Dynamic aperture limitation due to the synchrotron radiation from quadrupole magnets in very high energy colliders (FCC-ee)

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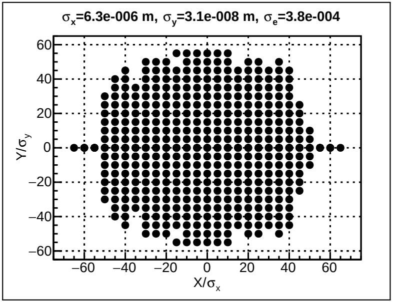
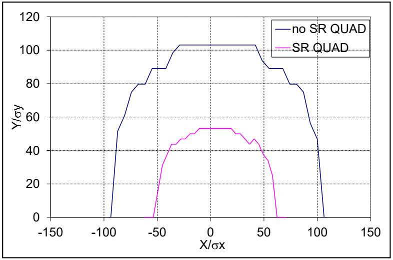
24-27 September, 2018
the 62nd ICFA Advanced Beam Dynamics Workshop on High Luminosity Circular e+e- Colliders (eeFACT2018)

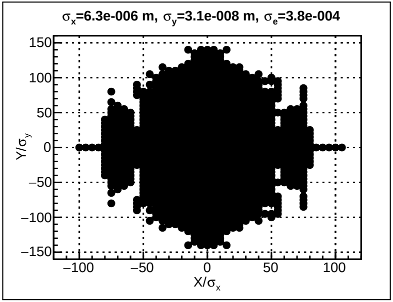
6d (SR from BEND, QUAD) and 6d tracking: *XY*

6d(SR): $R_x = 65\sigma_x \; R_y = 57\sigma_y$





6d: $R_x = 109\sigma_x \; R_y = 142\sigma_y$



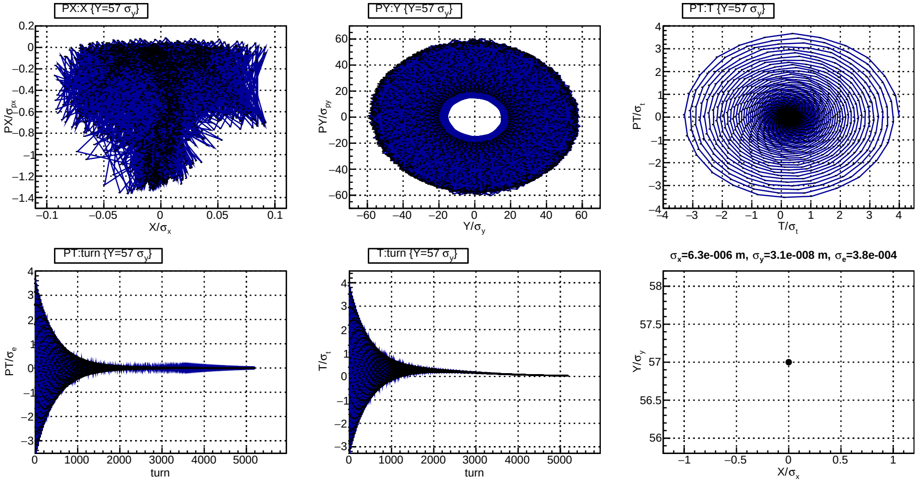
Introduction

Problem

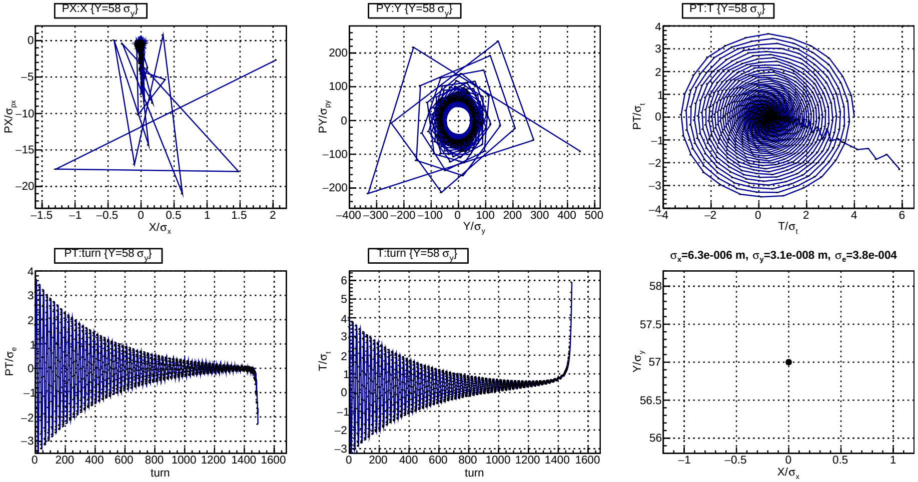
- Why vertical dynamic aperture drops from $R_y = 142\sigma_y$ to $R_y = 57\sigma_y$?
- Why horizontal dynamic aperture drops from $R_x = 109\sigma_x$ to $R_x = 65\sigma_x$?

