Low-emittance tuning for circular colliders

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With thanks to:
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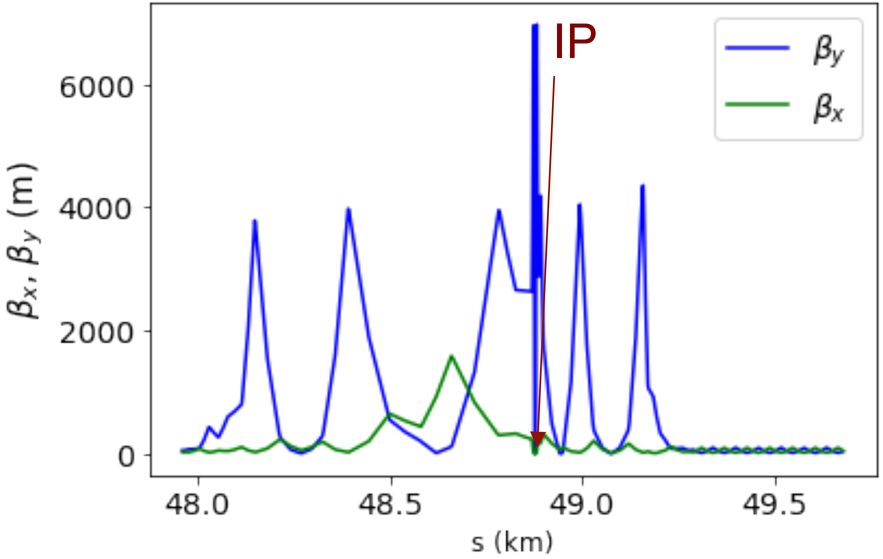
Challenges & constraints for FCC-ee (and CEPC) emittance tuning



$\beta_y^* = 1.6$ mm

$\beta_{x,\mathtt{max}} = 1587.97$ m
$\beta_{y,\mathtt{max}} = 6971.55$ m

Large emittance ratio, $\epsilon_y/\epsilon_x = 0.201\%$

Vertical dispersion & betatron coupling dominate ε_y growth

Horizontal emittance:

$$\epsilon_x = \frac{C_g}{J_x}\gamma^2\theta^3 F$$

$$F_{FODO} = \frac{1}{2\sin\psi}\frac{5+3\cos\psi}{1-\cos\psi}\frac{L}{l_B}$$

L: cell length
l_B: dipole length
ϕ: phase advance/cell

Vertical emittance:

$$\epsilon_y = \left(\frac{dp}{p}\right)^2(\gamma D_y^2 + 2\alpha D_y D_y' + \beta D_y'^2)$$

Sources of vertical emittance growth:

- vertical dispersion D_y
- betatron coupling
- opening angle ~ $1/\gamma$ (here negligible)

Correction methods used:

- **Orbit correction:**
  - MICADO & SVD from MADX
    - Hor. corrector at each QF, Vert. corrector at each QD

    1598 vertical correctors / 1590 horizontal correctors
    - BPM at each quadrupole

    1598 BPMs vertical / 1590 BPMs horizontal

- **Vertical dispersion and orbit:**
  - Orbit Dispersion Free Steering (DFS)

$$\begin{pmatrix} (1-\alpha)\vec{u} \\ \alpha \vec{D}_u \end{pmatrix} + \begin{pmatrix} (1-\alpha)\mathbf{A} \\ \alpha \mathbf{B} \end{pmatrix} \vec{\theta} = 0$$

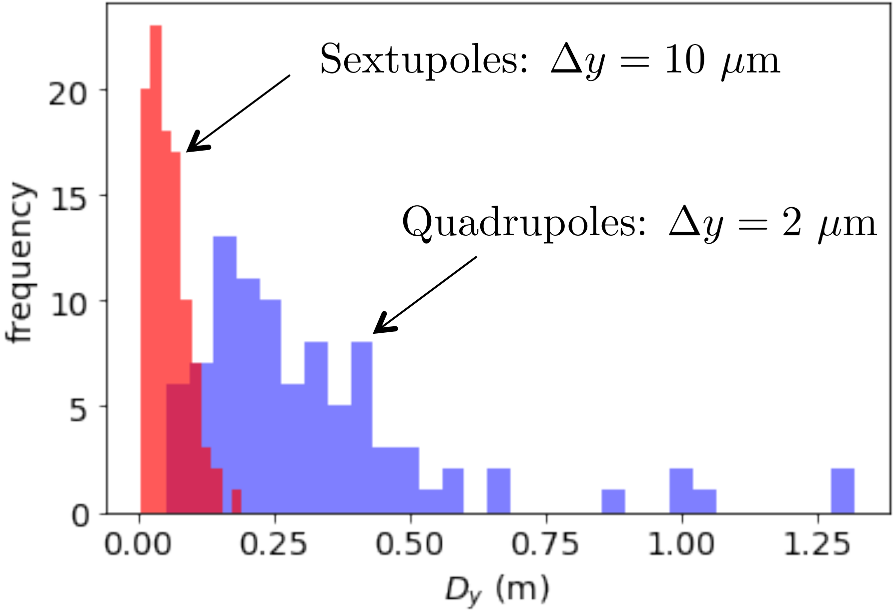
- **Linear coupling:**
  - Coupling resonant driving terms (RDT)
    - 1 skew at each sextupole + skews correctors at the IP

$$\begin{pmatrix} \vec{f}_{1001} \\ \vec{f}_{1010} \end{pmatrix}_{\text{meas}} = -\mathbf{M}\vec{J}_c,$$

