Optics aberration at IP and Beam-beam effects

K. Ohmi、Y. Funakoshi, H. Koiso, A. Morita, Y. Ohnishi,
D. Zhou (KEK)

eeFACT18@HKUST, HongKong

Introduction

- How to get the target luminosity
- Optics aberration degrade luminosity in beam-beam simulations.
- Optics correction at IP was one of key issues, since starting KEKB.
- The optics aberration is serious in SuperKEKB.
- The aberration is related to QCS mainly and also to other lattice magnets.

1. Design stage

2. Starting Phase II

3. Toward Phase III

Study in the design stage of SuperKEKB

- Weak-strong beam-beam simulation using SAD.
- Luminosity degradation has been seen from low bunch current.
- Interplay of beam-beam effect with lattice nonlinearity
- Skew sextupole component degrade luminosity (Y. Zhang).
- Where is the source of the nonlinearity.
- Focusing to QCS.

IR magnets and their nonlinearity

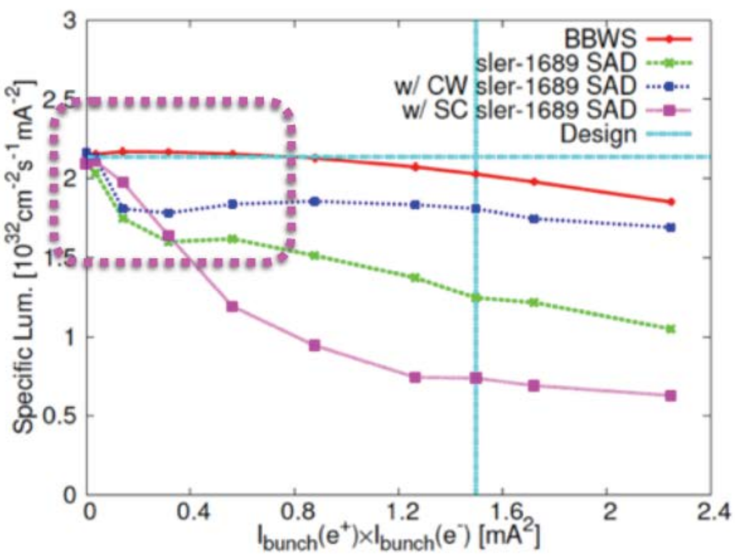
- There are many nonlinear field components in IR magnets.
- Chromatic coupling

➤ **Realistic lattice: lum. drops at low beam currents**

➤ **Crab-waist:**

- **To cancel beam-beam driven resonances**
- **Work well at high currents, but not well at low currents**

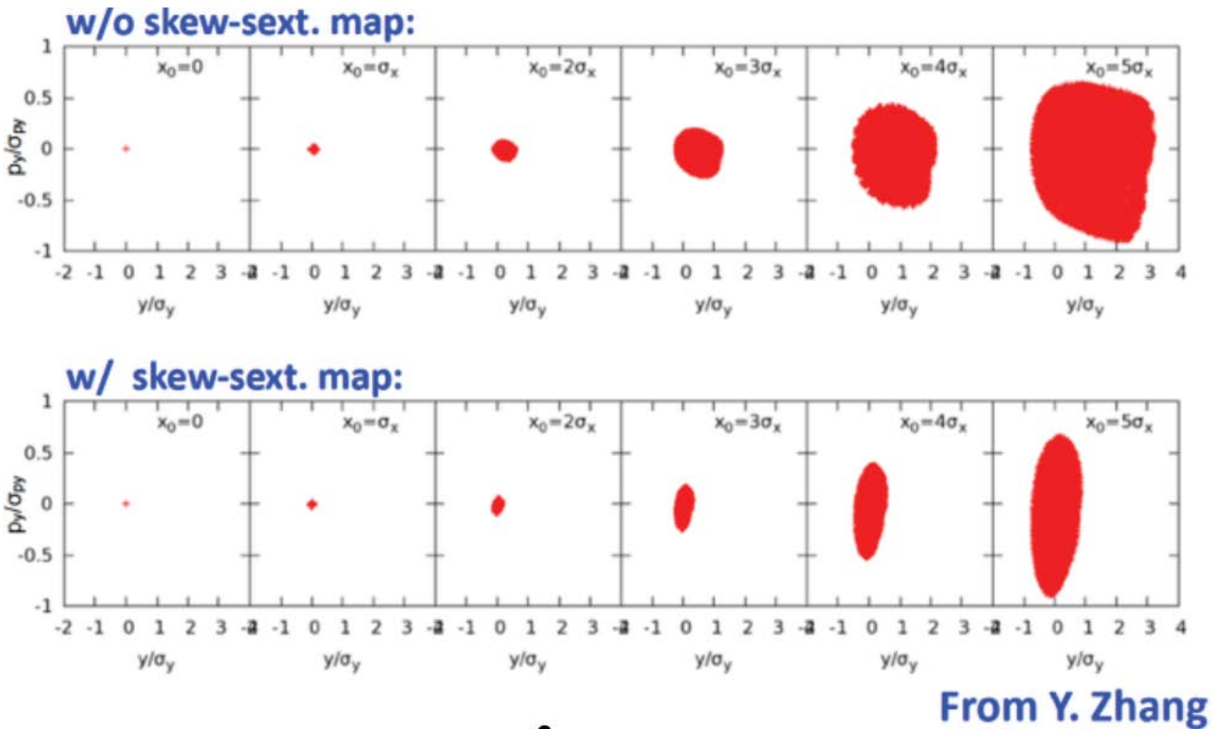
D. Zhou,
SKEKB MAC
2015



BBWS : arc expressed by simple transfer matrix
SAD: complex lattice structure

Y. Zhang’s (IHEP) work at KEK

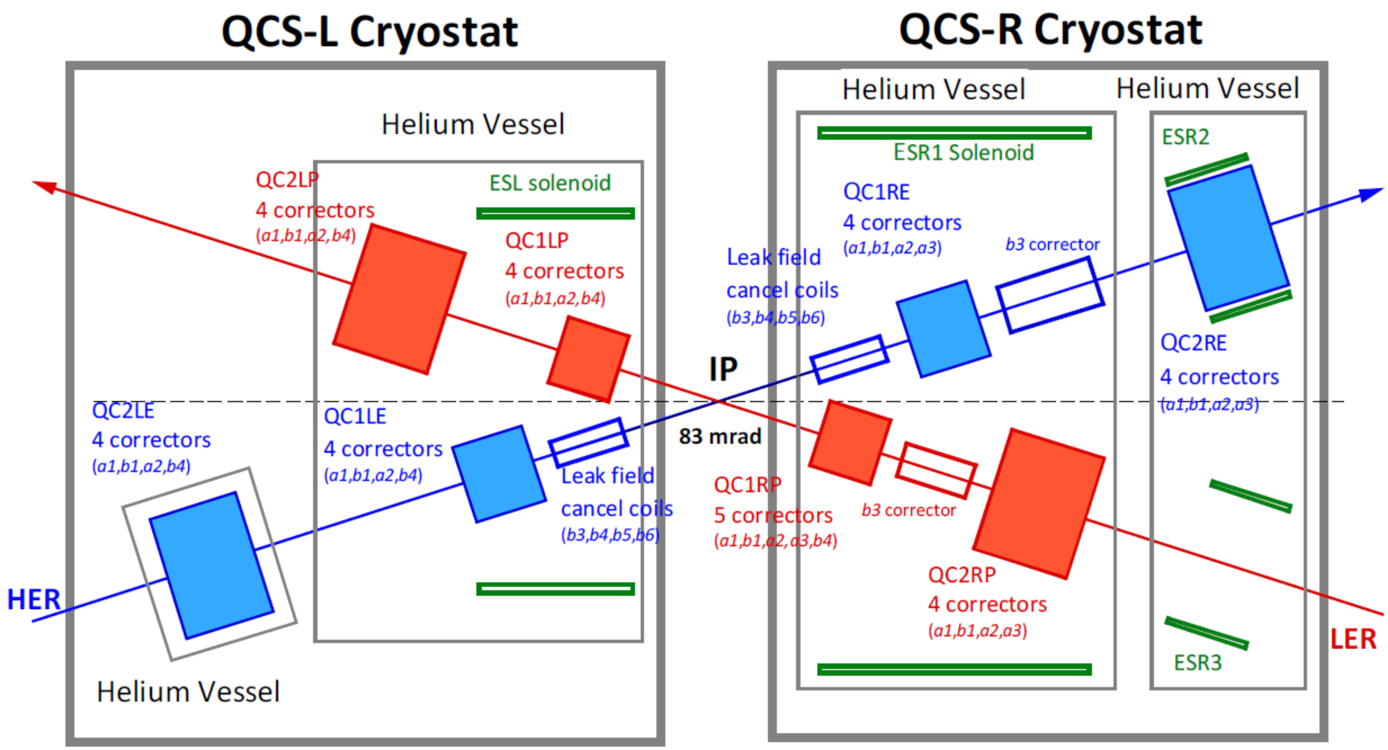
- Vertical orbit is induced by a large horizontal betatron oscillation.
- Skew sextupole term at IP, x^2y, is suspected for the luminosity degradation.



Nonlinear aberration at IP

QCS superconducting magnet system

N. Ohuchi et al.



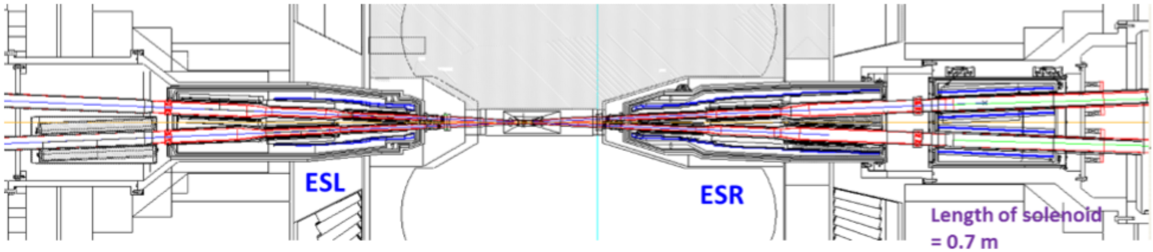
超伝導4極電磁石:4台
超伝導補正磁石(a1, b1, a2, b4):16台
QC1LP漏れ磁場キャンセル磁石(b3, b4, b5, b6):4台
超伝導補正ソレノイド:1台

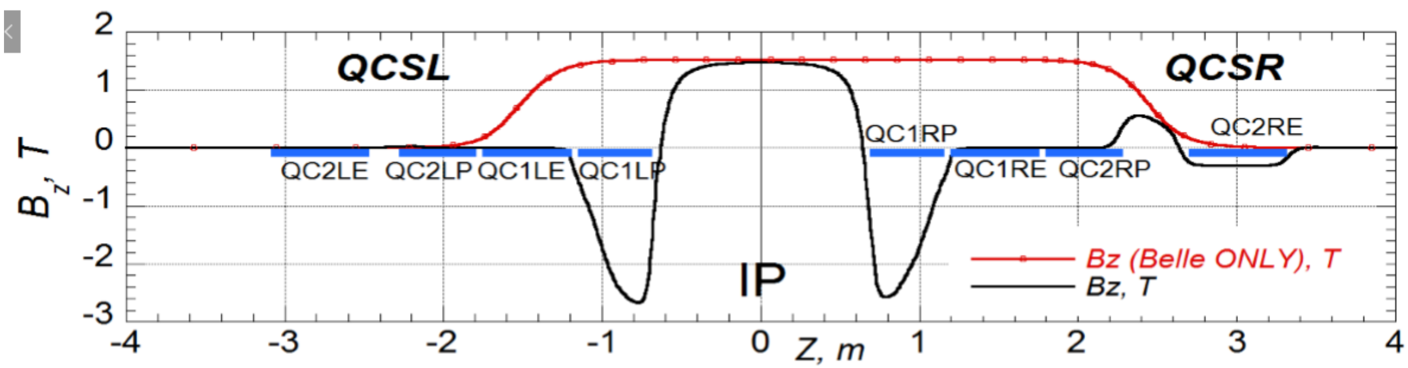
超伝導4極電磁石:4台
超伝導補正磁石(a1, b1, a2, a3, b3, b4):19台
QC1RP漏れ磁場キャンセル磁石(b3, b4, b5, b6):4台
超伝導補正ソレノイド:3台

Overview of IR magnets

N. Ohuchi et al.

- Compensation solenoids [ESL, ESR1, ESR2 and ESR3]

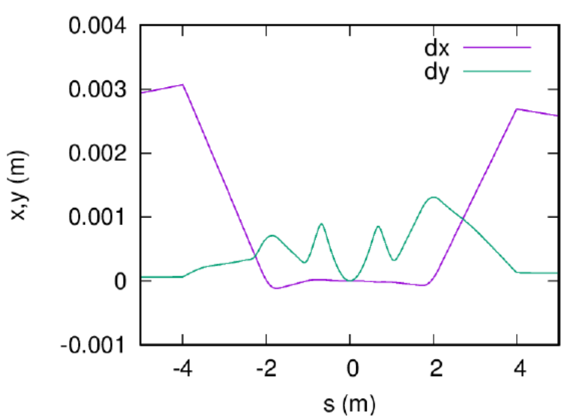




- In the left cryostat, $\boxed{\text{one solenoid (12 small solenoids) is overlaid on QC1LP and QC1LE.}}$
- In the right cryostat, the 1st solenoid (15 small solenoids) is overlaid on QC1RP, QC1RE and QC2RP.
  - The 2nd and 3rd solenoids on the each beam line in the QC2RE vessel.

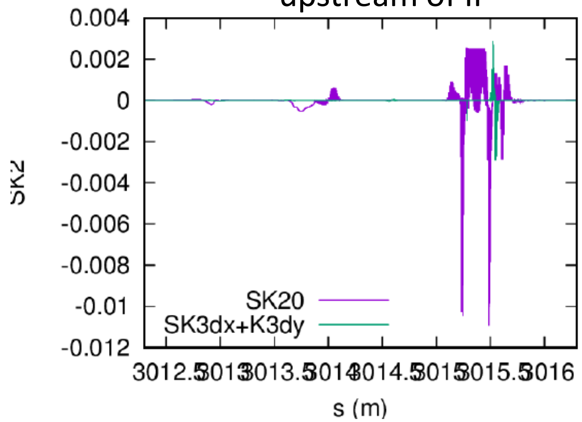
Evaluation of nonlinear term

- Focus on skew sextupole component.
- Reference axes in solenoid is chosen as a straight line with half crossing angle.
- Magnet components are defined on the reference orbit.
- Beam orbit deviate from the reference orbit.
- Skew sextupole component is induced by Skew sextupole and octupole with a vertical orbit.

