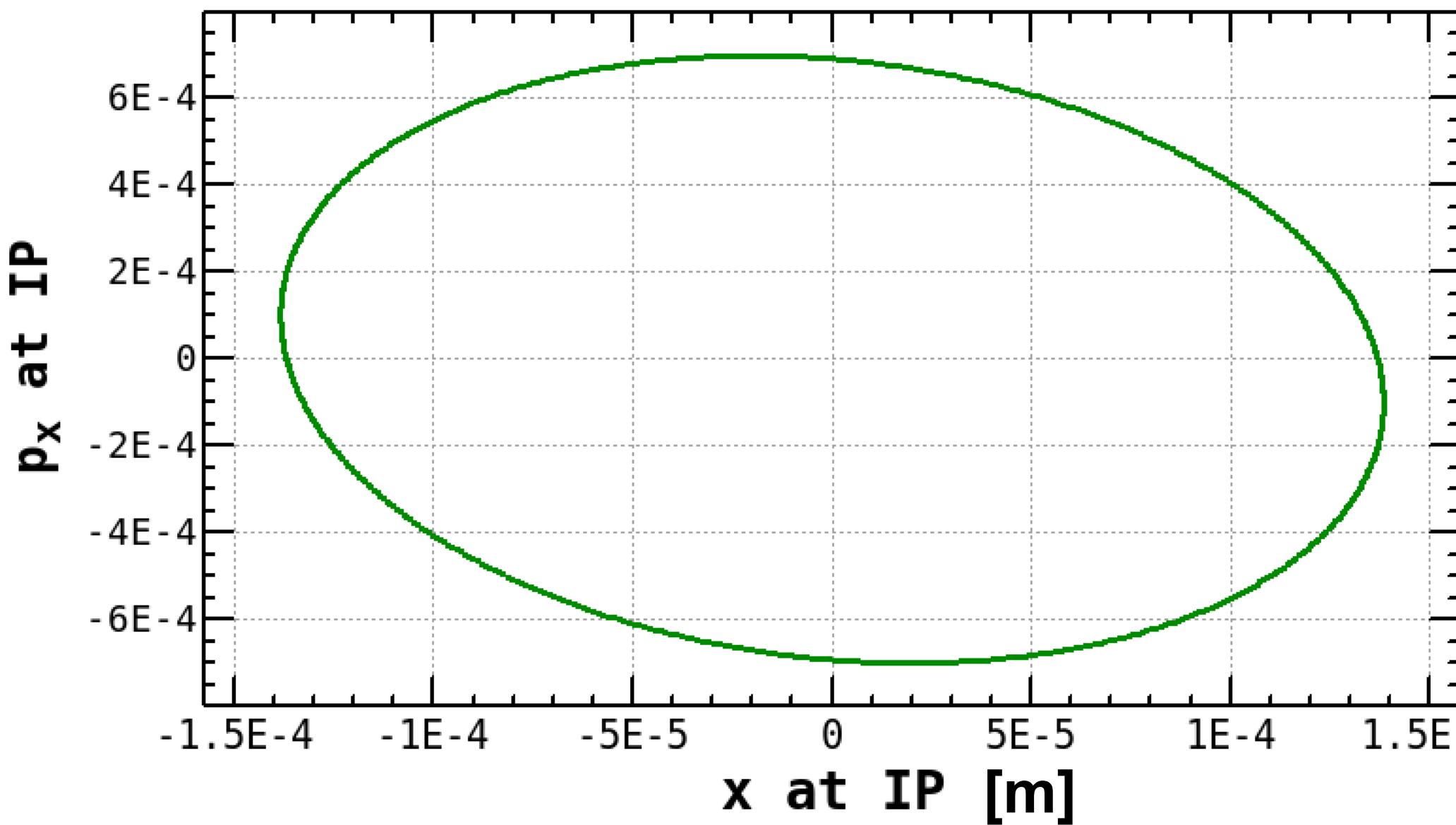
**Beam is kicked by a injection kicker.  
(horizontal)**

**XY couplings are extracted  