- **Beta beating correction & Horizontal dispersion via Response Matrix:**
  - Rematching of the phase advance at the BPMs
    - 1 trim quadrupole at each sextupole

$$(\Delta\phi_{xy}, \Delta D_x) = \mathbf{R}\Delta k_1$$

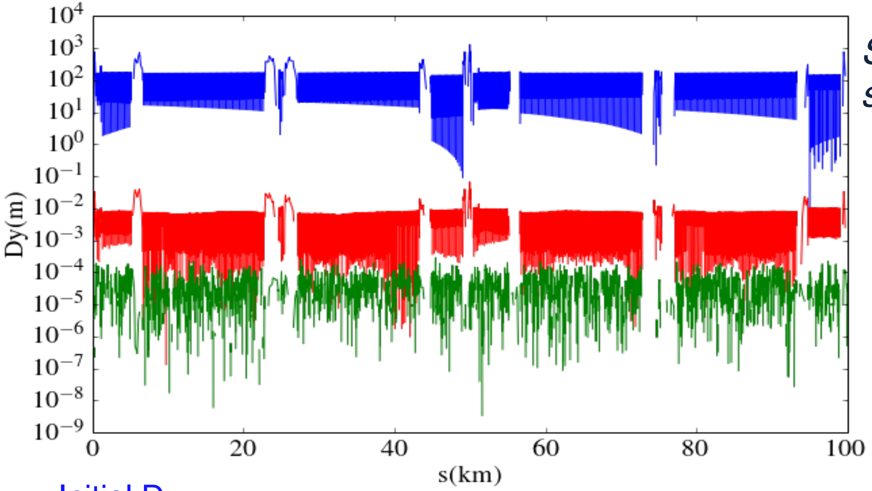
Initial assessment



Error type	y, rms (mm)	$D_{y,\mathrm{rms}}$ (mm)
quad arc ($\Delta y = 2\ \mu m$)	8.809	326.71
quad arc ($\Delta x = 10\ \mu m$)	0.0	0.0
quad arc ($\Delta \phi = 10\ \mu rad$)	0.0	2.677
sextupoles ($\Delta y = 10\ \mu m$)	0.0245	57.13
sextupoles ($\Delta x = 10\ \mu m$)	0.0	0.0
sextupoles ($\Delta \phi = 10\ \mu rad$)	0.0	0.004

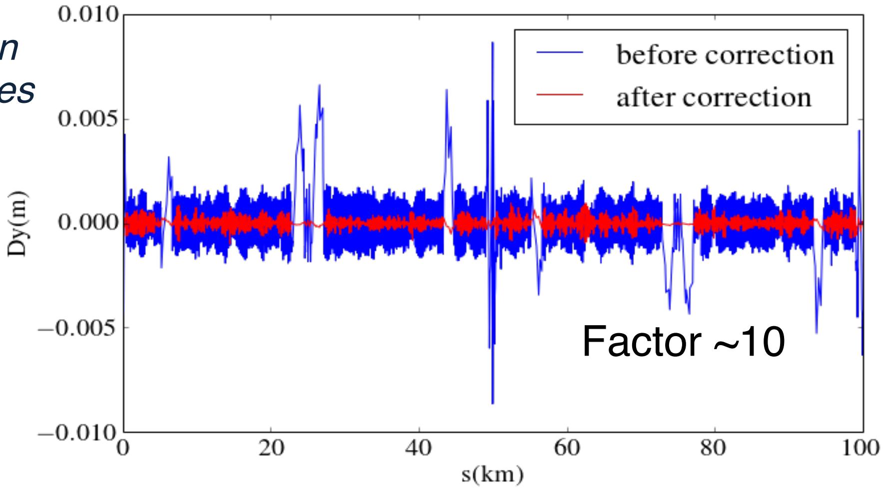
Correction methods applied to Vertical Quadrupole Misalignments (σ_y =100 µm)

Sextupoles turned off:



Switch on sextupoles

Dispersion correction during the coupling correction

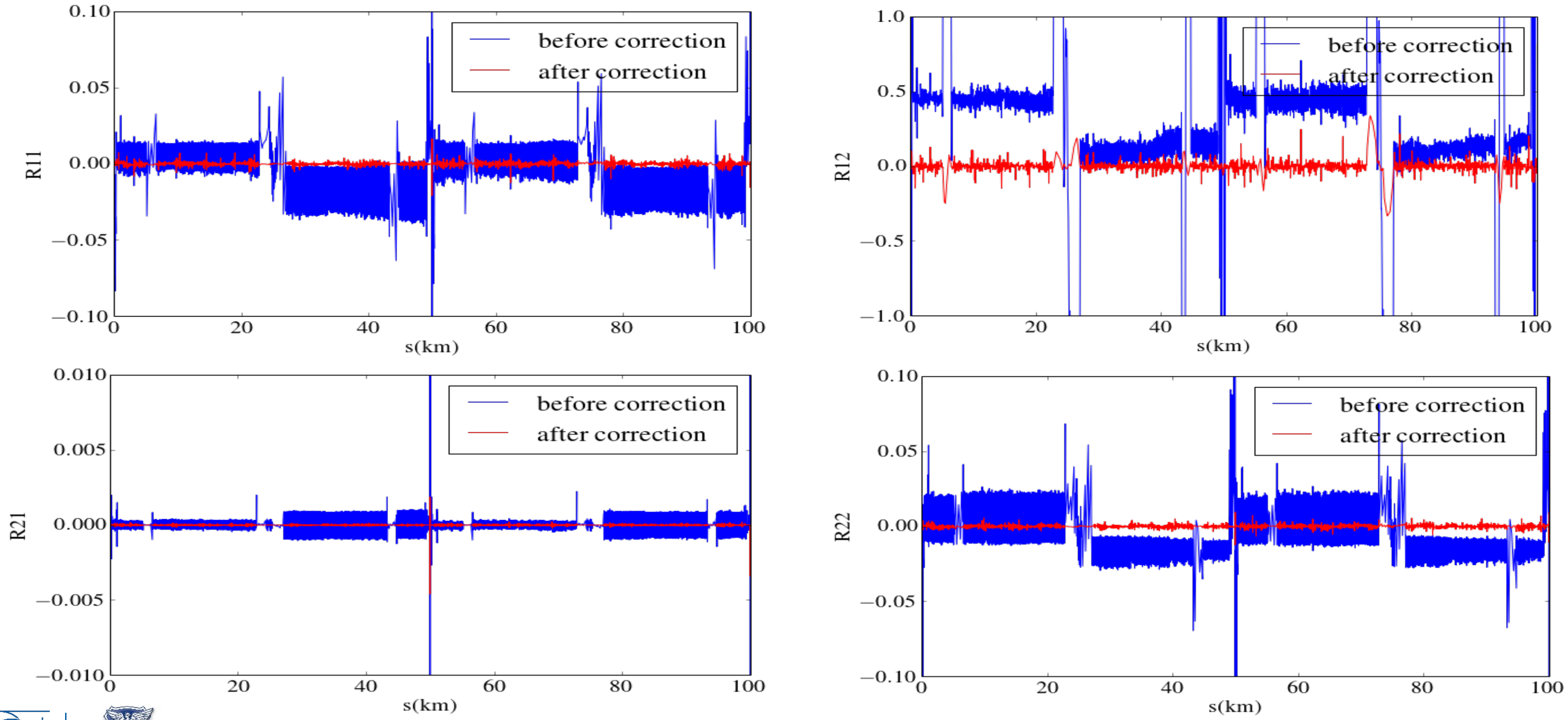


Initial D_y

After orbit correction - factor 2e4 improvement

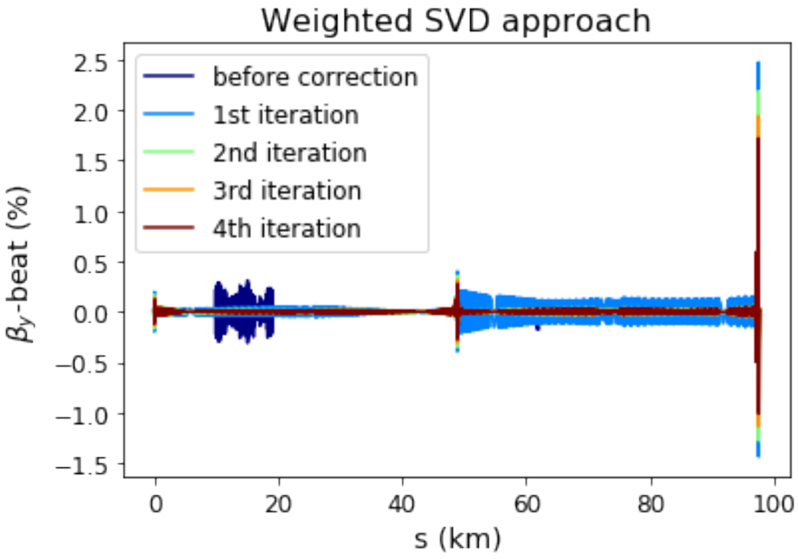
DFS - factor 50 improvement

Coupling matrix elements



Beta-beating Correction – Weighted SVD

Beta-beat introduced through: Arc quads: Δx = 100 µm, Δy = 100 µm, Δθ = 100 µm
Sextupoles: Δx = 100 µm, Δy = 100 µm, Δθ = 100 µm



Correction Strategy

7-8h up to one day of simulation/seed

- **Sextupoles strengths set to 0**
  - x-y orbits correction
  - Coupling correction
  - Tune matching
  - Beat-beat correction

  Loop 20 times

  - 1 step Dispersion Free Steering wo sextupole (Dy correction)
    +
    1 step coupling correction (kicker strength change the coupling configuration)
  - Save x,x',y,y' at the beginning of the machine

- **Set sextupoles strength to 10% of their design current**
  - orbit corrections
  - coupling correction, tune matching
  - beta beat correction
  - coupling + Dy correction
  - increase by 10% the sextupole strength

This avoid the tunes run of to resonance and maximize the number of seeds

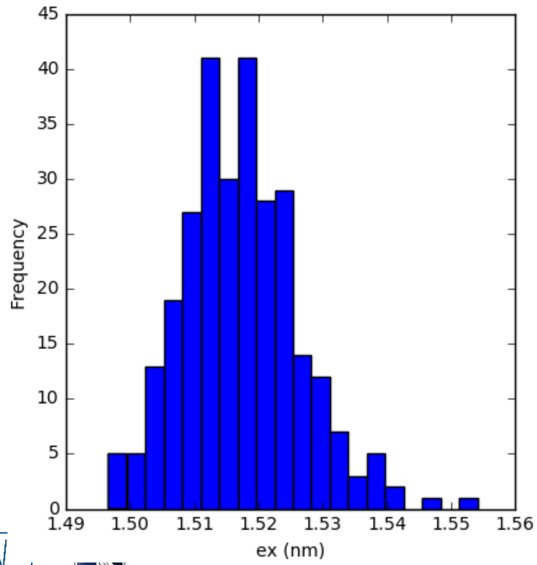
- **Final correction**
  - Coupling corrections
  - Weighted beat-beat correction