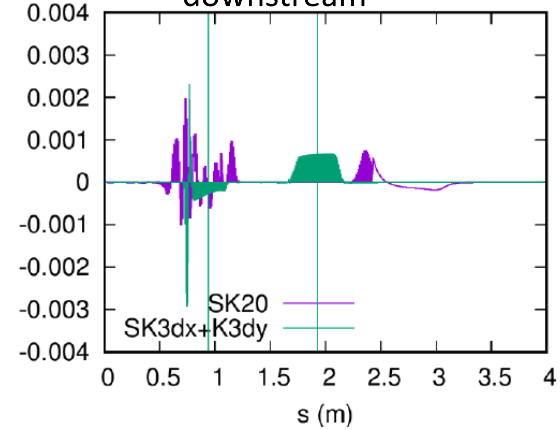


Skew sextupole component

upstream of IP



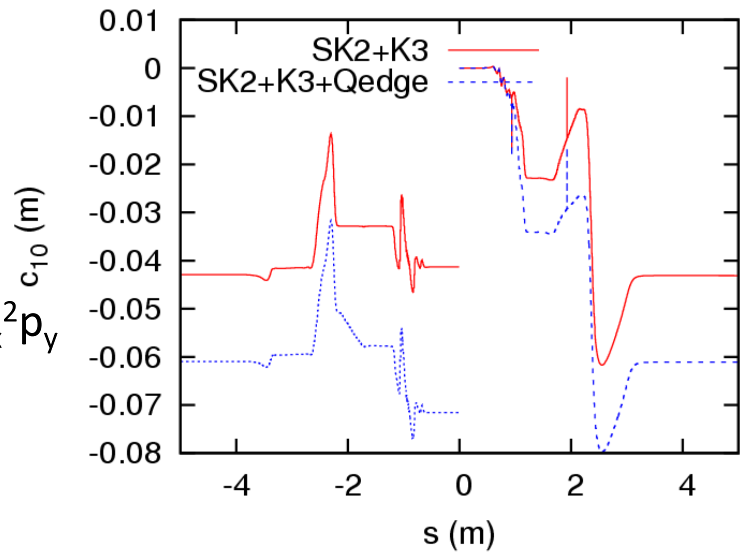
downstream



C_{10} from SK2 and K3+yCOD

- Contribution to SK2 is coming from explicit Skew Sext $SK2_0$ and octupole, K3+COD
- No contribution from higher order than K3.

$H=c_{10}p_x^2p_y$



There are 10 skew components.

$y^3, y^2p_y, yp_y^2, p_y^3$

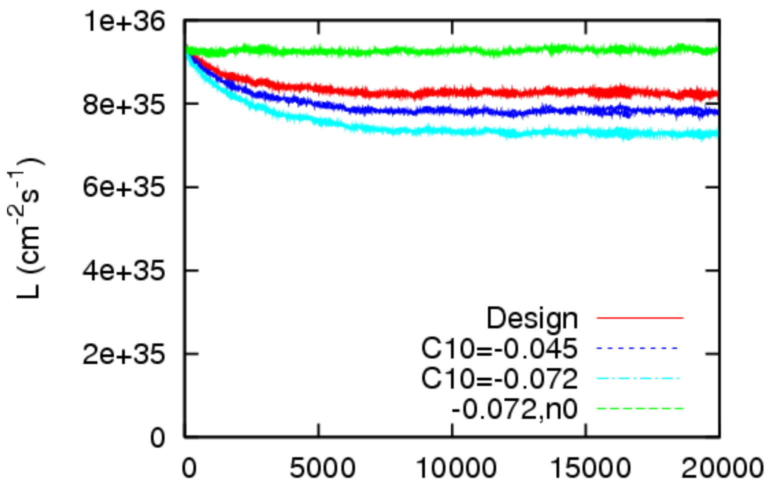
$x^2y, x^2p_y, xp_xy, xp_xp_y, p_x^2y, p_x^2p_y$

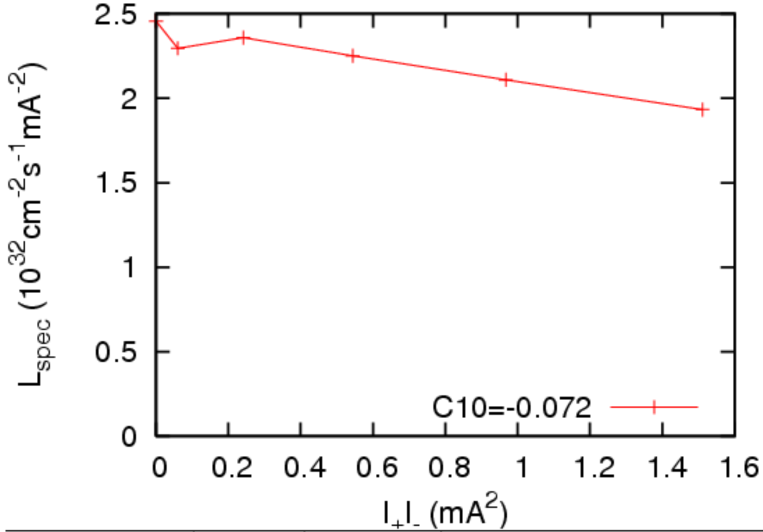
$H = c_{10}p_x^2p_y$

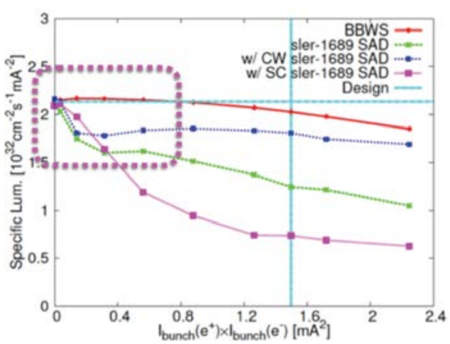
$$
\begin{aligned}
\mathcal{M}(s) &= \prod_{i=0}^{N-1} e^{-\mathcal{H}(x,s_i)} M(s_i, s_{i+1}) \\
&= \left\{ \prod_{i=0}^{N-1} M^{-1}(s_i, s) e^{-\mathcal{H}(x,s_i)} M(s_i, s) \right\} M(s) \\
&= \left\{ \prod_{I=0}^{N-1} e^{-\mathcal{H}(M(s,s_i)x, s_i)} \right\} M(s) \\
&\approx \underline{e^{-\oint \mathcal{H}(M(s,s')x, s')ds'} M(s)}
\end{aligned}
$$

- Skew sextupole coming from higher order nonlinearity is small.

Luminosity for $H=c_{10}p_x^2p_y$







- The luminosity degradation due to c10 is weaker than that of beam-beam simulation with SAD.
- There may be still unknown nonlinearity?

Commissioning of SuperKEKB β^* is squeezed step-by-step

- c_{10}=0.072 m is kept for β^*change, because IR magnets are fixed in SuperKEKB.
- For normalized coordinates, $P_i = \sqrt{\beta_i} p_i, X_i = x_i/\sqrt{\beta_i}$

$$C_{10} = \frac{c_{10}}{\beta_x^* \sqrt{\beta_y^*}} \qquad H = c_{10} p_x^2 p_y \qquad H_N = C_{10} P_x^2 P_y$$

- C_{10}=136.9 $m^{-1/2}$ for β_x*=3.2cm, β_y*=0.27mm
- Normalized C_{10} directly affects the beam dynamics. $\Delta Y = C_{10} P_x^2$

$$\Delta Y = C_{10} P_x^2 \approx 136.9\varepsilon_x \approx 0.15\sqrt{\varepsilon_y}$$ for β_x*=3.2cm, β_y*=0.27mm

- The effect is reduced by Detune of β^*.
- C_{10} is 4.4% for 8x8, 8.8% for 4x8.

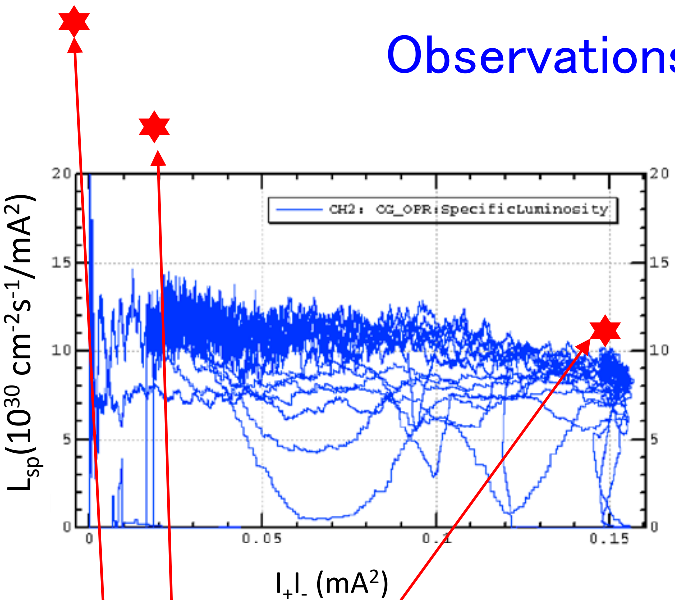
- This nonlinearity does not affect commissioning stage. (MAC2018)

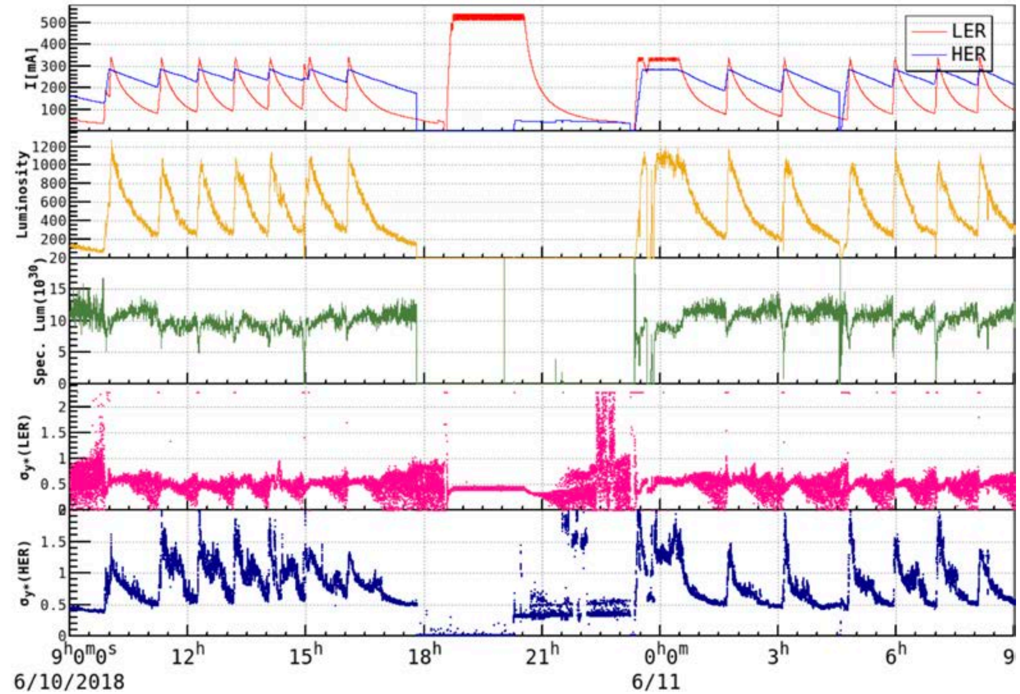
Phase II commissioning stage

- Collision starts the end of April 2018.
- Beta was squeezed to 8->6->4->3mm. Clear luminosity gain did not have the first 1.5 month of the beam-beam commissioning. Rather it worsened.
- Collision tools (offset, optics/emittance, waist, x-y coupling …) are developed during the period.
- Many works were done simultaneously
  - Develop machine protection interlock.
  - Injection tuning. Linac tuning. Back ground.
  - Beam current increase.
  - ………………

Lspec at June 10, 2018

Observations





- 0mA, σ_{y0}=0.3μm, 0.4μm, L_{sp}=35
- 200x80mA, σ_{y0}=0.5μm, 0.6μm, L_{sp}=23
- 285x340mA, σ_{y0}=1.5μm, 0.6μm, L_{sp}=11

Lsp agrees with geo value at high current

L_{peak}=1.2x10^{33} $cm^{-2}s^{-1}$,
285x340mA, N_b=788

Blow-up of e- beam was serious.

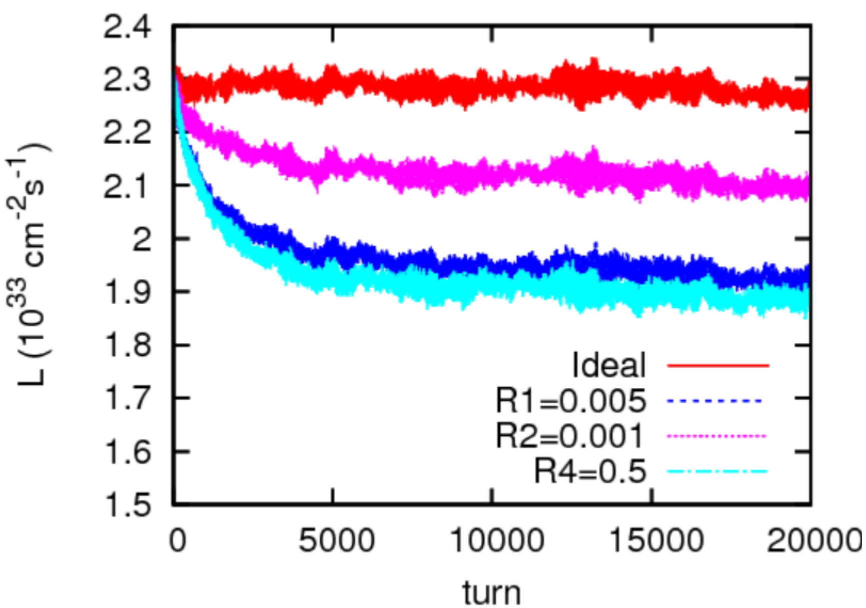
$$L_{sp} = \frac{1}{2\pi\sigma_{xc}\sigma_{yc}e^2 f_0}$$