from H-mode analysis**

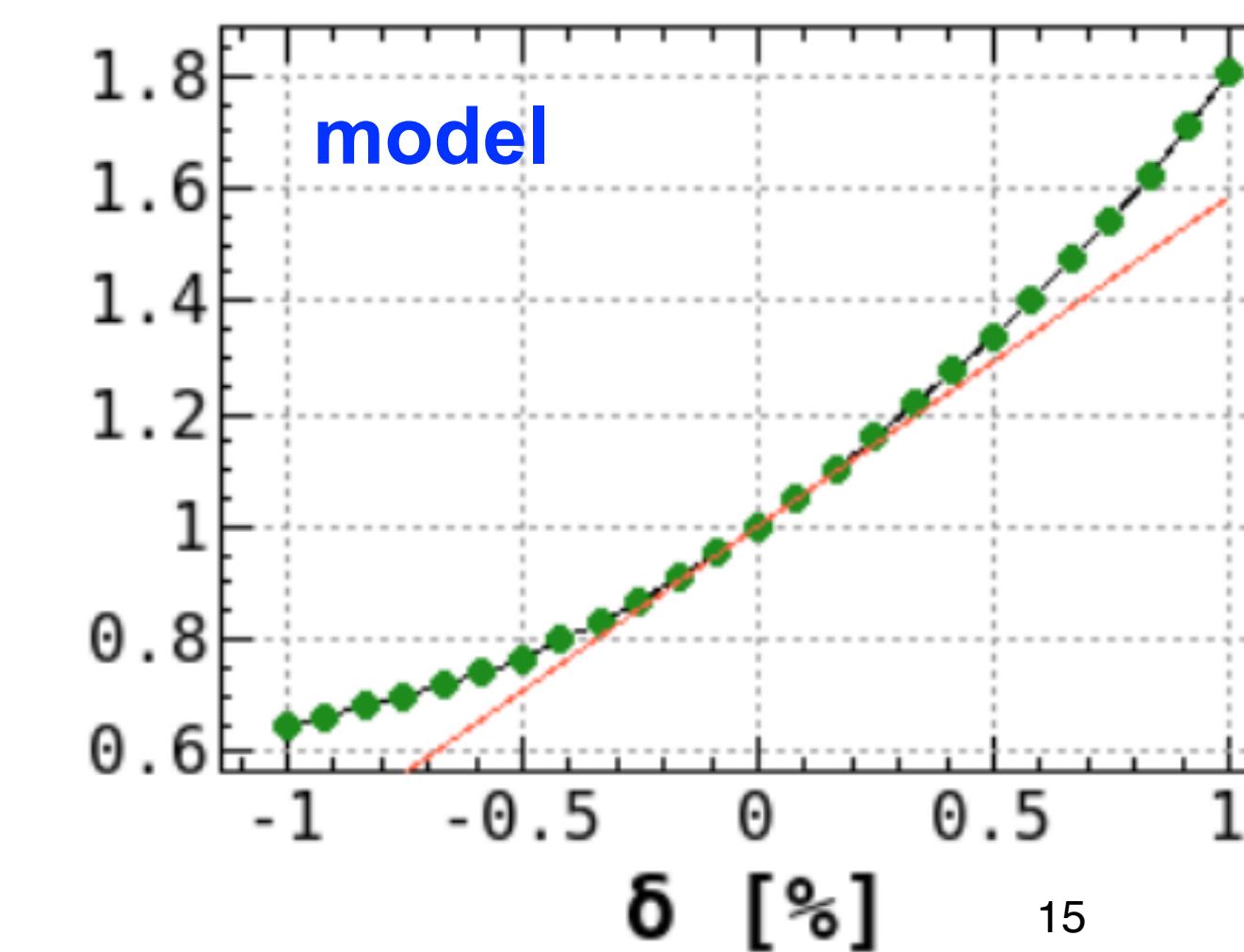
**Measured phase space plot at IP**



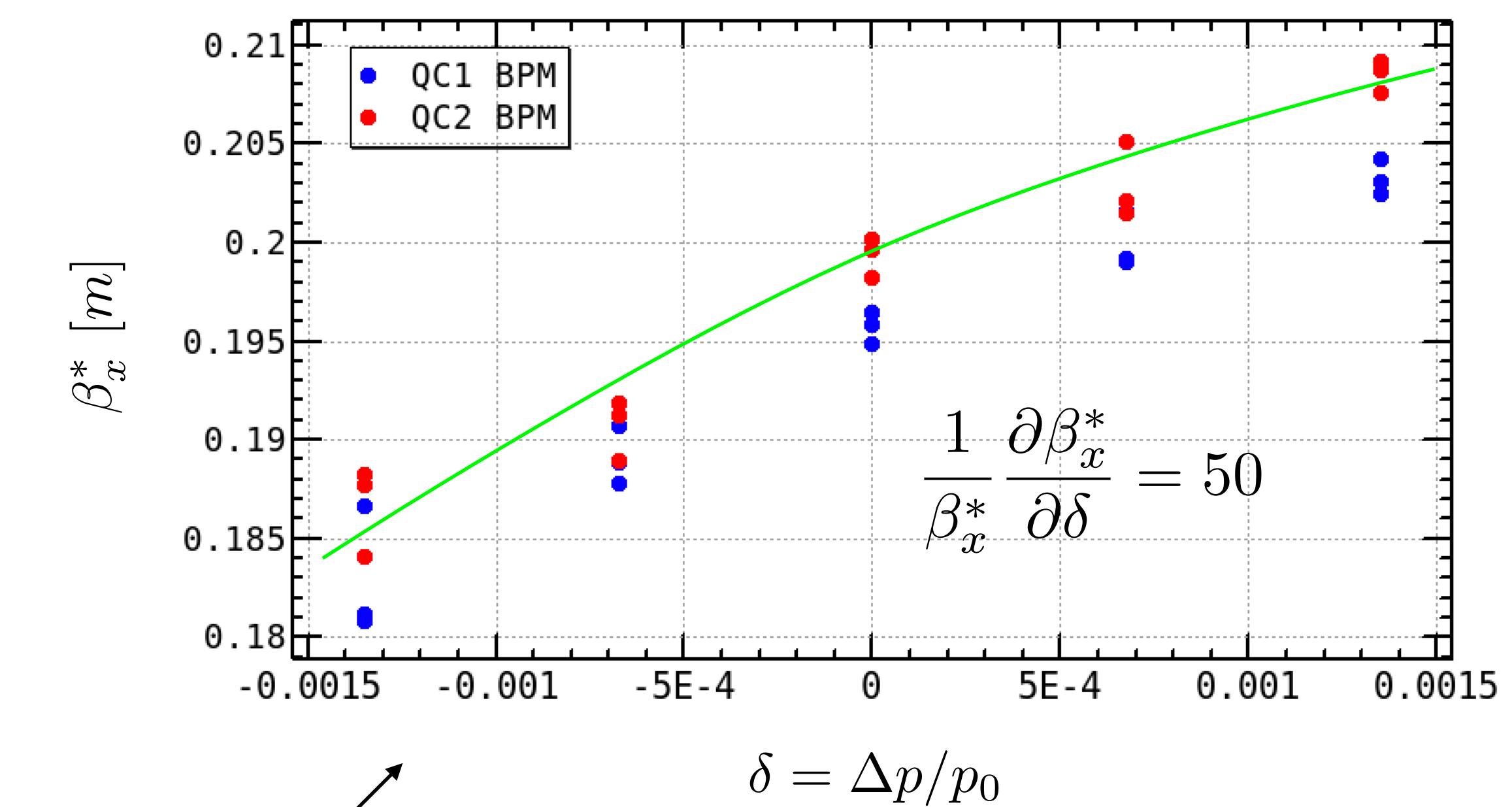