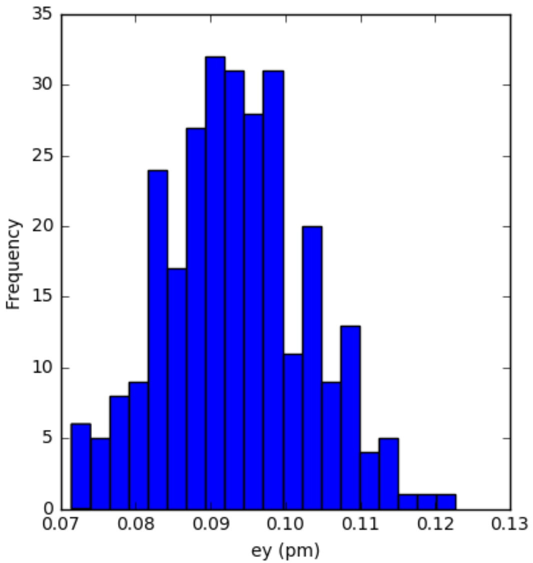
Corrected Lattice

- Misaligned arc quads & sextupoles

	σ_x (μm)	σ_y (μm)	σ_θ (μrad)
arc quadrupoles	100	100	100
IP quadrupoles	0	0	0
sextupoles	100	100	0

IP quads perfectly aligned (for now)



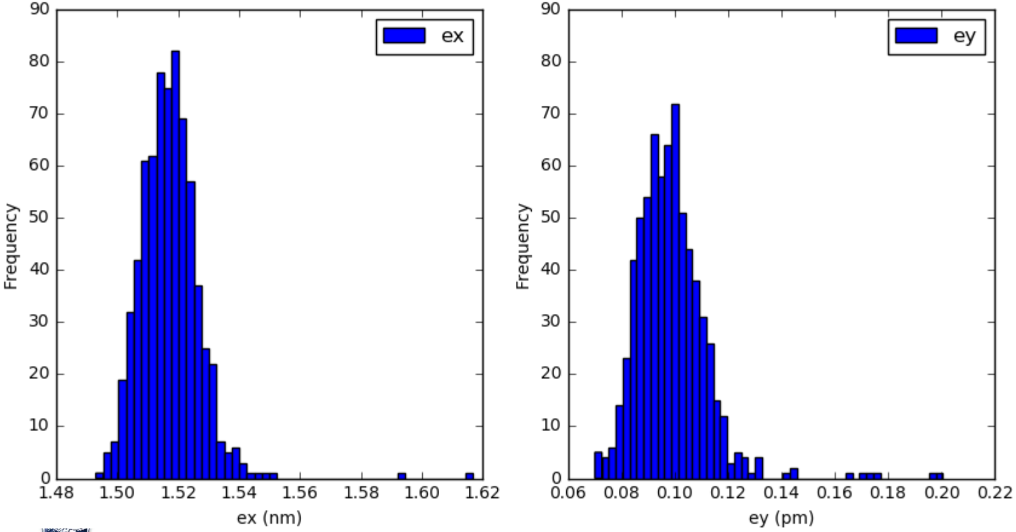


436 out of 500 seeds converged

ε_y = 0.093 pm +/- 0.01
ε_x = 1.520 nm +/- 0.009
$\varepsilon_y/\varepsilon_x$= 0.006% (limit 0.1%)

Corrected Lattice
- Misaligned arc and IP quads & sextupoles

	σ_x (μm)	σ_y (μm)	σ_θ (μrad)
arc quadrupoles	100	100	100
IP quadrupoles	50	50	50
sextupoles	100	100	0

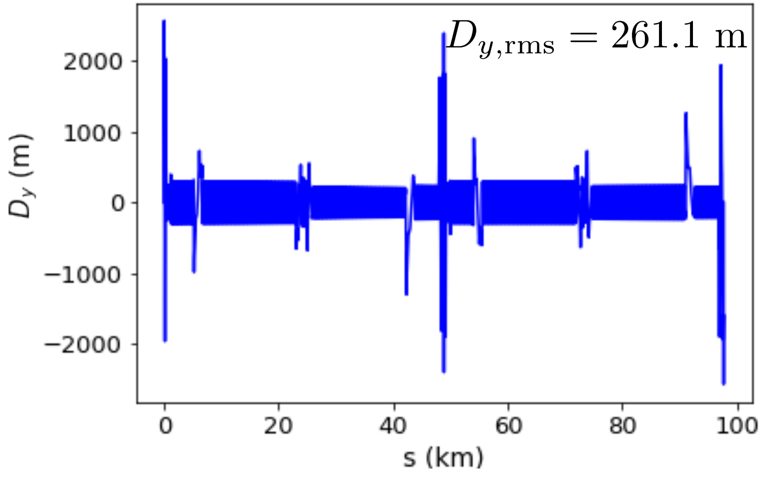


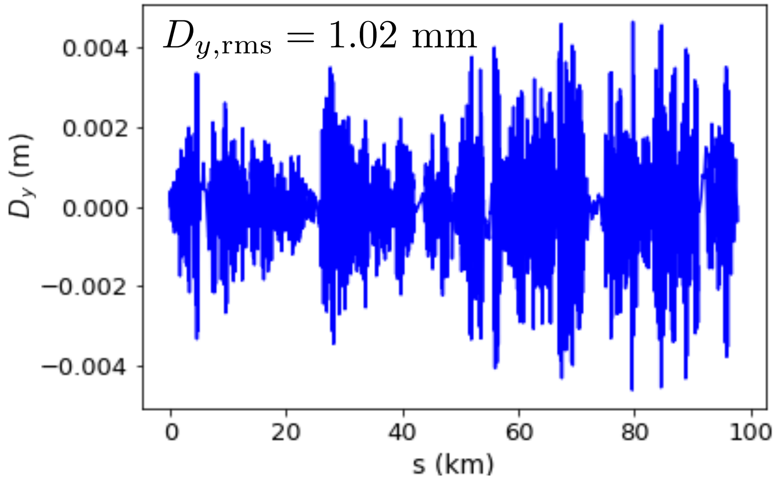
700 out of 1000 seeds converged

ε_y = 0.099 pm +/- 0.013
ε_x = 1.52 nm +/- 0.01
$\varepsilon_y/\varepsilon_x$ = 0.0065% (limit 0.1%)