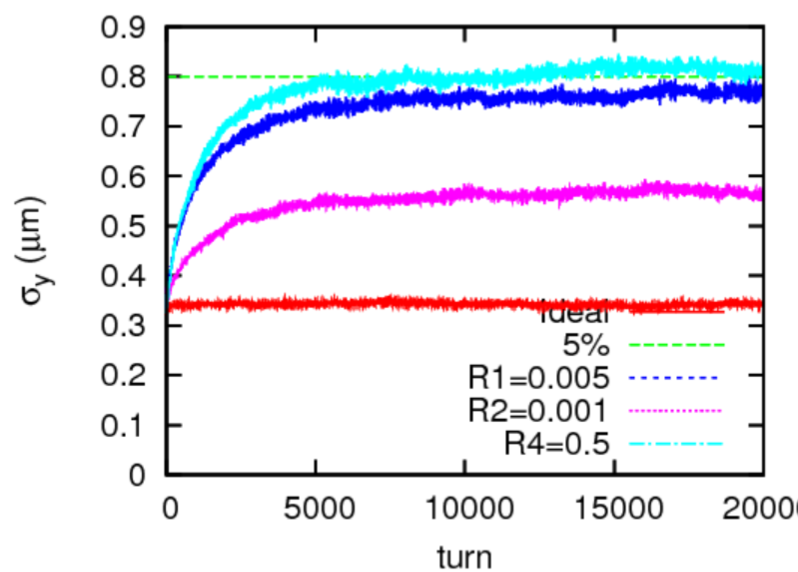
$$\sigma_{yc} = \sqrt{\sigma_{y+}^2 + \sigma_{y-}^2}$$

Luminosity in a weak-strong simulation

- BBWS, strong e- beam 5% coupling 285mA, βx=200mm, βy=4mm, early stage of parameters

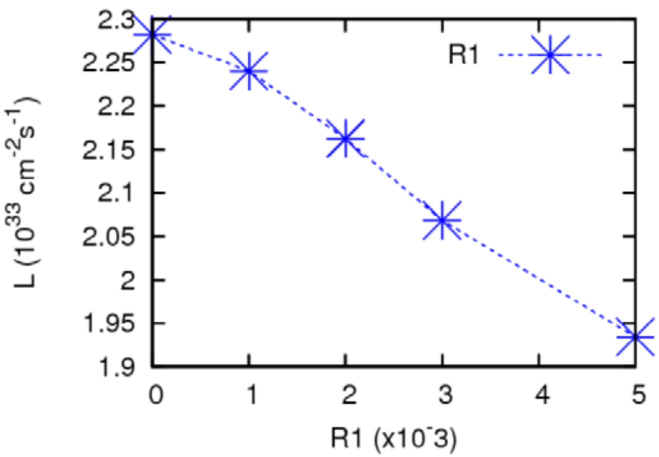
  weak e+ beam 1% coupling 340mA

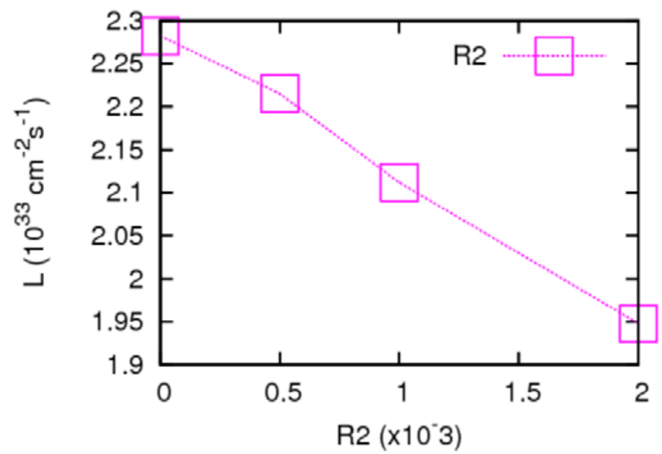




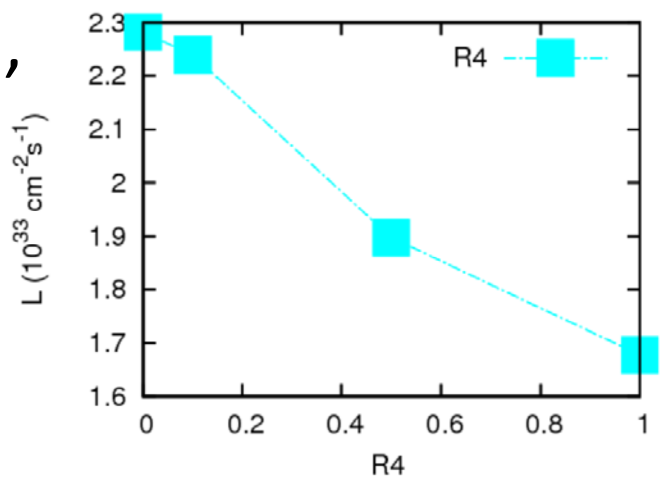
Even in very conservative condition of the simulation, measured luminosity was half of simulation.

R scan in the simulation

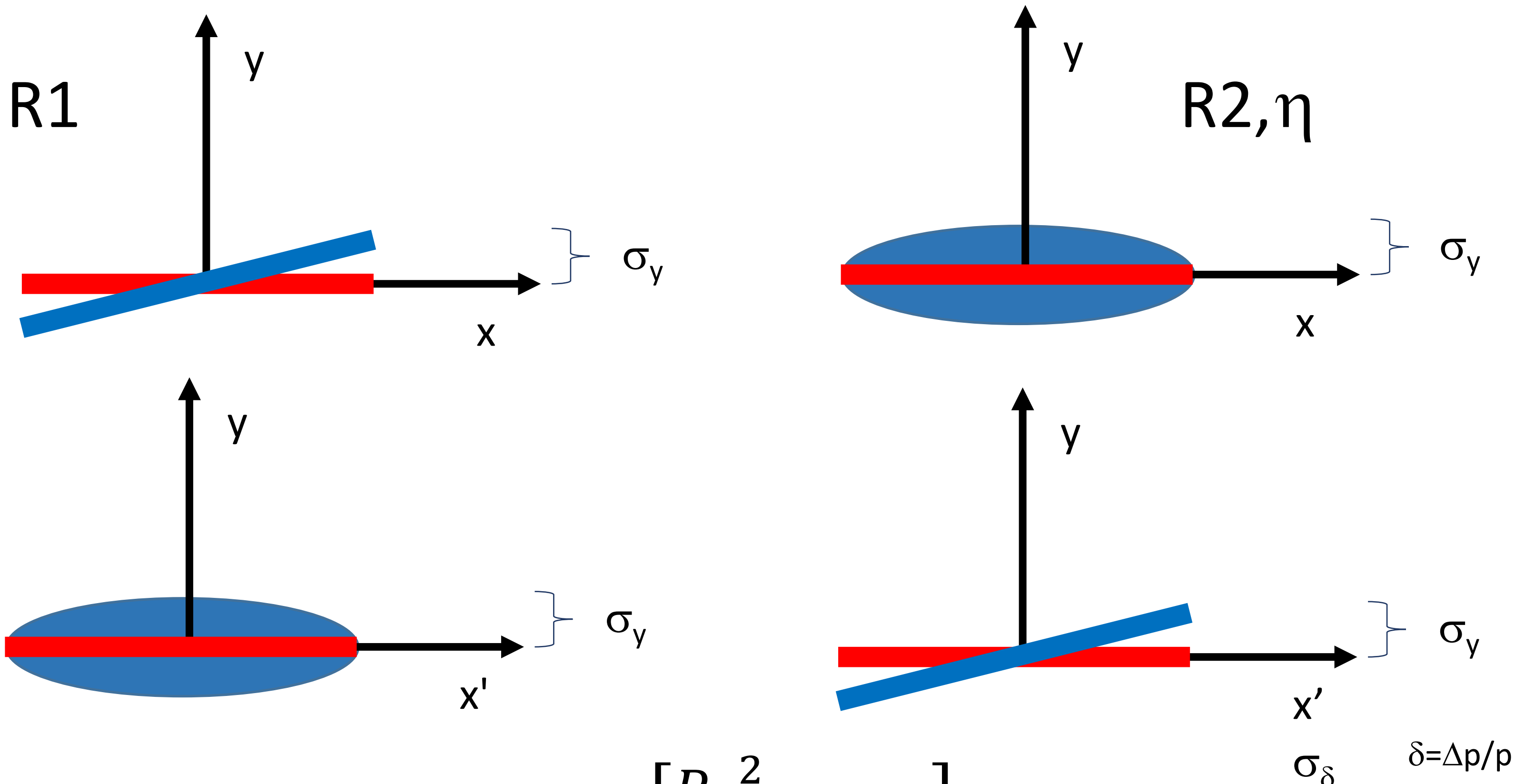




- Required tuning range R1 O(mrad), R2 O(mm), R3 O(1m^{-1}),R4 O(0.1)
- R2 scanned O(0.01-0.1mm)
- Lack of tuning range especially in R2.



IP coupling and beam distribution at IP



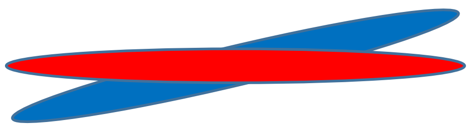
$$\sigma_y^2(s=0) \approx \sigma_{y,0}^2 + \sigma_x^2\left[\frac{{R_2}^2}{\beta_x^2} + {R_1}^2\right] + \left(\eta_y\sigma_\delta\right)^2$$

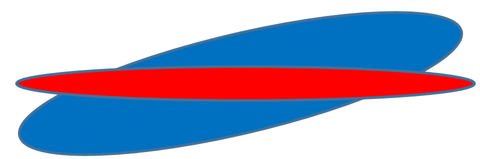
We do not change IR magnets for squeezing β*, R2 is kept.
Effect of R2 is enhanced for squeezing β*.

Beam shape at IP with IP coupling

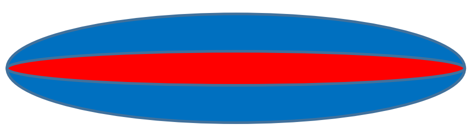
low current high current

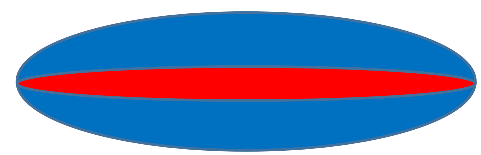
- R1





- R2





Discrepancy from L calculated by the measured beam size

Better agreement with L calculated by the measured beam size

- In either case, luminosity better agree with that given by the measured beam size at high current,
- Emittance growth is remarkable for beam with coupling.

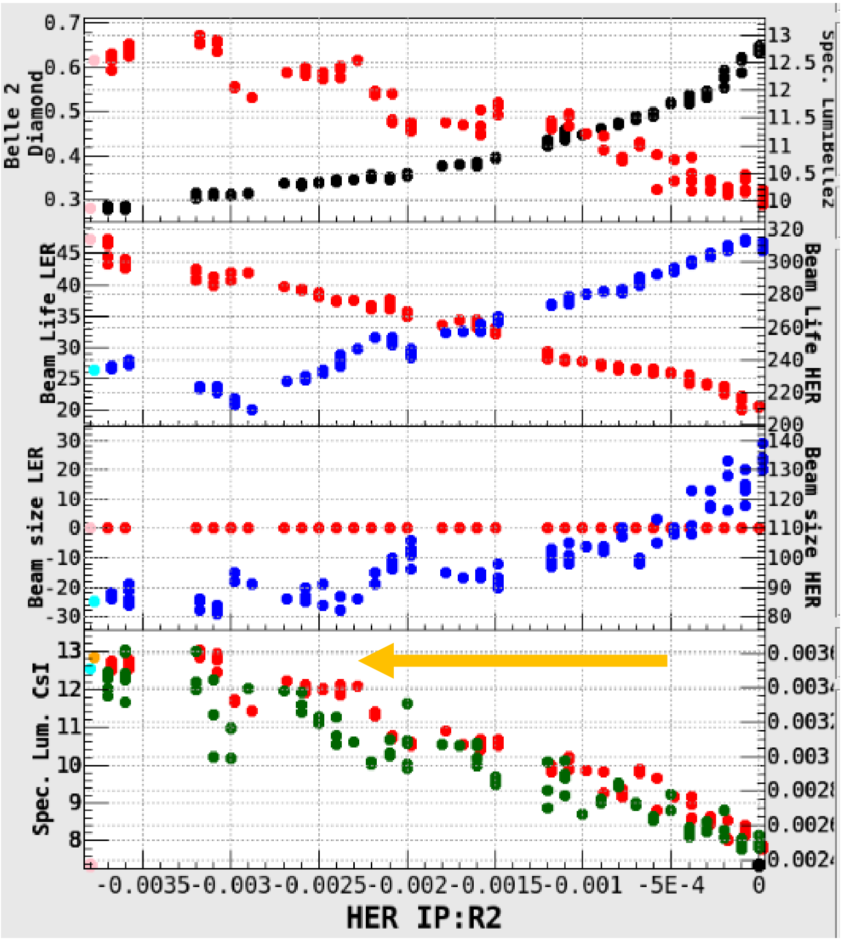
HER R2 scan in June 15, 2018

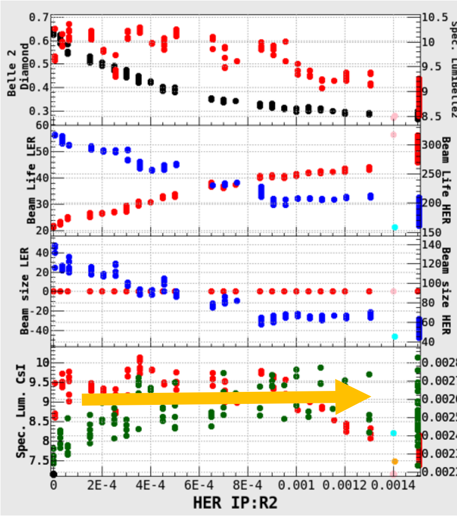
Increase tuning range of R2, R2 correction scheme is changed so using sextupole bump as is done in KEKB, although there are side effects.