**LER**



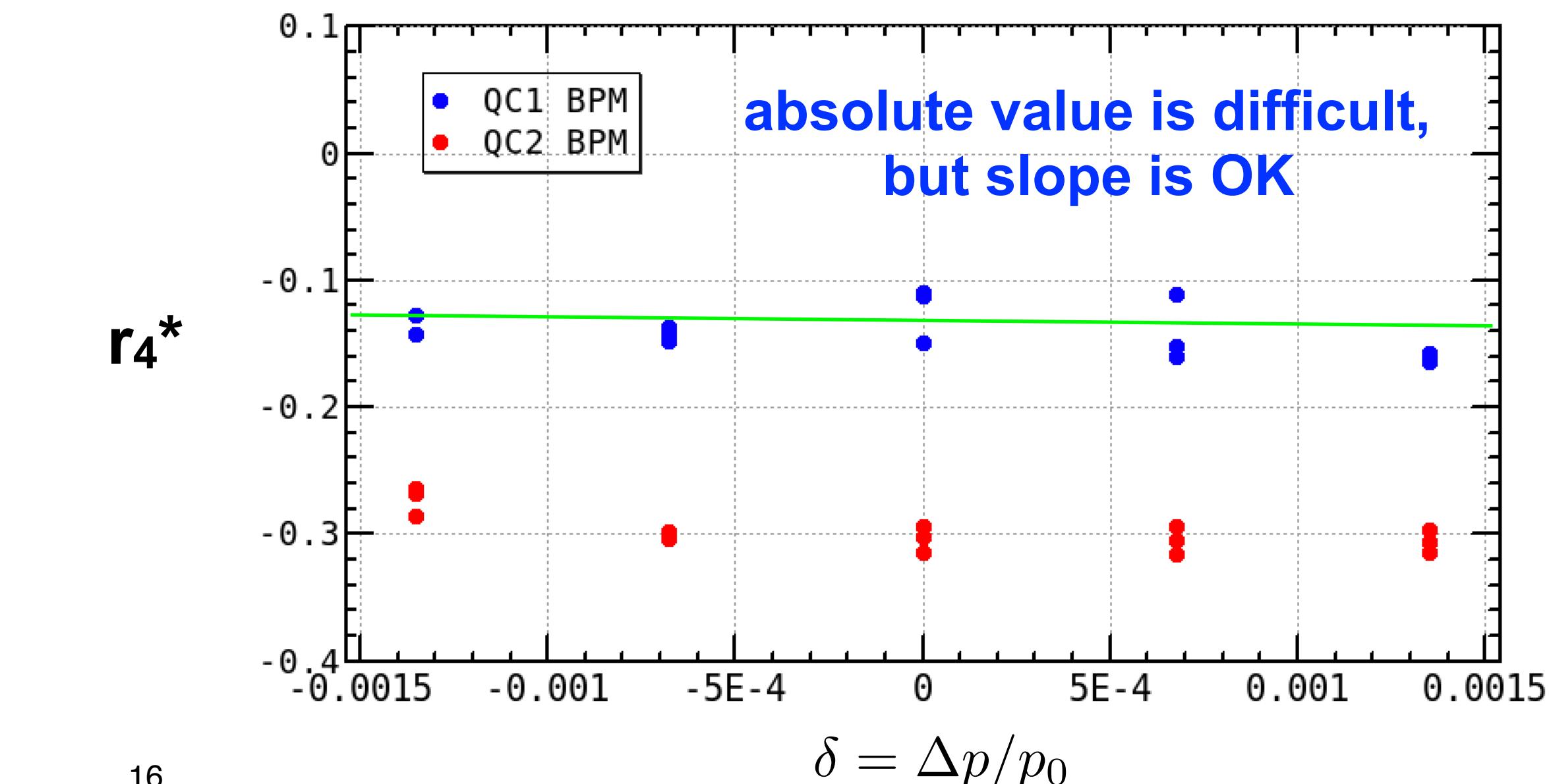
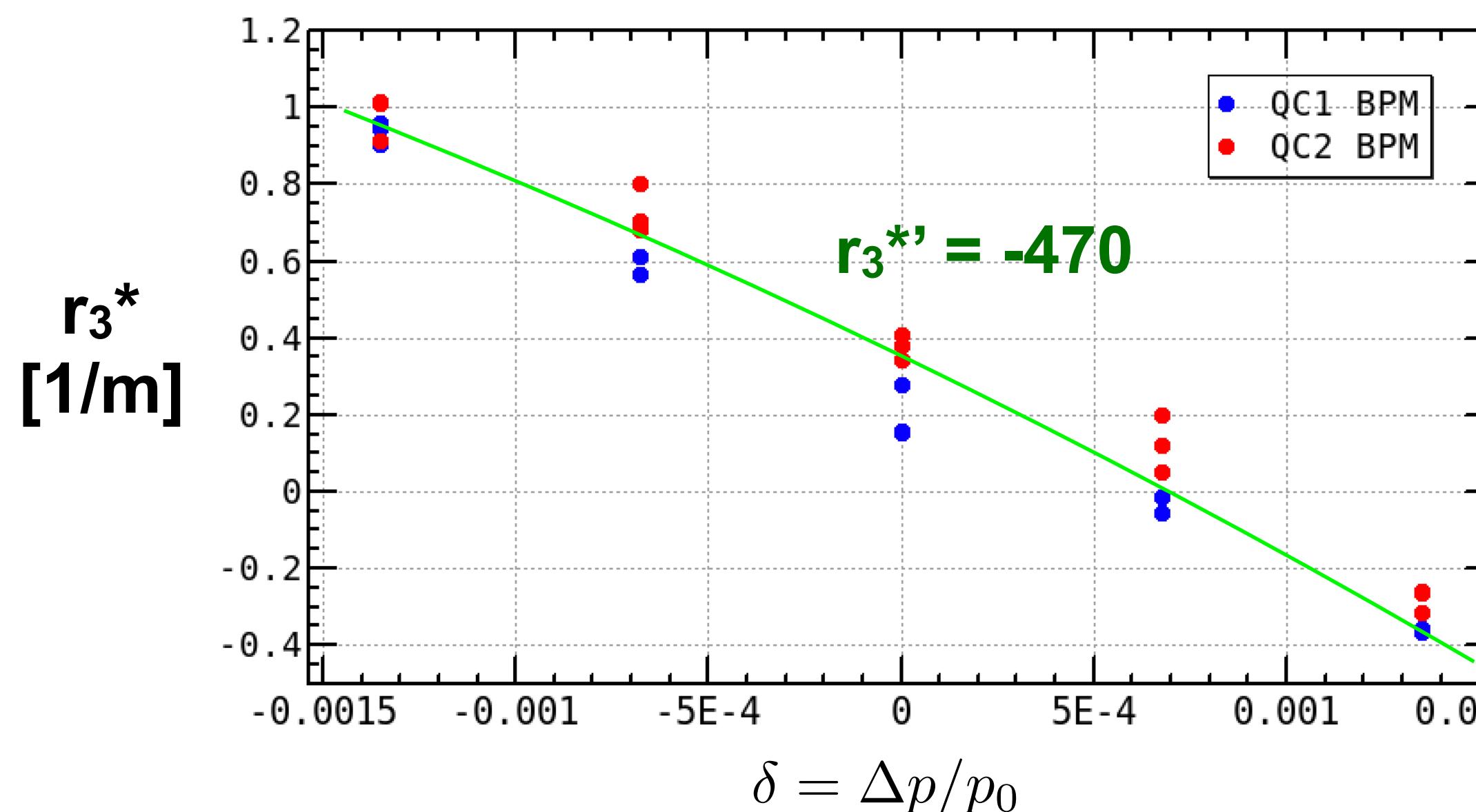