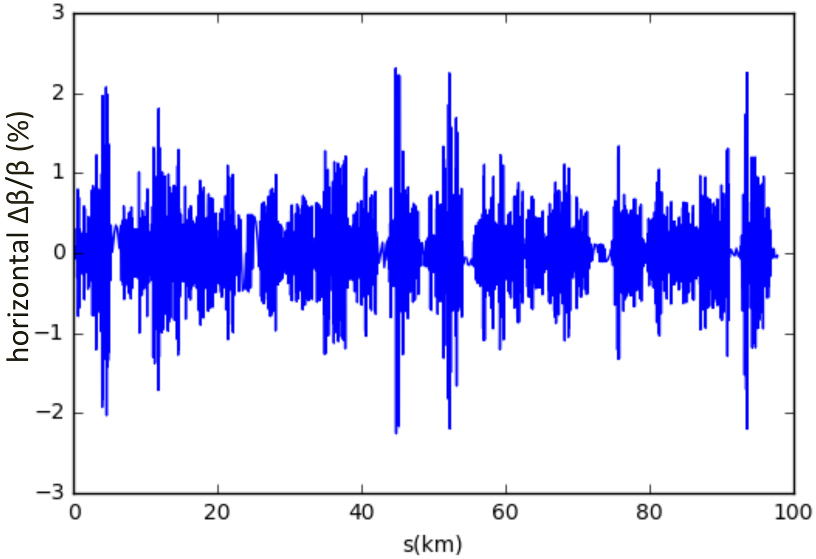
Vertical dispersion highly susceptible misalignments

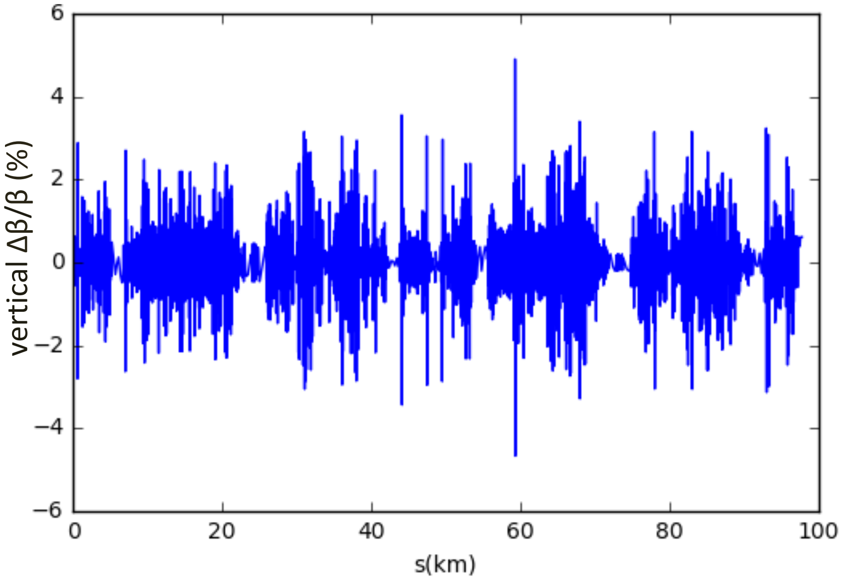
Dispersion introduced through: quadrupoles: Δx = 100 μm, Δy = 100 μm, Δθ = 100 μm
Sextupoles: Δx = 100 μm, Δy = 100 μm, Δθ = 100 μm





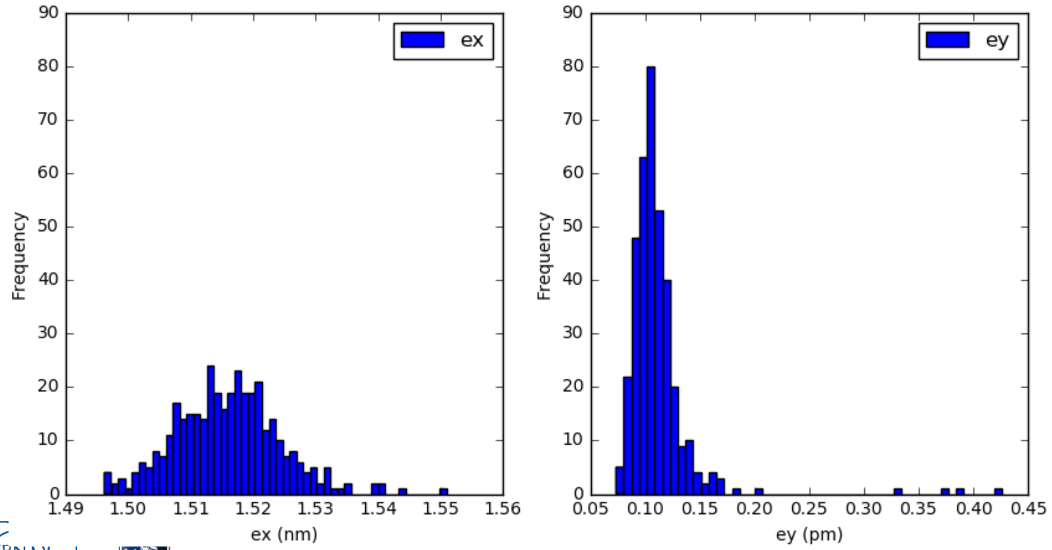
Beta beat after correction





Corrected Lattice
- Misaligned arc and IP quads & sextupoles

	σ_x (μm)	σ_y (μm)	σ_θ (μrad)
arc quadrupoles	100	100	100
IP quadrupoles	100	100	100
sextupoles	100	100	