- R2=-3.9mm
- I+=340mA
- I-=285mA
- 789 bunch
- no inj

$$\Delta\sigma_y = \frac{R_2}{\beta_x}\sigma_x = 0.8\mu m$$

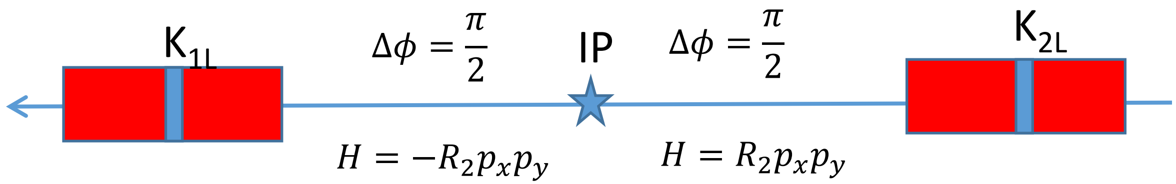
$=2\sigma_y$





Relation of R and skew strength of QC1 in a simple model

- Transformation of R2,



$$H = \pm R_2 p_x p_y \qquad\qquad y = y \pm R_2 p_x$$

- Assume π/2 for phase difference between IP to both QC1.

$$H = \pm \frac{R_2}{\sqrt{\beta_x^* \beta_{x,1}} \sqrt{\beta_y^* \beta_{y,1}}} xy \approx \pm R_2 xy$$

- Skew quad at QC1 is B'L/Bρ=R_2, which is independent of β*.
- Deviation from π/2 induces R3.
- Control of inside of π section is hard from outside. It should be corrected by both side of skew. (like waist correction)

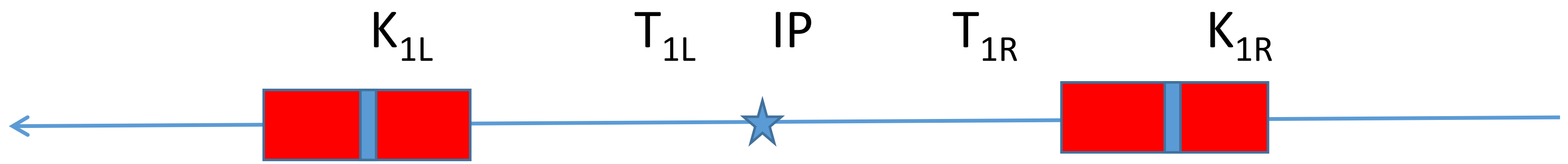
$$RM_{2\times2}R^{-1} = e^{-R_2 p_x p_y} M_{2\times2} e^{R_2 p_x p_y}$$

- We do not change IR magnets for squeezing β*, R2 is kept.
- Effect of R2 is enhanced for squeezing β*.

$H = ds p_y^2$

waist shift

Skew Q component at QC1



$M_{rev}(k_{1L},k_{1R})= T_{1R}K_{1R}T_{1R}^{-1}M_0T_{1L}^{-1}K_{1L}\,T_{1L}$ K_1: skew thin matrix with k_1.

$M_{rev}(R)=RM_0R^{-1}$ M_0: rev. matrix w/o coupling

Solve[$M_{rev}(k_{1L},k_{1R})$==$M_{rev}(R)$,{R}] focus off-diagonal 2x2 matrix

$R_{1\text{-}4}$ are represented by k_{1L},k_{1R}.

$M_{rev,R}(k_{1L},k_{1R})= M_0T_{1L}^{-1}K_{1L}\,T_{1L}T_{1R}K_{1R}T_{1R}^{-1}$ $R_{1\text{-}4,L}$ are also given. $R_{1\text{-}4,L}$

Skew correction at realistic IR

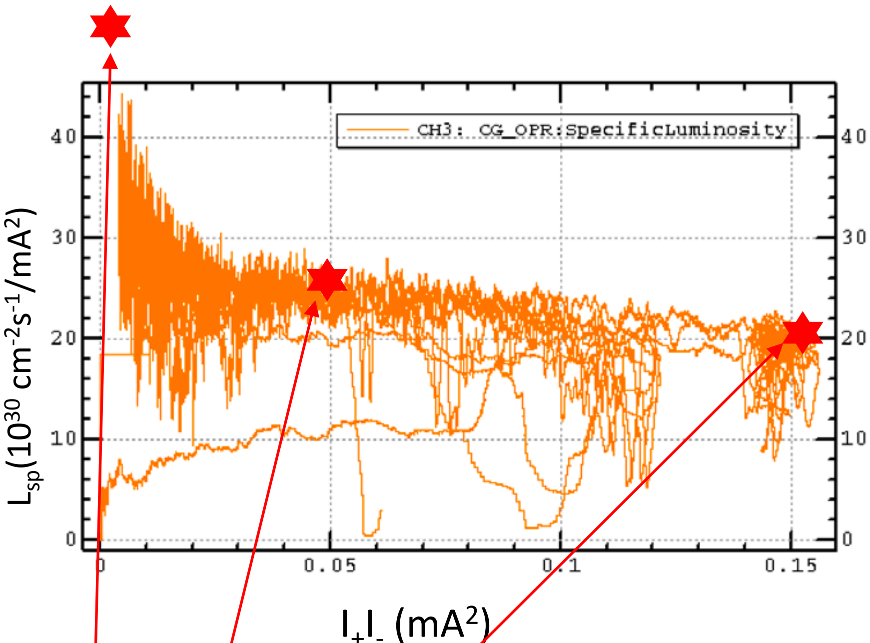
- $\beta_x^*=0.1$, $\beta_y^*=0.003$, (MKS)
- $\beta_{x,1}=4.46$, $\alpha_{x,1}=-7.52$, $\phi_{x,1}=0.236$, $\beta_{y,1}=329$, $\alpha_{y,1}=-12.3$, $\phi_{y,1}=0.2495$

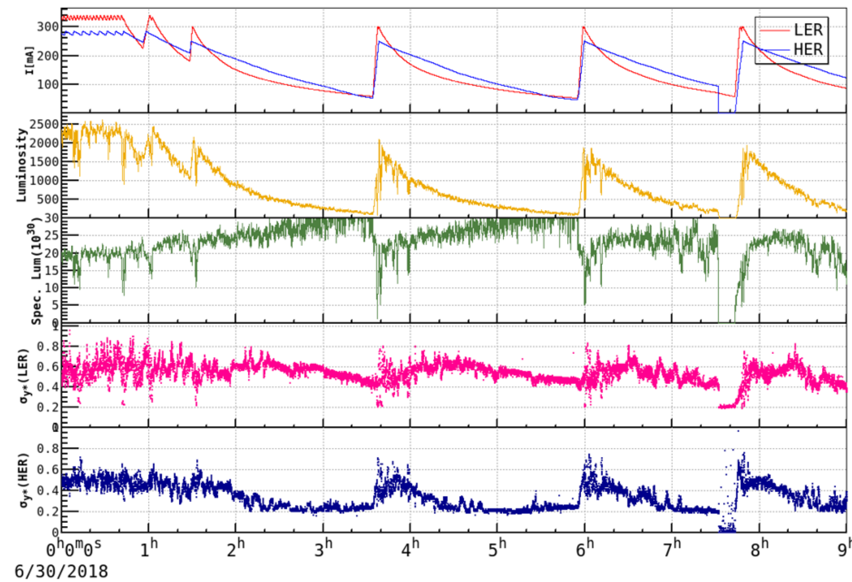
- $R_1=-14.9\ k_{L1}-14.9\ k_{R1}$, $R_2=-0.716\ k_{L1}+0.716\ k_{R1}$,
- $R_3=-487\ k_{L1}+487\ k_{R1}$, $R_4=-1156\ k_{L1}-1156\ k_{R1}$

- For $k_{L1}= -k_{R1} =0.0021$, $R_1=R_4=0$, $R_2=0.003$, $R_3=-2.05$.
- R_3 leaks outside of IR due to the deviation of betatron phase from $\pi/2$.
- Correct x-y coupling due to the leakage of R_3 globally.
- Detailed values are determined by SAD (Ohnishi).

June 30, 2018

Observations





- 0mA, σ_{y0}=0.25μm, 0.25μm, Lsp=49
- 200x160mA, σ_{y0}=0.4μm, 0.6μm, Lsp=24.4
- 285x340mA, σ_{y0}=0.6μm, 0.6μm, Lsp=20.7

$$L_{sp} = \frac{1}{2\pi\sigma_{xc}\sigma_{yc}e^2 f_0}$$

10^{30} cm^{-2}s^{-1}/mA^2

$$\sigma_{yc} = \sqrt{\sigma_{y+}^2 + \sigma_{y-}^2}$$

Lsp agrees with geo value at every current

6/29 21:00- R2 using QCS corrector

L_{peak}=2.5x10^{33} cm^{-2}s^{-1} , (2 times higher)

285x340mA, N_b=788

Blow-up of e+ beam was serious.

TbT measurement

$r_i = R_i$

- y motion in X mode.

$$\boldsymbol{x} = RB\boldsymbol{X}$$

$$B = \begin{pmatrix} B_X & 0 \\ 0 & B_Y \end{pmatrix}$$

$$R = \begin{pmatrix} r_0 & 0 & r_4 & -r_2 \\ 0 & r_0 & -r_3 & r_1 \\ -r_1 & -r_2 & r_0 & 0 \\ -r_3 & -r_4 & 0 & r_0 \end{pmatrix}$$

$$B_X = \begin{pmatrix} \sqrt{\beta_X} & 0 \\ -\alpha_X/\sqrt{\beta_X} & 1/\sqrt{\beta_X} \end{pmatrix}$$

r1: cos component of y for x betatron motion ,r2: sin component

$$y = -r_1 x - r_2 p_x = -r_1 a \cos\phi(s) + r_2 \left[\frac{a}{\beta}\sin\phi(s) + \frac{\alpha}{\sqrt{\beta}} a \cos\phi(s)\right]$$

$$= c\cos(2\pi n \nu_x + \phi_y)$$

$$\phi(s) = 2\pi n \nu_x + \phi_x$$

$$\frac{c}{a}\cos(\phi_y - \phi_x) = \left(-r_1 + r_2 \frac{\alpha}{\sqrt{\beta}}\right) \qquad \frac{c}{a}\sin(\phi_y - \phi_x) = \frac{r_2}{\beta}$$

r3: cos component of y for px betatron motion ,r4: sin component

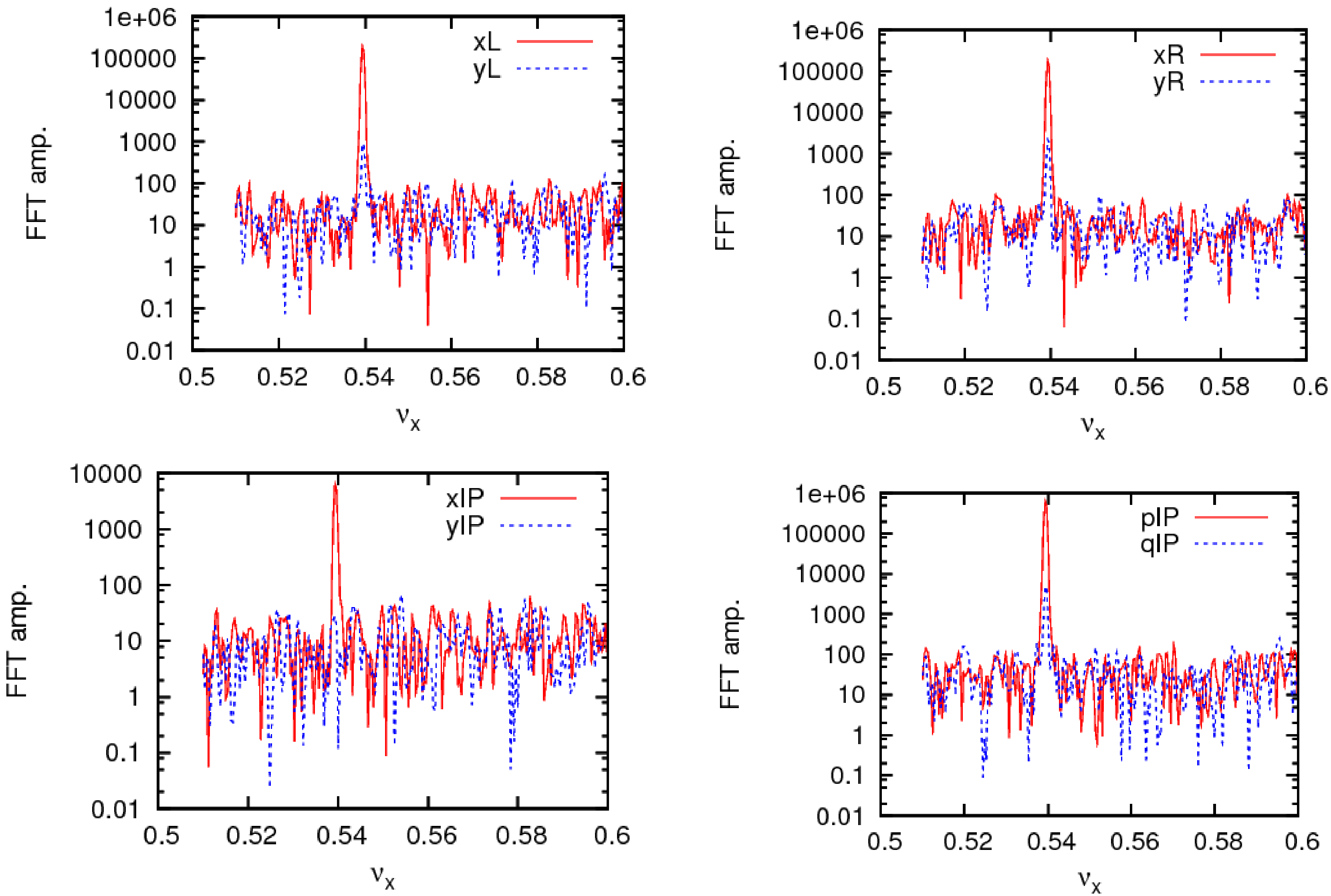
$$p_y = r_3 x - r_4 p_x = r_3 a \cos\phi(s) + r_4 \left[\frac{a}{\beta}\sin\phi(s) + \frac{\alpha}{\sqrt{\beta}} a \cos\phi(s)\right]$$

$$= d\cos(2\pi n \nu_x + \phi_q)$$

$$\frac{d}{a}\cos(\phi_q - \phi_x) = \left(r_3 + r_4 \frac{\alpha}{\sqrt{\beta}}\right) \qquad \frac{d}{a}\sin(\phi_q - \phi_x) = -\frac{r_4}{\beta}$$