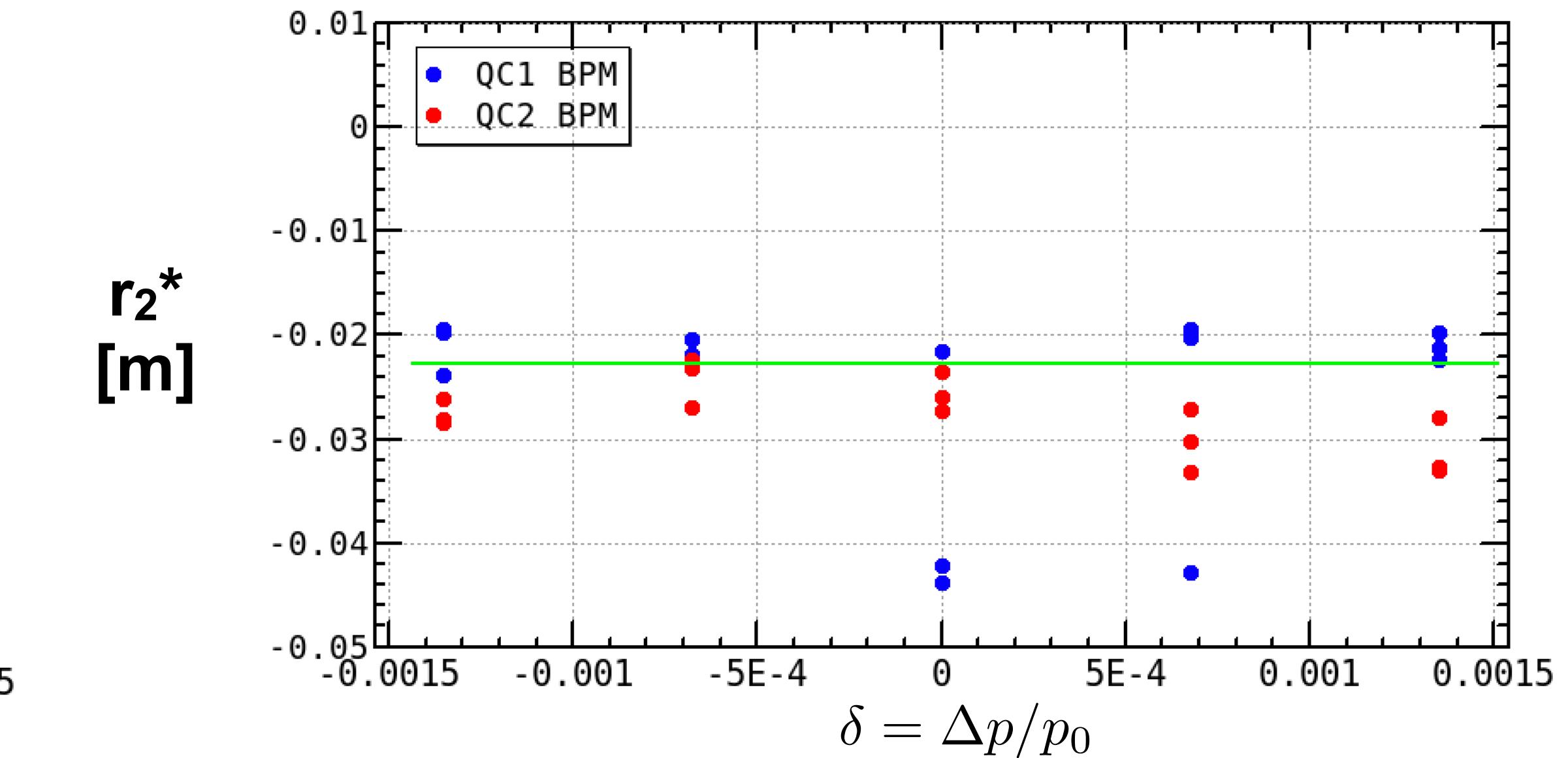
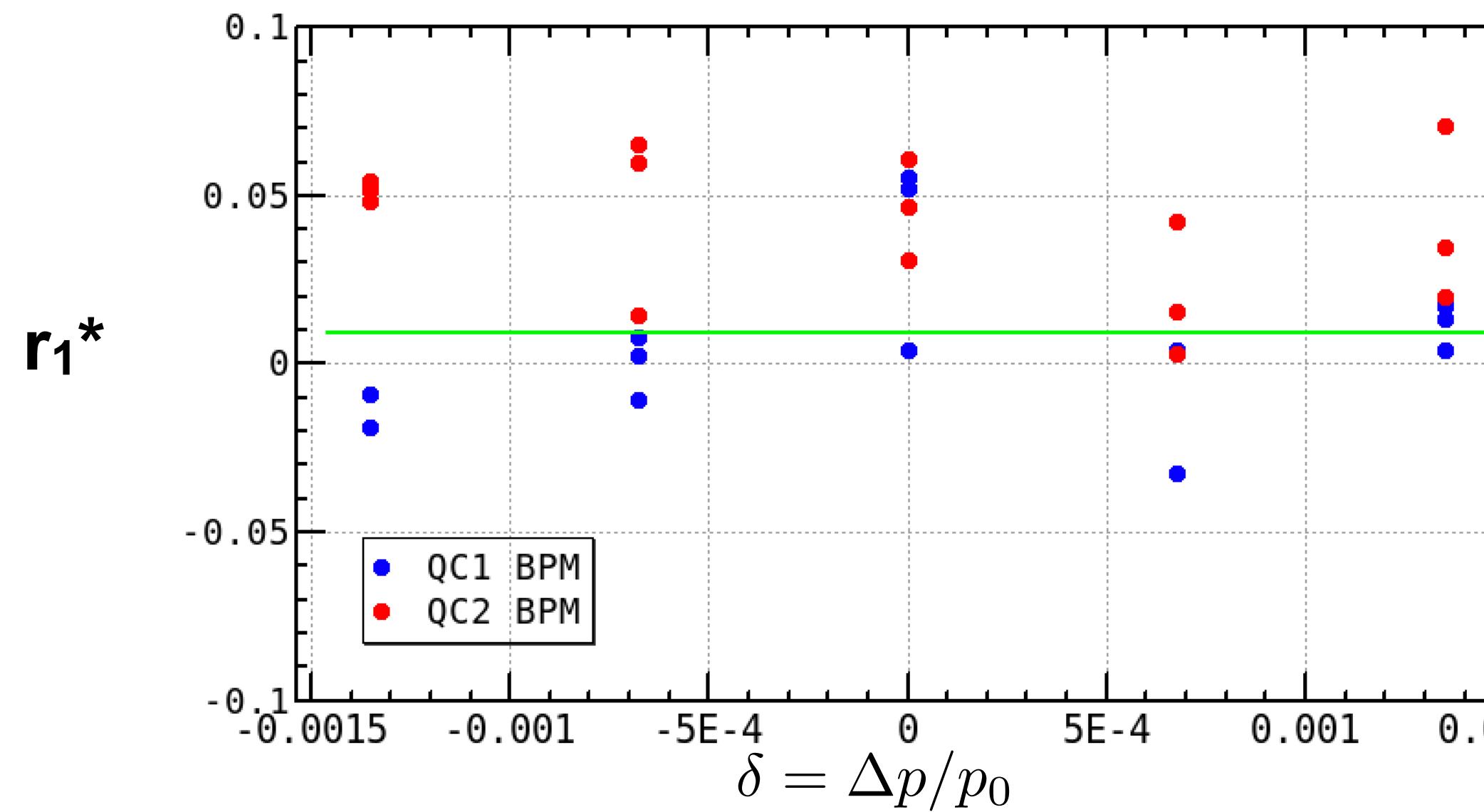
$$(\partial \beta_x / \beta_x) / \partial \delta: 58.27$$



**LER**



Horizontal beta\* chromaticity



- We have measured chromatic phase advance and correct them by sextupole magnets.
  - comparison between Phase 1(w/o QCS) and Phase 2(with QCS)
  - Chromatic XY coupling at the IP also measured by using TbT BPMs.
  - Measurement and correction of chromatic optical functions will be important to improve dynamic aperture and luminosity performance.