369 out of 1000 seeds converged

ε_y = 0.11 pm +/- 0.03
ε_x= 1.52 nm +/- 0.01
$\varepsilon_y/\varepsilon_x$= 0.0073% (limit 0.1%)

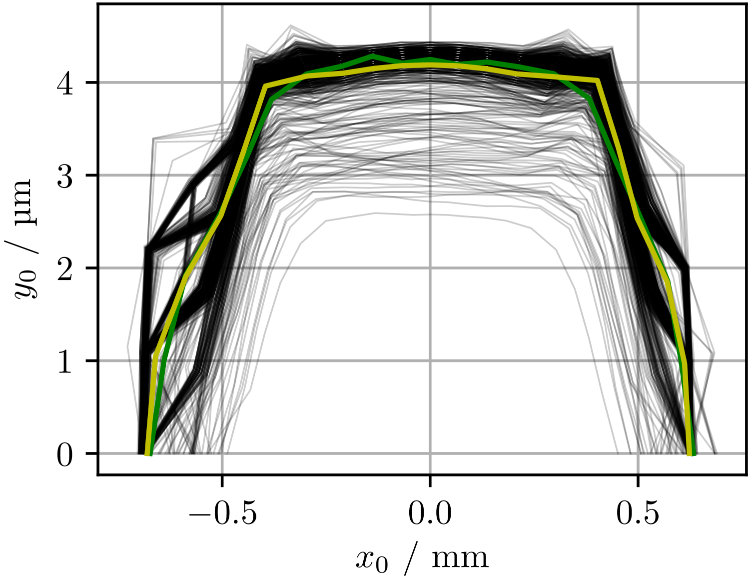
To increase the number of successful seeds:

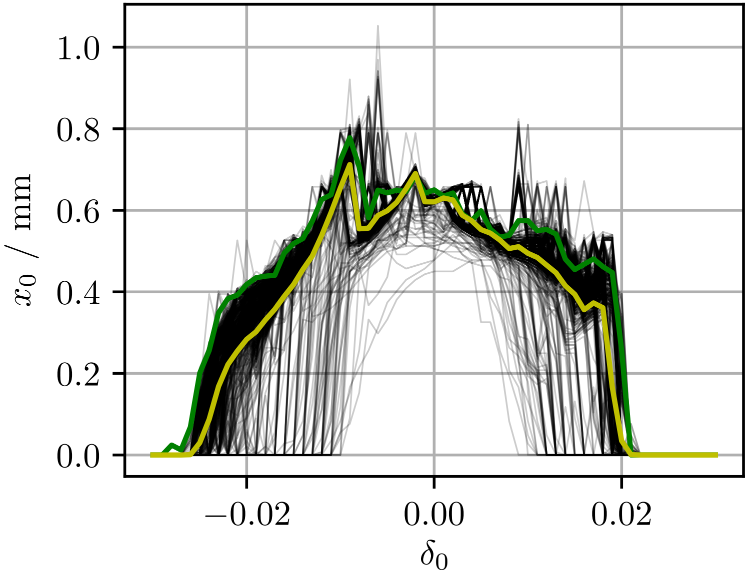
- Add more tune matching to strategy.
- Start with relaxed optics, before reducing β^*.
- Place limit on maximum trim and skew quad strength that can be applied any given iteration step.

Tracking done by T. Tydecks

Dynamic / Momentum aperture

with radiation damping only





Apertures sufficient for beam storage and injection.

	σ_x (μm)	σ_y (μm)	σ_θ (μrad)
arc quadrupoles	100	100	100
IP quadrupoles	50	50	50
sextupoles	100	100	0