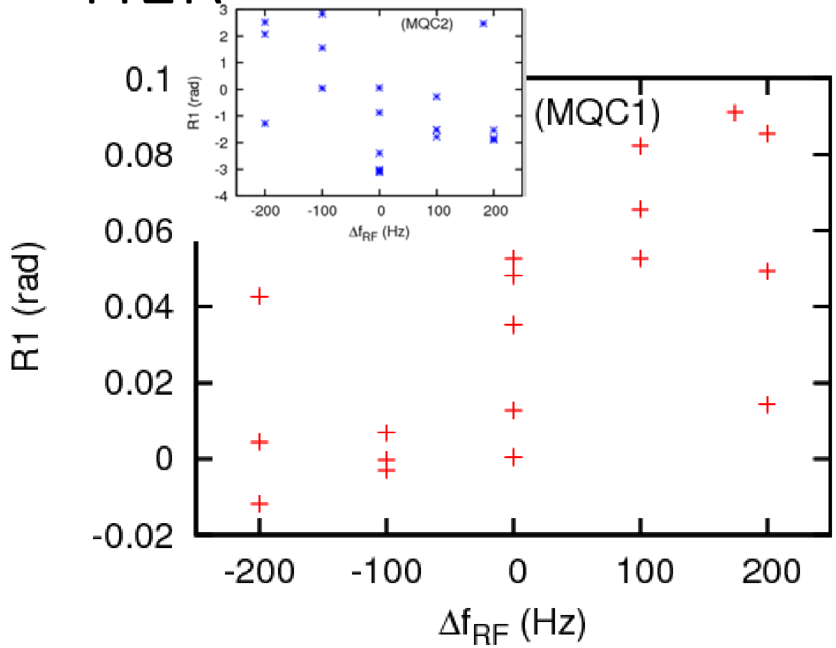
FFT of BPM data

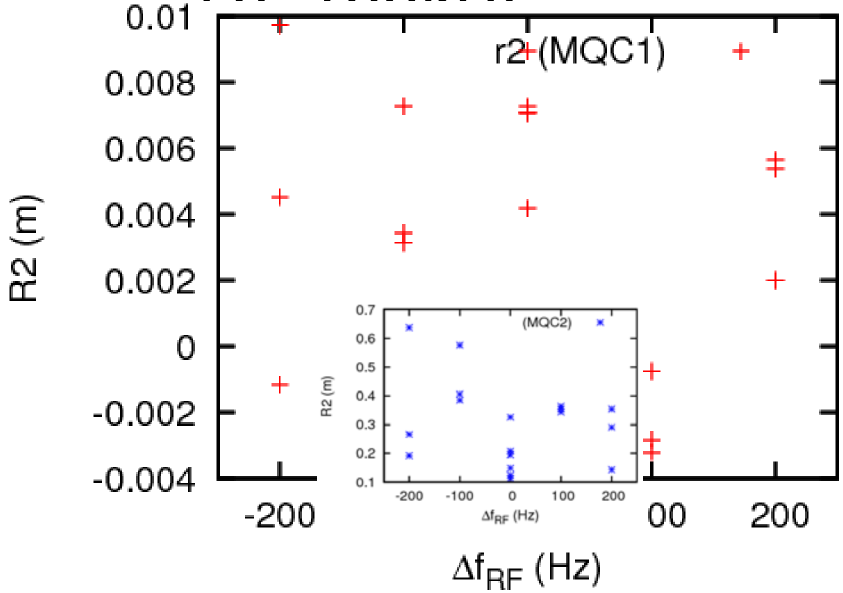
- Small y_{IP}, but enough $p_{yIP}=q_{IP}$.

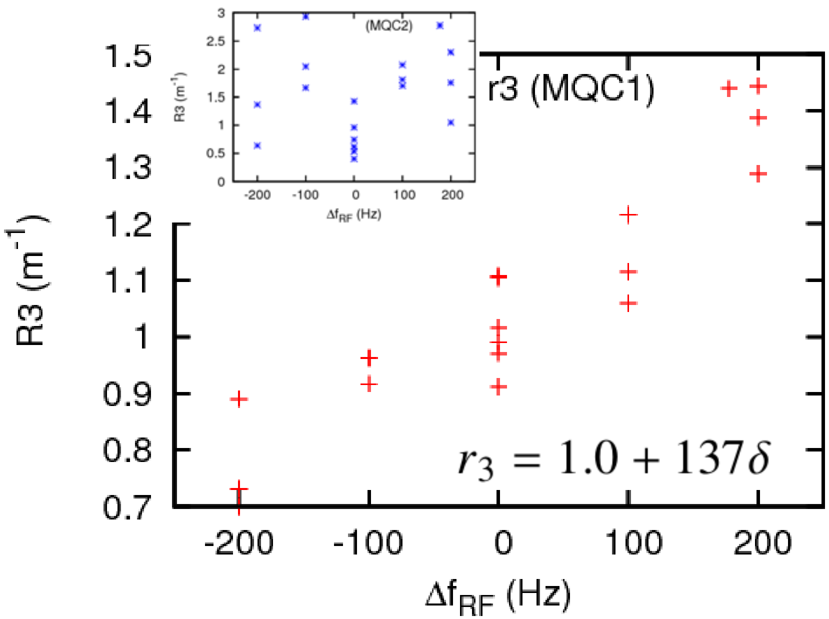


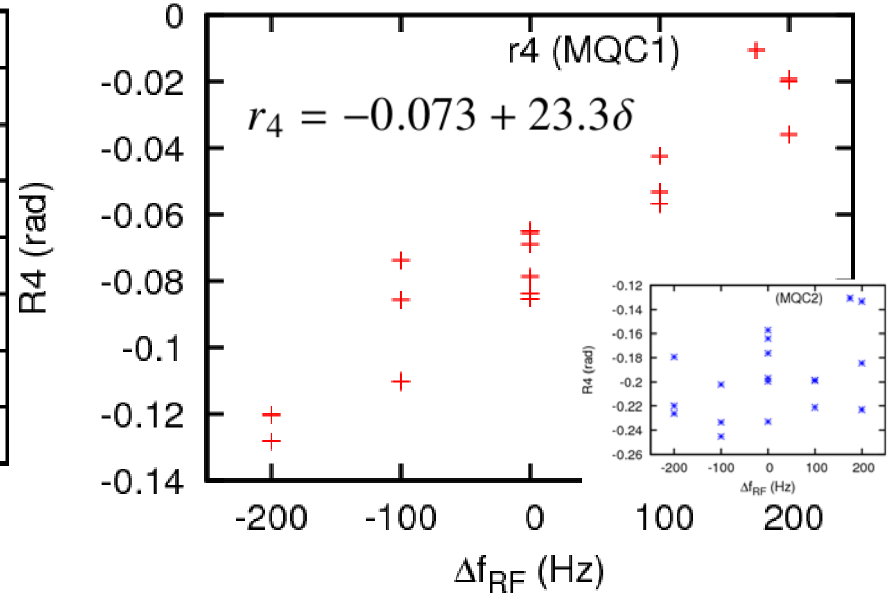
HER

- F0=100kHz

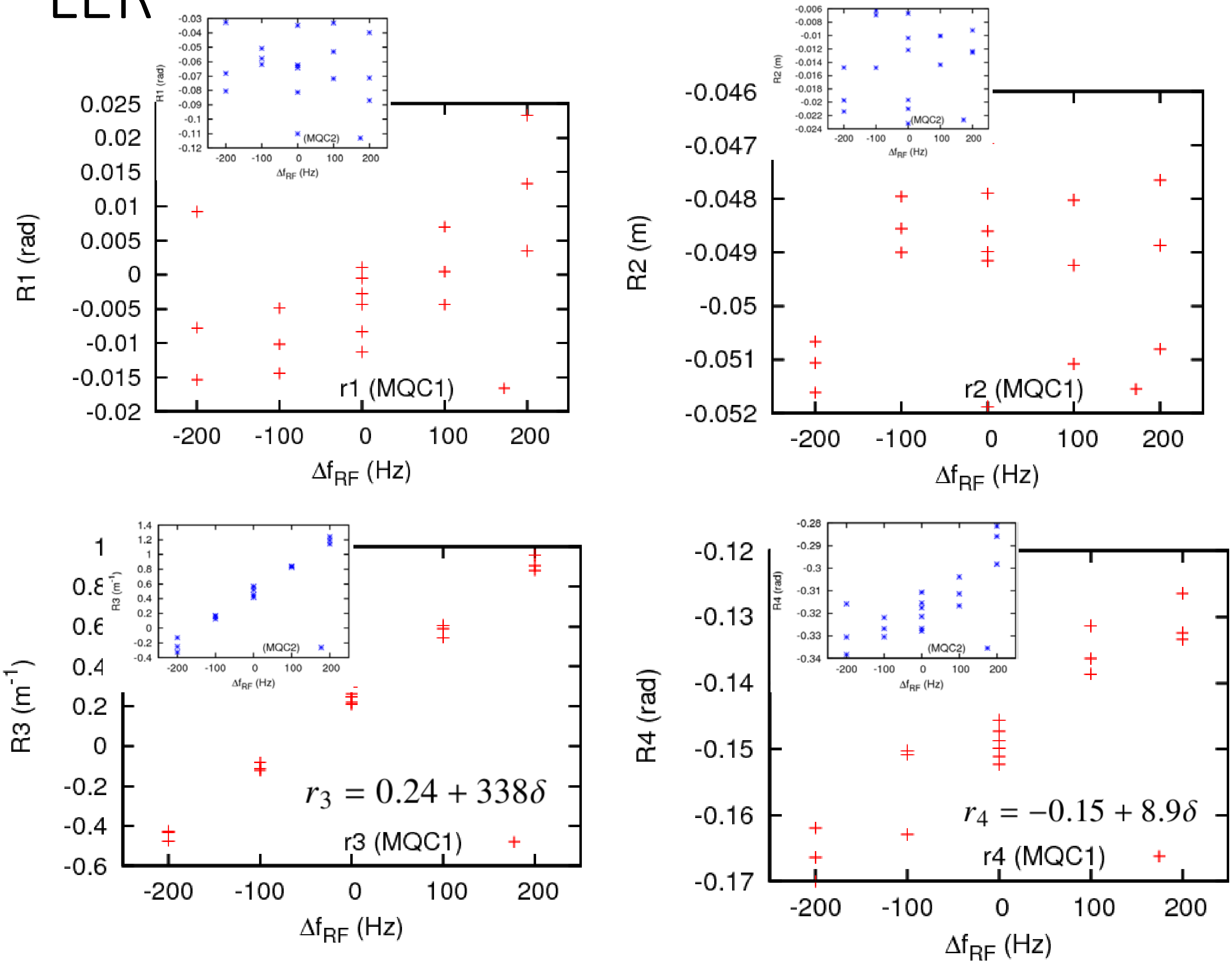








LER

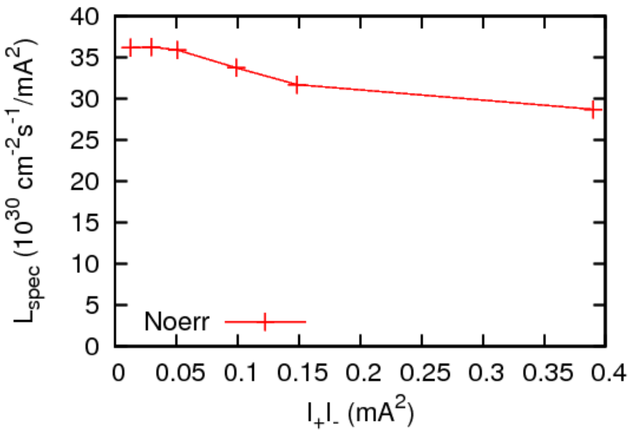

R1 (rad)
Δf_RF (Hz)
r1 (MQC1)
R1 (rad)
(MQC2)
R2 (m)
r2 (MQC1)
(MQC2)
R3 (m^-1)
$r_3 = 0.24 + 338\delta$
r3 (MQC1)
(MQC2)
R4 (rad)
$r_4 = -0.15 + 8.9\delta$
r4 (MQC1)
(MQC2)

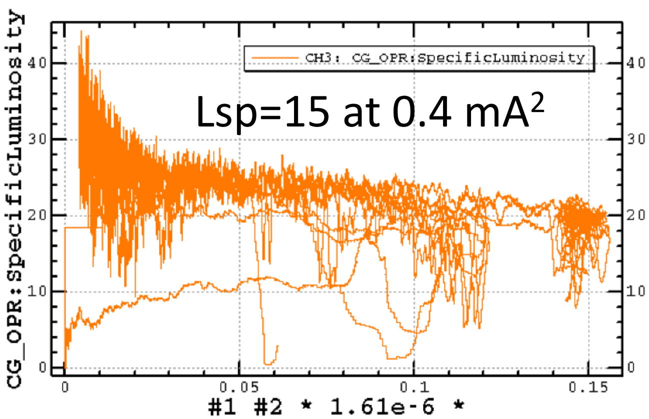
Toward Phase III

- Squeezing beta*, Luminosity increase is not trivial at all without IP optics tuning.

Lspec without error

Measured Lspec





Luminosity is half at I_+I_-=0.4mA^2.

Design 1.5 mA^2. β_y* 1/10

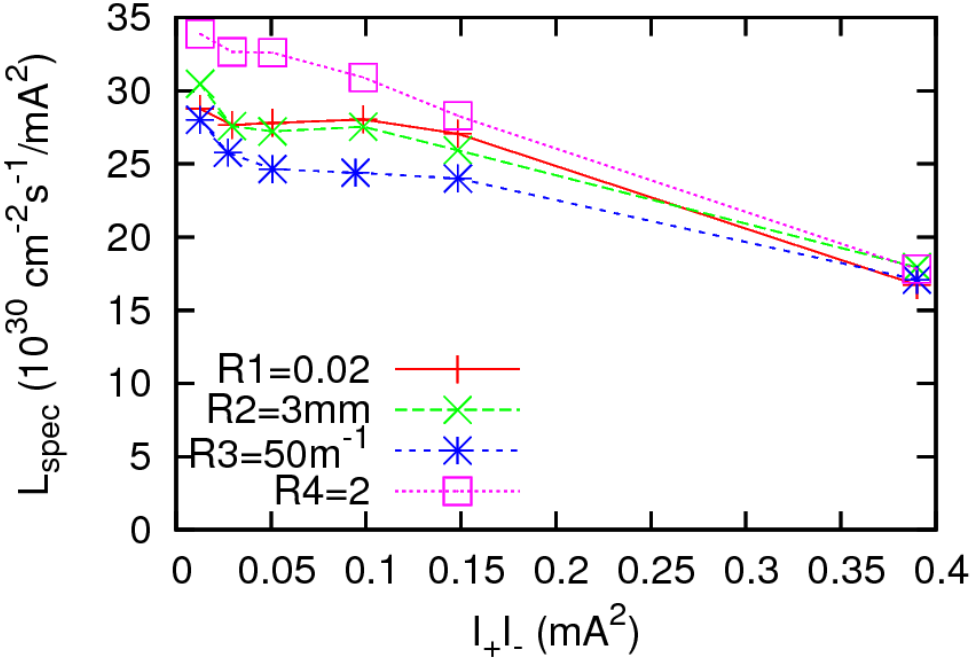
Beam-beam simulation considering optics aberrations at IP

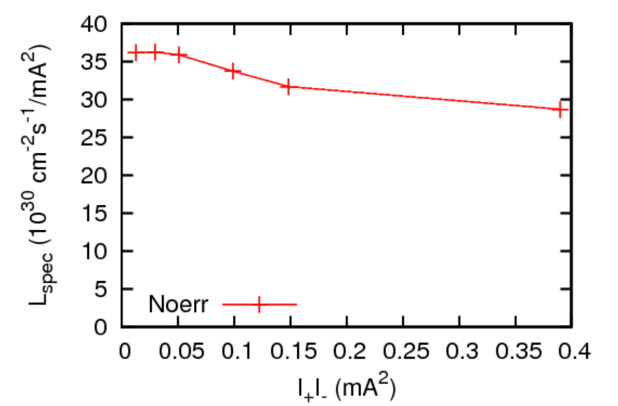
- Linear
- Nonlinear
- Chromatic

- Recent operation showed e+ beam is weaker than e- beam. Weak(e+)-strong(e-) simulation is performed.

Weak(e+)-strong(e-) simulation with errors

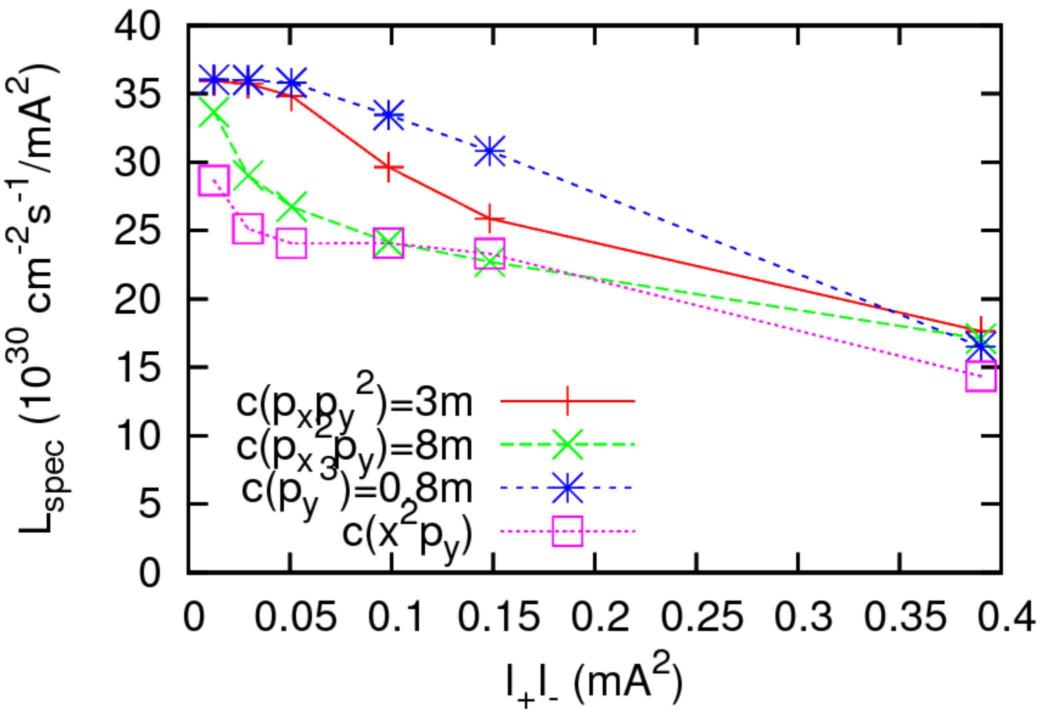
- Error strengths of R3 and R4 are much larger than measurement. Discard.
- R1 and R2 were already scanned and given optimum.
- We cleared linear aberrations in Phase-II.





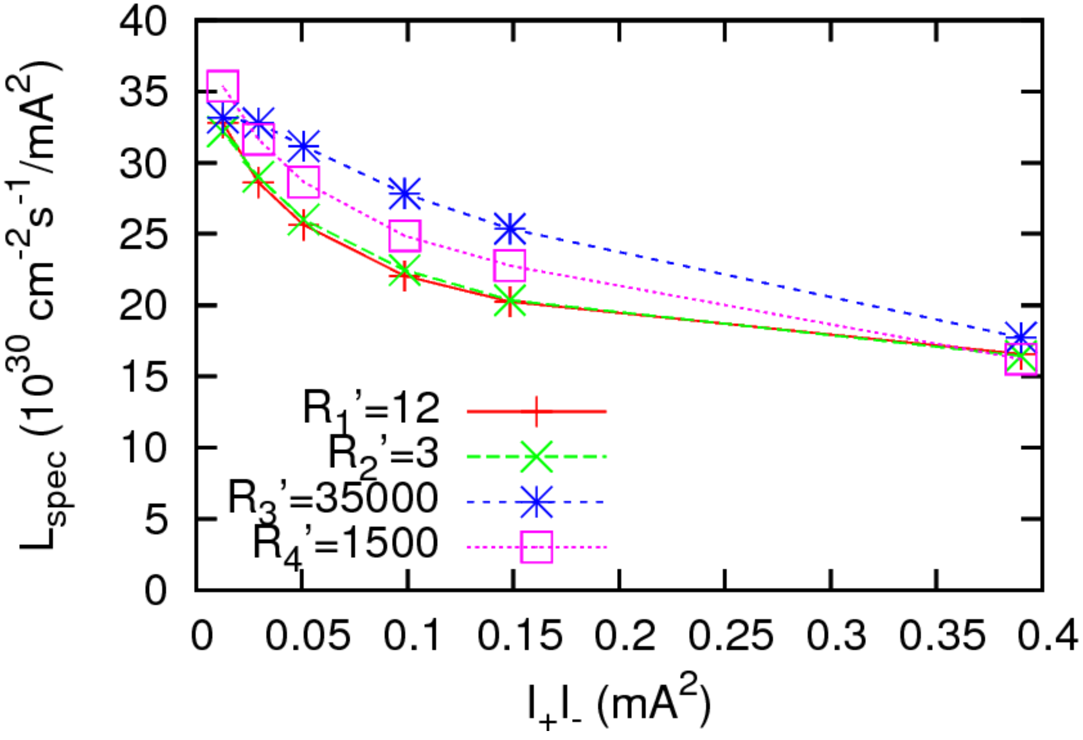
Nonlinear aberrations

- $p_x^2 p_y$ term was studied before commission.
- $p_x^2 p_y$ term well reproduces measured L_{sp}.
- The strength is 100 times larger than the value given by design of QCS. $c_{10}=c(p_x^2 p_y)=0.07$.



Chromatic coupling

- R3’ and R4’ were measured to be R3’=300, R4’=20.
- The behaviors for R1’ and R2’ are plausible.
- R1’ and R2’ are hard to be measured in the present monitor. R1’ ~-10 in measurement?



Summary

- SuperKEKB is squeezing β^* step-by-step in the commissioning.
- Luminosity increase proportional to β_y^* is not trivial at all.
- High Luminosity is only achieved, when the optics aberration at IP are perfectly corrected.
- QCS as error source and corrector is key component.
- Errors induced at QCS are enhanced for squeezing β^*.
- Correction of nonlinear aberration is next target in Phase-III commissioning.
- Final target, Lsp=220x10^{30}cm^{-2}s^{-1}mA^{-2}.

Thank you for your attention

Transfer matrix, M

$$M = RBUB^{-1}R^{-1} = RM_{2\times2}R^{-1}$$

- Matrix transformation for R.

$$R = \begin{pmatrix} r_0 & 0 & r_4 & -r_2 \\ 0 & r_0 & -r_3 & r_1 \\ -r_1 & -r_2 & r_0 & 0 \\ -r_3 & -r_4 & 0 & r_0 \end{pmatrix} \quad B = \begin{pmatrix} B_X & 0 \\ 0 & B_Y \end{pmatrix}$$

$$\bar{y} = r_0 y - r_1 x - r_2 p_x$$

$$\bar{p}_y = r_0 p_y - r_3 x - r_4 p_x$$

$$U = \begin{pmatrix} U_X & 0 \\ 0 & U_Y \end{pmatrix} \quad B_X = \begin{pmatrix} \sqrt{\beta_X} & 0 \\ -\alpha_X/\sqrt{\beta_X} & 1/\sqrt{\beta_X} \end{pmatrix}$$

$$U_X = \begin{pmatrix} \cos\phi_X & \sin\phi_X \\ -\sin\phi_X & \cos\phi_X \end{pmatrix}$$

- Corresponding canonical transformation for R.

$$G_2(x, \bar{p}_x, y, \bar{p}_y) = x\bar{p}_x + y\bar{p}_y + axy + b\bar{p}_x y - cx\bar{p}_y - d\bar{p}_x\bar{p}_y$$

$$\bar{y} = \frac{\partial G_2}{\partial \bar{p}_y} = y - cx - d\bar{p}_x \qquad p_y = \frac{\partial G_2}{\partial y} = \bar{p}_y + ax + b\bar{p}_x$$

$$\bar{x} = \frac{\partial G_2}{\partial \bar{p}_x} = x + by - d\bar{p}_y \qquad p_x = \frac{\partial G_2}{\partial x} = \bar{p}_x + ay - c\bar{p}_y$$

$$c(\delta) \approx r_1(\delta) \quad d(\delta) \approx r_2(\delta) \qquad a(\delta) \approx r_3(\delta) \quad b(\delta) \approx r_4(\delta)$$

6D transfer map for chromatic coupling

- 4D transfer for a(δ), b(δ), c(δ), d(δ)

$$R = \begin{pmatrix} 1+\frac{ad}{1+bc} & \frac{bd}{1+bc} & b\left(1-\frac{ad}{1+bc}\right) & -\frac{d}{1+bc} \\ -\frac{ac}{1+bc} & \frac{1}{1+bc} & -\frac{a}{1+bc} & \frac{c}{1+bc} \\ -c\left(1-\frac{ad}{1+bc}\right) & -\frac{d}{1+bc} & 1+\frac{ad}{1+bc} & \frac{cd}{1+bc} \\ -\frac{a}{1+bc} & -\frac{b}{1+bc} & \frac{ab}{1+bc} & \frac{1}{1+bc} \end{pmatrix}$$

or

$$\bar{p}_x = \frac{p_x - ay + cp_y + acx}{1+bc}$$

$$\bar{p}_y = p_y - ax - b\bar{p}_x$$

$$\bar{x} = x + by - d\bar{p}_y$$

$$\bar{y} = y - cx - d\bar{p}_x$$

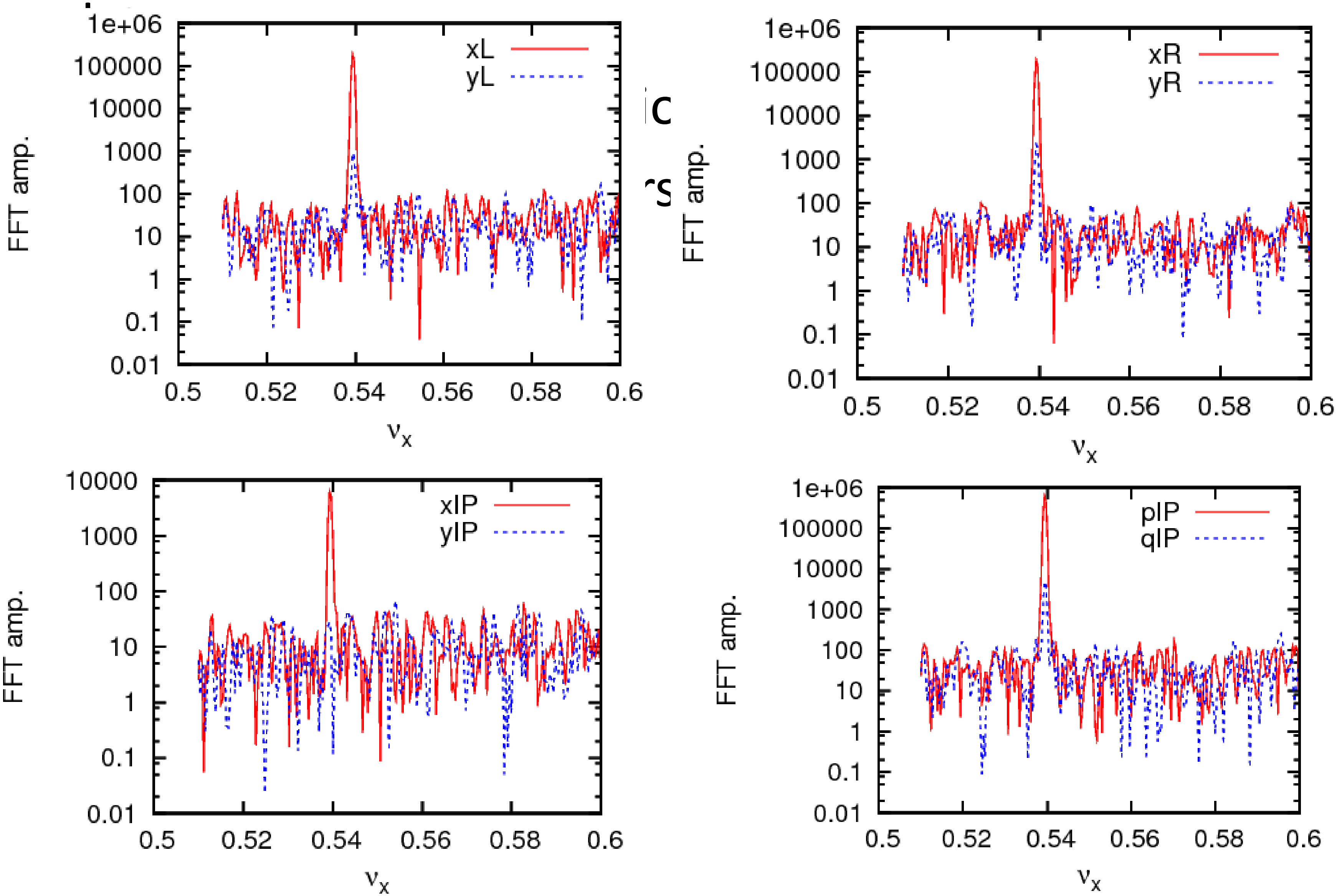
$$c(\delta) \approx r_1(\delta) \quad d(\delta) \approx r_2(\delta)$$