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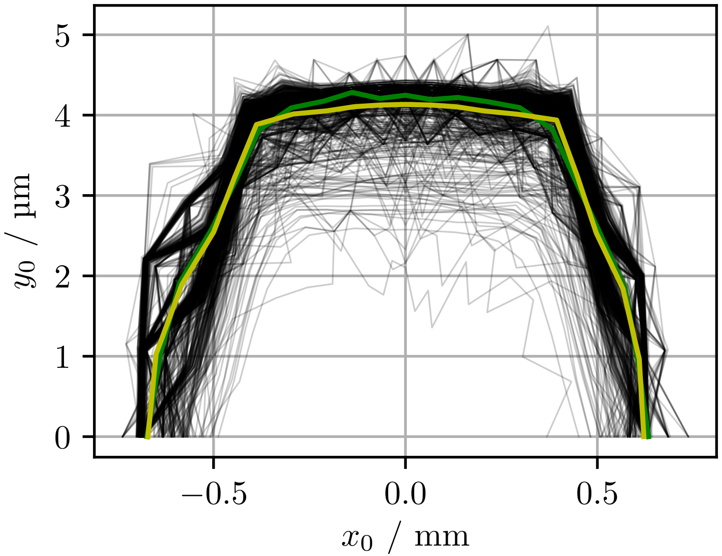
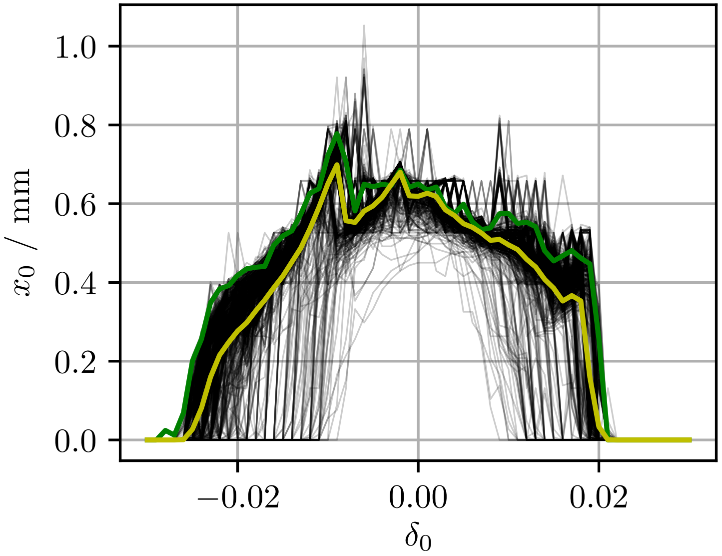
Tracking done by T. Tydecks

Dynamic / Momentum aperture

with radiation damping and quantum excitation



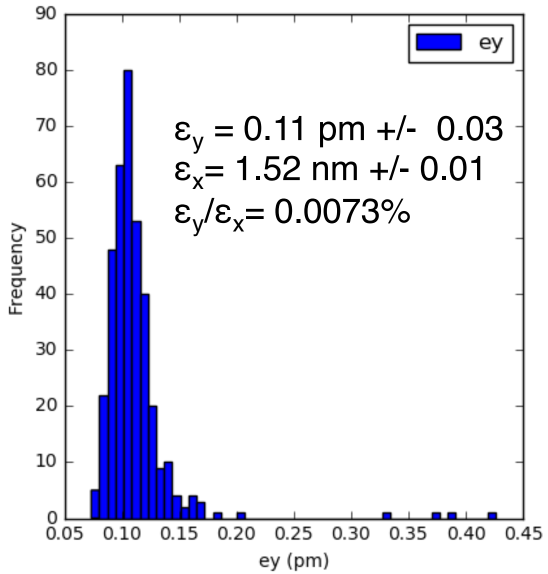


Apertures sufficient for beam storage and injection.

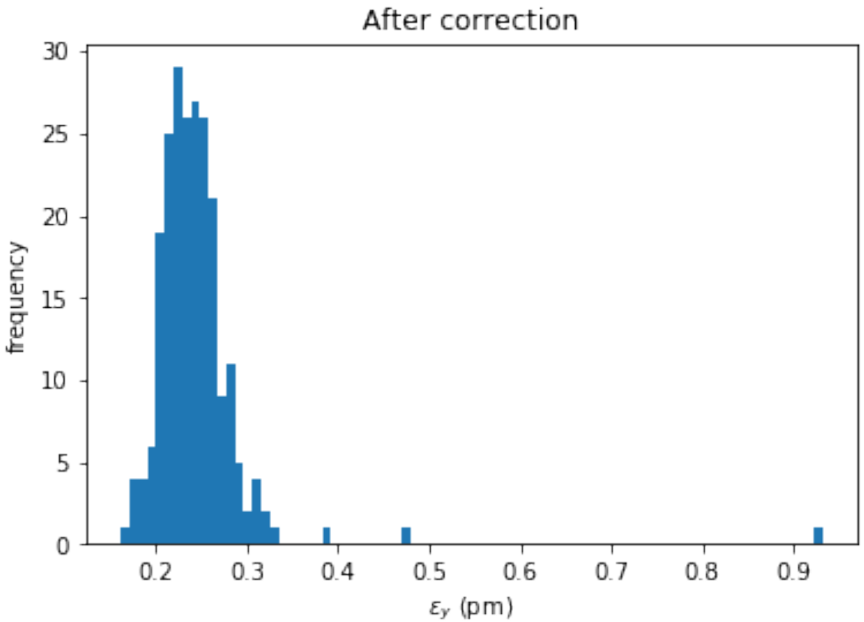
	σ_x (μm)	σ_y (μm)	σ_θ (μrad)
arc quadrupoles	100	100	100
IP quadrupoles	50	50	50
sextupoles	100	100	0