$$a(\delta) \approx r_3(\delta) \quad b(\delta) \approx r_4(\delta)$$

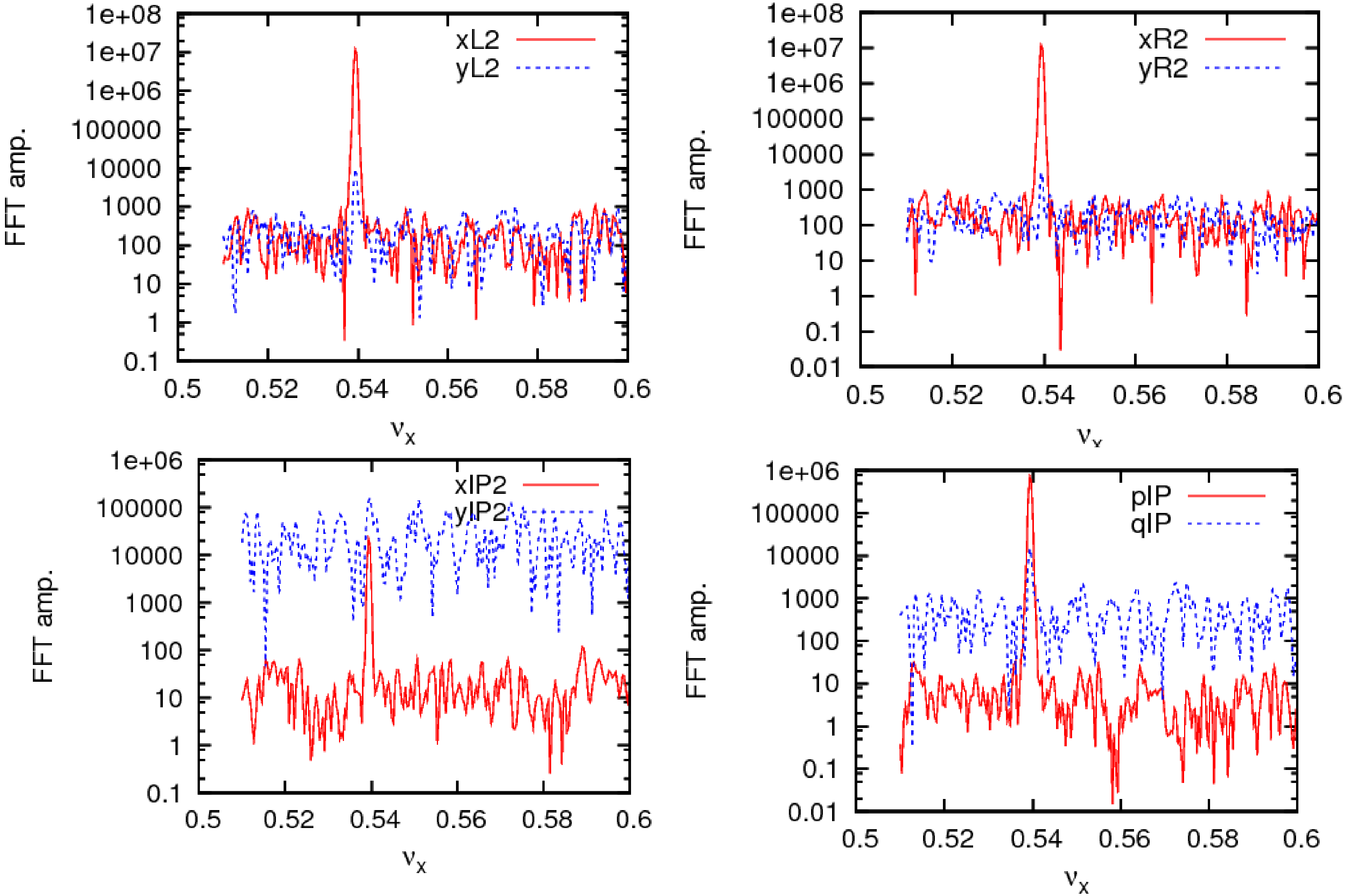
- Z transfer

$$\bar{z} = \frac{\partial G_2}{\partial \bar{p}_z} = z + a'xy + b'\bar{p}_x y - c'x\bar{p}_y - d'\bar{p}_x\bar{p}_y$$

- Take Fourier transformation of the BPM position

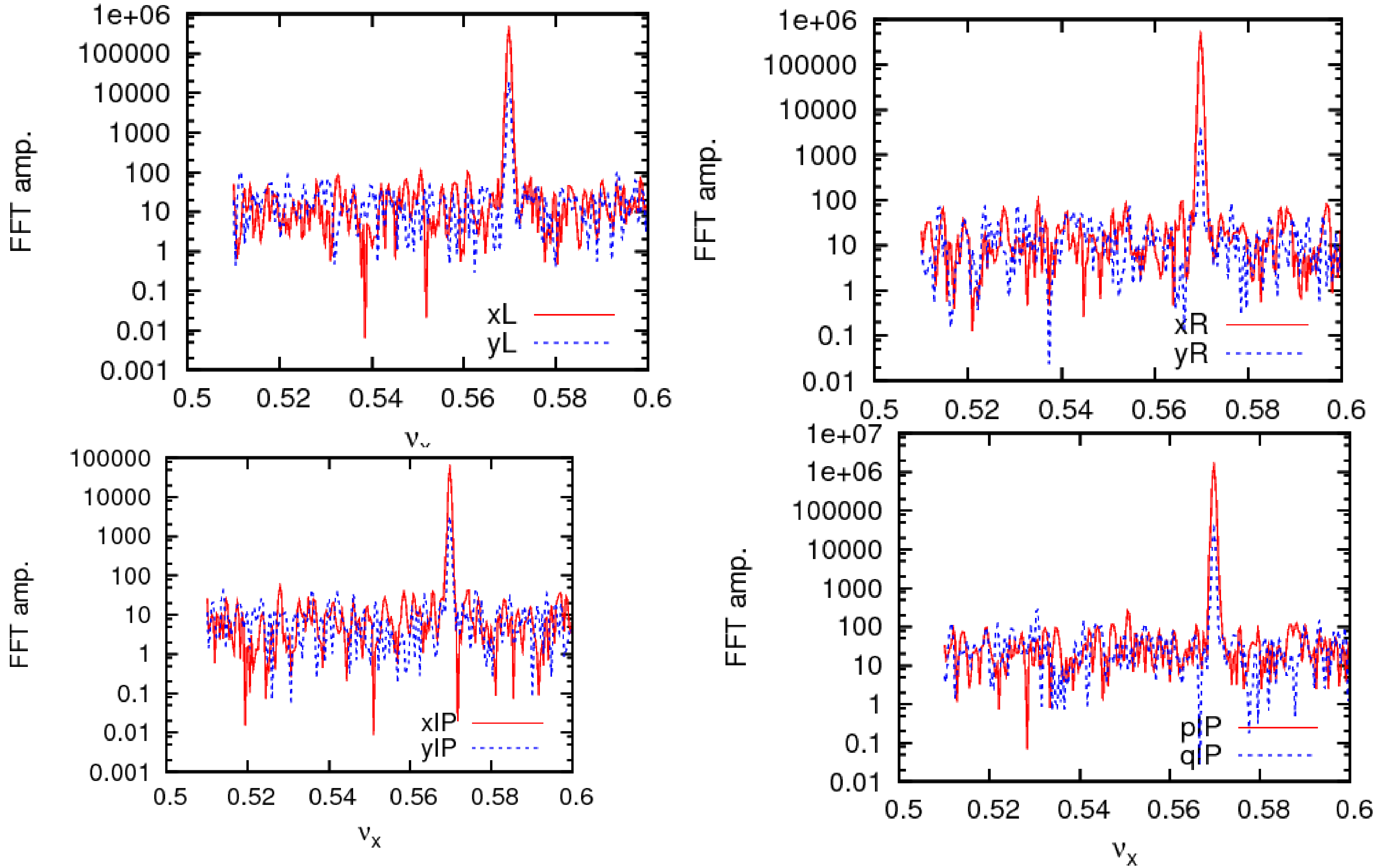


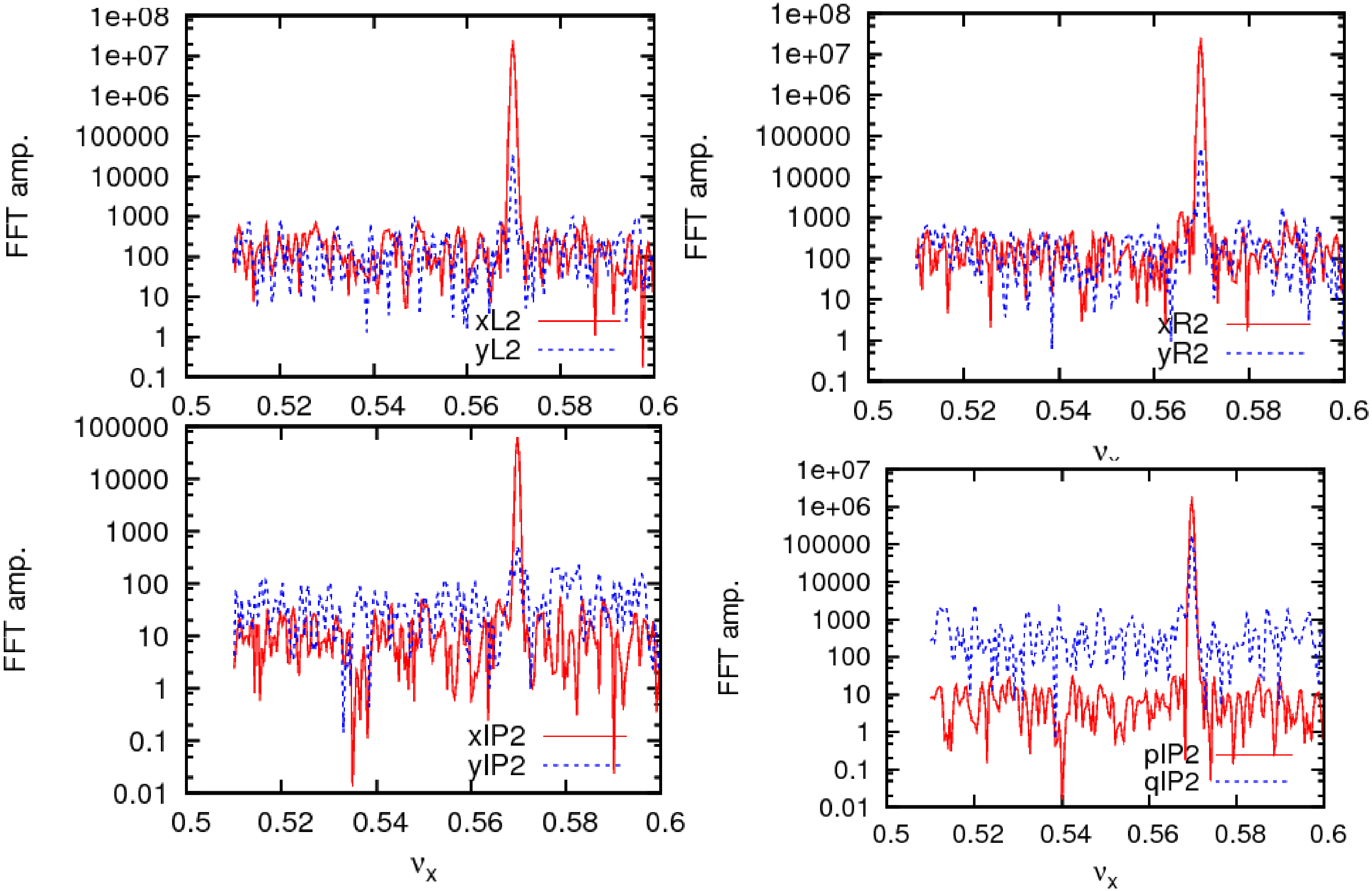
HER



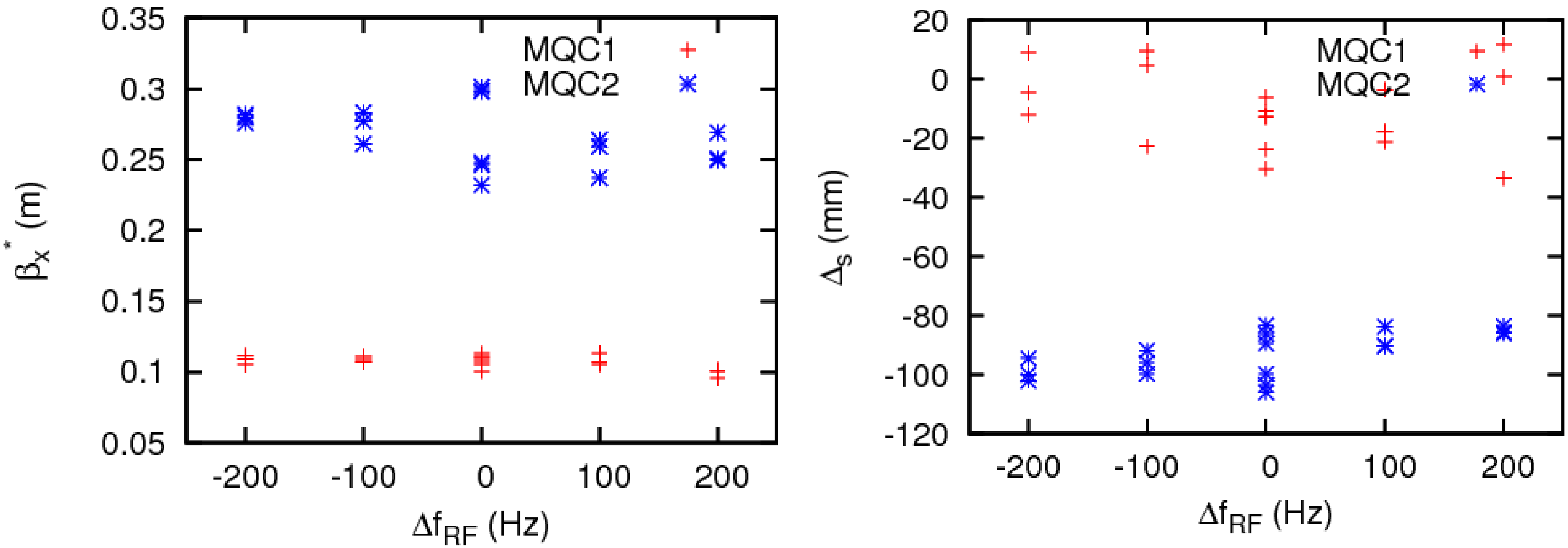
xL2
yL2
FFT amp.
νx
xR2
yR2
FFT amp.
νx
xIP2
yIP2
FFT amp.
νx
pIP
qIP
FFT amp.
νx

LER

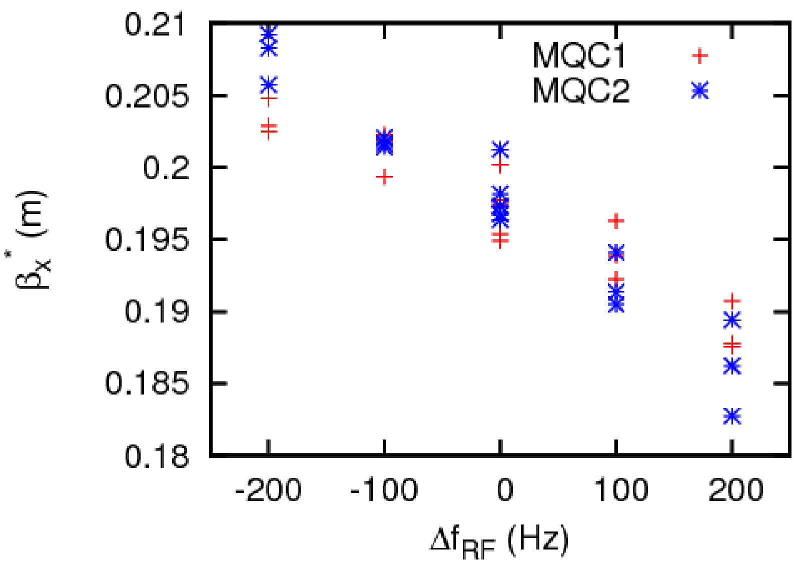

FFT amp.
xL
yL
ν_x
FFT amp.
xR
yR
FFT amp.
xIP
yIP
ν_x
FFT amp.
pIP
qIP
ν_x



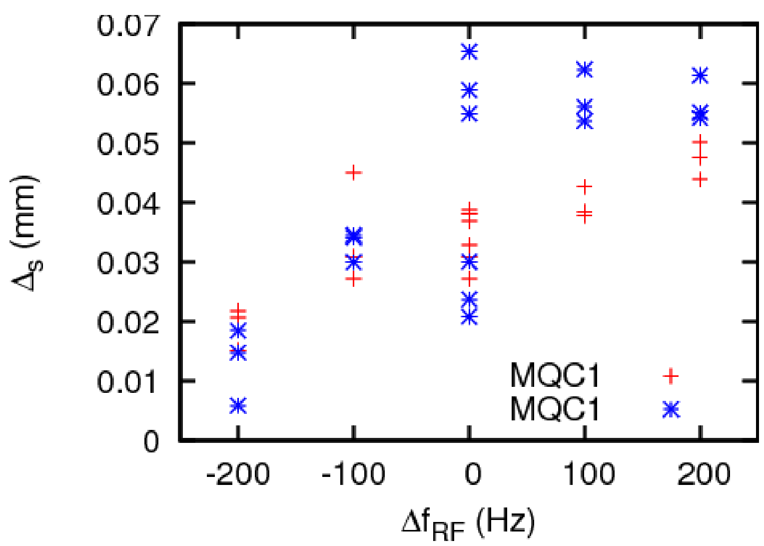
FFT amp.
1e+08
1e+07
1e+06
100000
10000
1000
100
10
1
0.1
0.01
0.5
0.52
0.54
0.56
0.58
0.6
xL2
yL2
xR2
yR2
xIP2
yIP2
pIP2
qIP2
ν_x



0.35
0.3
0.25
0.2
0.15
0.1
0.05
β_x^* (m)
-200
-100
0
100
200
Δf_{RF} (Hz)
MQC1
MQC2
20
0
-20
-40
-60
-80
-100
-120
Δ_s (mm)
-200
-100
0
100
200
Δf_{RF} (Hz)
MQC1
MQC2

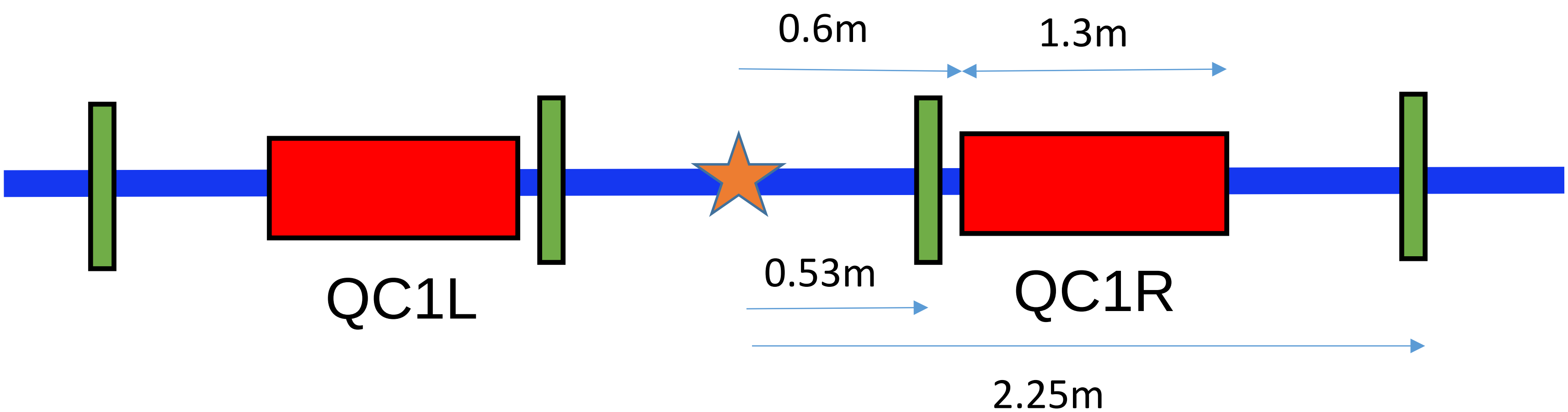


0.21
0.205
0.2
0.195
0.19
0.185
0.18
β_x^* (m)
-200
-100
0
100
200
Δf_RF (Hz)
MQC1
MQC2



0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Δ_s (mm)
-200
-100
0
100
200
Δf_RF (Hz)
MQC1
MQC1

Beam motion at Interaction Region (IR)



AX	BX	NX	EX	Element	s(m)	AY	BY	NY	#
118.364	190.713	-.2414	.00147	MQC2RE	3013.81	-135.99	263.746	-.2501	5691
5.29905	2.90873	-.2204	1.95E-6	MQC1RE	3015.78	176.638	93.6284	-.2491	6474
-9.E-13	.10000	.00000	1.2E-13	IP.1	.000000	3.0E-12	.00300	.00000	1
-5.2990	2.90874	.22032	-1.9E-6	MQC1LE	.53000	-176.63	93.6286	.24910	112
-106.71	142.968	.24102	-.00130	MQC2LE	2.2500	168.532	291.140	.24998	793

Betatron phase, betatron tune

- Beam position variation

$$x_n = a\cos(2\pi n\nu_x + \phi_x)$$

ϕ_x: Initial betatron phase at a position.

$$\boldsymbol{X} = \sqrt{W}\begin{pmatrix}\cos\phi(s) \\ -\sin\phi(s)\end{pmatrix} = \begin{pmatrix}\dfrac{x}{\sqrt{\beta}} \\ \dfrac{\alpha x + \beta p_x}{\sqrt{\beta}}\end{pmatrix}$$

$$\phi(s) = 2\pi n\nu_x + \phi_x\text{c}$$

- Fourier transformation of beam position

$$x_\nu = \sum_{n=0}^{N} x_n \exp(2\pi i\nu n) \qquad x_\nu = \frac{a}{2}\exp(-i\phi_x)$$

- Betatron amplitude and phase

$$a = \sqrt{\beta W} = 2|x_\nu| \qquad \phi_x = -\tan^{-1}\left(\frac{\mathrm{Im}\, x_\nu}{\mathrm{Re}\, x_\nu}\right)$$

- α,β are determined by Fourier transformation of p_x.

$$p_n = -\frac{a}{\beta}\sin(2\pi n\nu_x + \phi_x) - \frac{\alpha}{\sqrt{\beta}}a\cos(2\pi n\nu_x + \phi_x)$$

$$p_\nu = \frac{b}{2}\exp(-i\phi_p) = \frac{a}{2}\left(-\frac{i}{\beta} + \frac{\alpha}{\sqrt{\beta}}\right)\exp(-i\phi_x)$$

$$\boxed{\left(-\frac{i}{\beta} + \frac{\alpha}{\sqrt{\beta}}\right) = \frac{b}{a}\exp\left(-i(\phi_p - \phi_x)\right)}$$

$$\beta = \frac{a}{b\sin(\phi_p - \phi_x)}$$

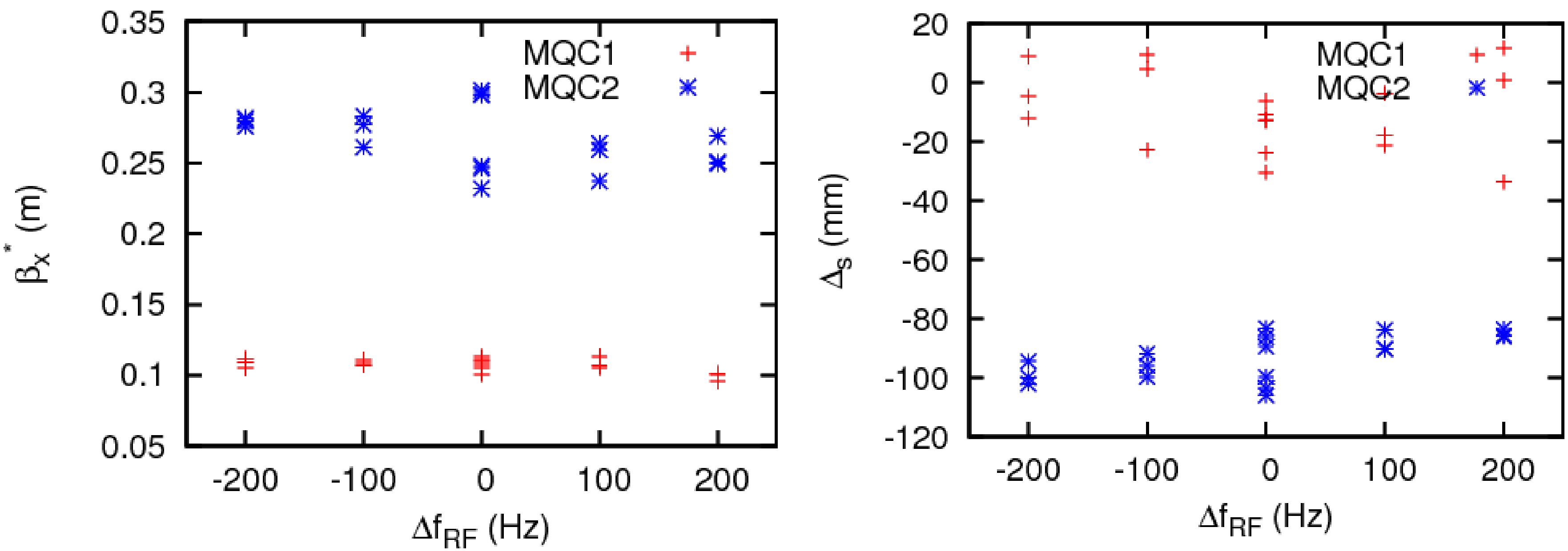
$$\alpha = \frac{a\sqrt{\beta}}{b}\cos(\phi_p - \phi_x)$$

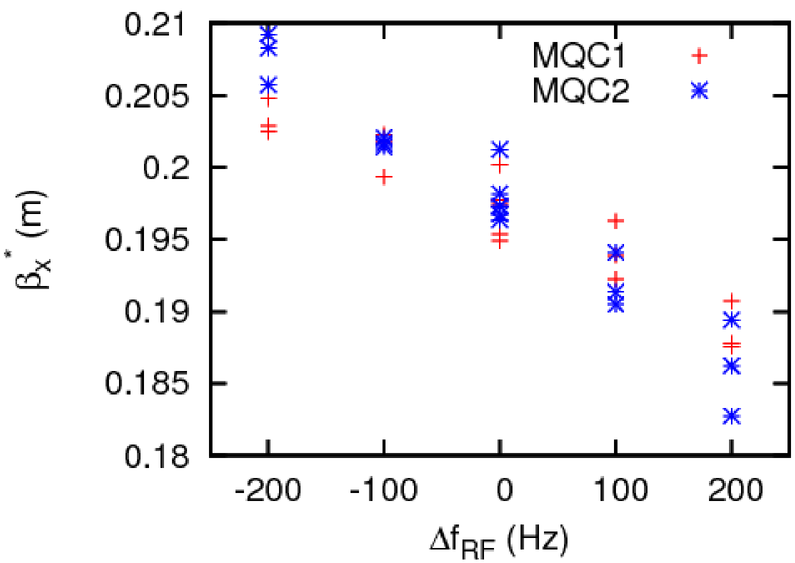
Tune 75 nu= 0.539297

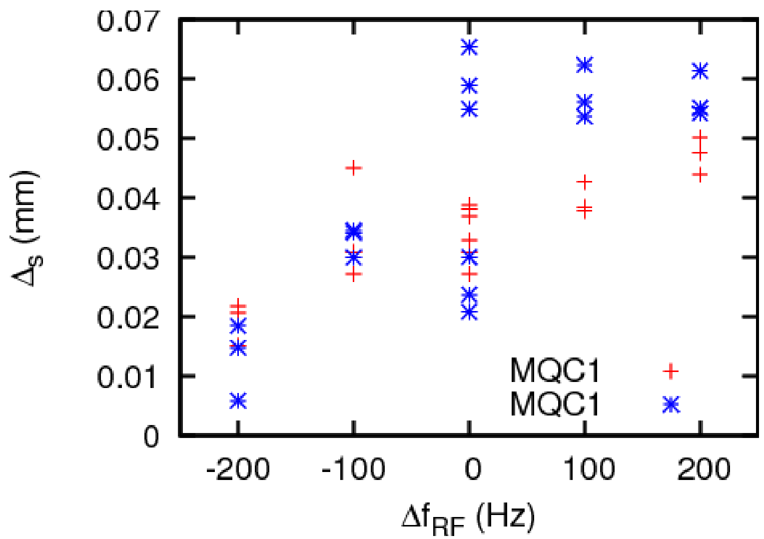
#	real	imag	phx/2p	real	imag	phy/2p
L	-47.348487	436.662161	0.267190	-28.252748	-10.712758	-0.442318
R	-110.801772	-417.576000	-0.291280	39.633016	26.146045	0.092814
L2	2.944297	3478.970627	0.249865	-97.595958	-12.765645	-0.479300
R2	-421.541901	-3431.803912	-0.269452	54.409751	1.071608	0.003134

Tune 75 nu= 0.539297

x	-79.613824	9.250941	0.481589	5.179247	0.947493	0.028797
p	60.033105	806.052591	0.238168	-61.666732	-34.973067	-0.417892
x2	-86.52538	121.238292	0.348652	318.590162	184.654037	0.083601
p2	49.458256	853.005395	0.240782	-119.215526	-21.533290	-0.471559


0.35
0.3
0.25
0.2
0.15
0.1
0.05
-200
-100
0
100
200
β_x^* (m)
Δf_{RF} (Hz)
MQC1
MQC2
20
0
-20
-40
-60
-80
-100
-120
Δ_s (mm)





- No dependence in R3