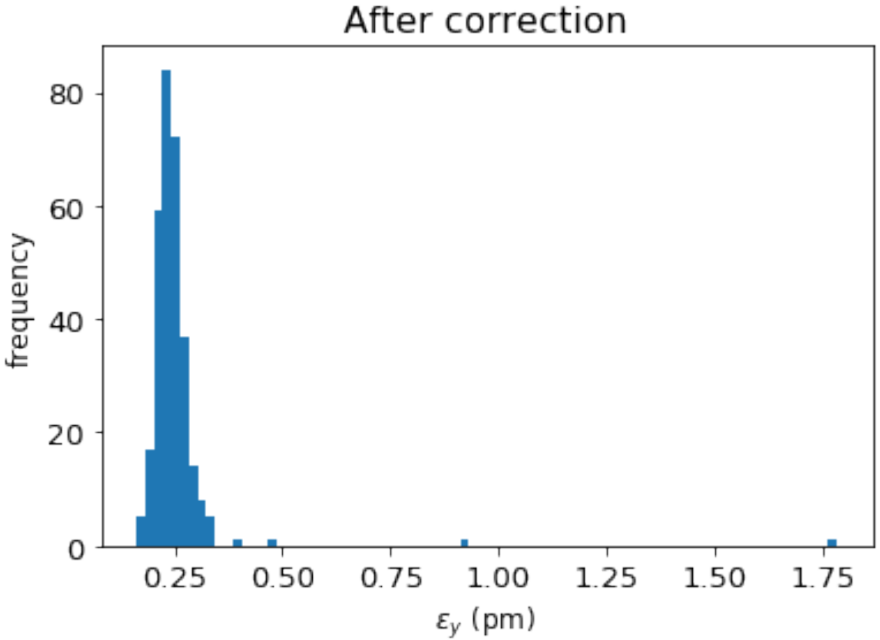
Conclusions

- FCC-ee poses a unique challenge for emittance tuning
- With 100 µm, 100 µrad misalignments in arc quads & sextupoles and 50µm and 50 µrad misalignments in IP quads, the mean vertical emittance achieved after correction schemes applied is ε_y = 0.11 pm rad



Back up slides – top v213 lattice





	σ_x (μm)	σ_y (μm)	σ_θ (μrad)
arc quadrupoles	100	100	100
IP quadrupoles	0	0	0
sextupoles	100	100	100
dipoles	100	100	100

	σ_x (μm)	σ_y (μm)	σ_θ (μrad)
arc quadrupoles	100	100	100
IP quadrupoles	50	50	50
sextupoles	100	100	100
dipoles	100	100	100

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Back up slides - DFS

- Build numerically a matrix for vertical orbit (u) & dispersion (D_u) response under a corrector kick (al)

$$\begin{pmatrix} (1-\alpha)\vec{u} \\ \alpha\vec{D}_u \end{pmatrix} + \begin{pmatrix} (1-\alpha)\mathbf{A} \\ \alpha\mathbf{B} \end{pmatrix}\vec{\theta} = 0$$

- Orbit response

$$A_{i,j} = \frac{\sqrt{\beta_i\beta_j}}{2\sin(\pi Q_y)}\cos(|\mu_i-\mu_j|-\pi Q_y)$$

- Dispersion response

$$\begin{aligned} B_{ij} \;=\; & \{\sum_l^{quad}\frac{K_lL_l\beta_l}{4\sin(\pi Q)^2}\cos(|\mu_i-\mu_l|-\pi Q)\cos(|\mu_l-\mu_j|-\pi Q) \\ & -\sum_m^{sext}\frac{K_{2,m}D_{x,m}L_m\beta_m}{4\sin(\pi Q)^2}\cos(|\mu_i-\mu_m|-\pi Q)\cos(|\mu_m-\mu_j|-\pi Q) \\ & -\frac{\cos(|\mu_i-\mu_j|-\pi Q)}{\sin(\pi Q)}\}\sqrt{\beta_l\beta_j} \end{aligned}$$

- SVD analysis to solve the system and find a solution

"Emittance optimization with dispersion free steering at LEP"
R. Assmann et al. Phys. Rev. ST Accel. Beams 3, 121001

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Back up slides - coupling

Coupling RDT f_{1001}-f_{1010} are related to the coupling parameter via:

$$\Delta Q_{\min} = |C^-| = \left|\frac{4\Delta}{2\pi R}\oint ds f_{1001} e^{-i(\phi_x-\phi_y)+is\Delta/R}\right|,$$

References:
-Vertical emittance reduction and preservation in electron storage rings via resonance driving terms correction, A. Franchi et al, PRSTAB 14, 034002

f_{1001}-f_{1010} can be computed via analytical formulas,
or via a matrix formalism with the coupling matrix:

$$f_{\substack{1001\\1010}} = \frac{\sum_w^W J_{w,1}\sqrt{\beta_x^w \beta_y^w} e^{i(\Delta\phi_{w,x} \mp \Delta\phi_{w,y})}}{4(1 - e^{2\pi i(Q_u \mp Q_v)})},$$

$$\Delta D_y = -(\Delta J_w) D_x \frac{\sqrt{\beta_y \beta_{y0}}}{2sin(\pi Q)} cos(\pi Q - |\phi_{y0} - \phi_y|)$$

A response matrix can be written to measure the the response of the RDTs to a skew quadrupole field, J_c. The system, which can be inverted via SVD:

$$\begin{pmatrix} \vec{f}_{1001} \\ \vec{f}_{1010} \end{pmatrix}_{\text{meas}} = -\mathbf{M}\vec{J}_c,$$

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Back up slides – beta-beating

For n trim quadrupoles which can exercise a small field strength k_1, the weighted SVD can be applied through adding weighting factors f to each measurement of the beta-beat.

$$\begin{pmatrix} f_1\left(\frac{\beta_1-\beta_{y0}}{\beta_{y0}}\right) \\ f_2\left(\frac{\beta_2-\beta_{y0}}{\beta_{y0}}\right) \\ \dots \\ f_m\left(\frac{\beta_m-\beta_{y0}}{\beta_{y0}}\right) \end{pmatrix}_{meas} = \begin{pmatrix} f_1\ (R_{11}, R_{12}, R_{13}, \dots, R_{1n}) \\ f_2\ (R_{21}, R_{22}, R_{23}, \dots, R_{1n}) \\ \dots \\ f_m\ (R_{m1}, R_{m2}, R_{m3}, \dots, R_{mn}) \end{pmatrix} * \begin{pmatrix} k_1 \\ k_2 \\ \dots \\ k_n \end{pmatrix}$$

where β_{y0} is the ideal beta function at the given BPM, $R_{i,j}$ are elements in the response matrix.

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