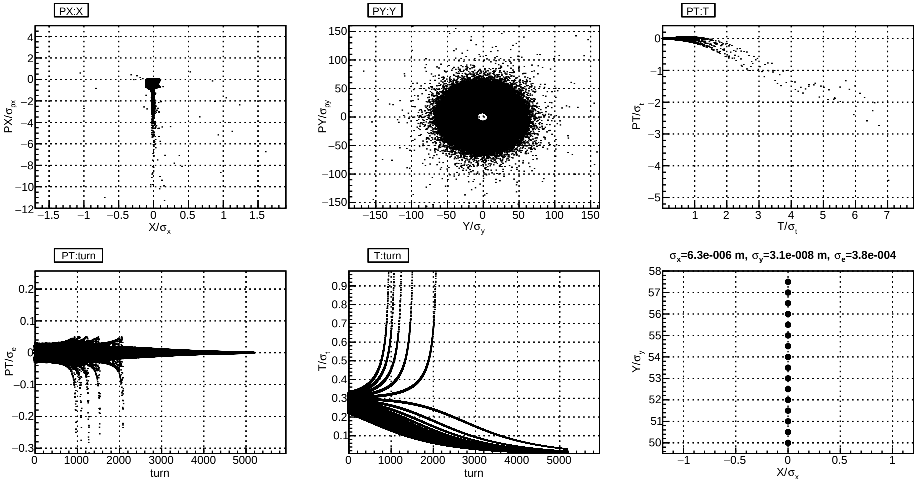
in the FCCee_z_202_nosol_13.seq lattice at 45 GeV

6d (SR from BEND, QUAD; last stable): $Y_0 = 57\sigma_y$



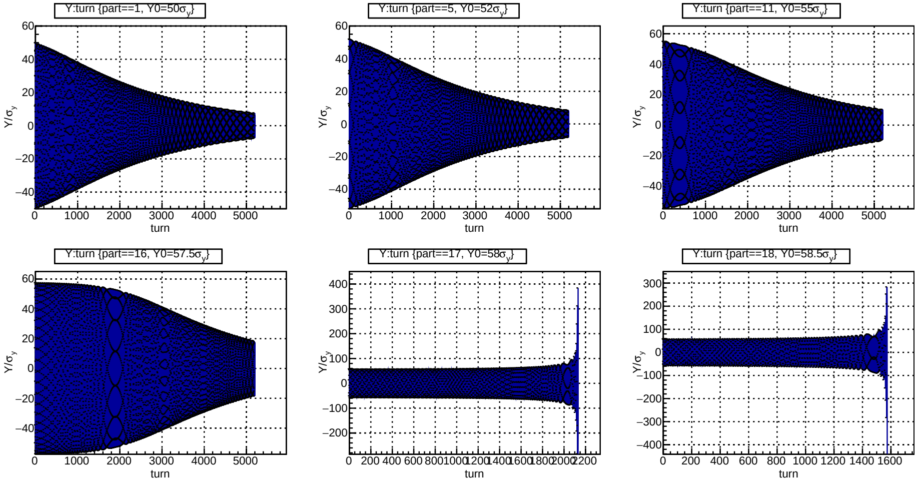
6d (SR from BEND, QUAD; first unstable): $Y_0 = 58\sigma_y$





PX:X
PX/σpx
X/σx
PY:Y
PY/σpy
Y/σy
PT:T
PT/σt
T/σt
PT:turn
PT/σe
turn
T:turn
T/σt
turn
σx=6.3e-006 m, σy=3.1e-008 m, σe=3.8e-004
Y/σy
X/σx

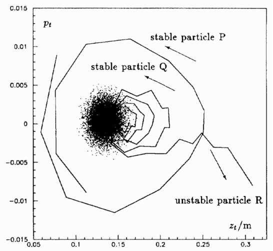
6d (SR from BEND, QUAD) damping



History

References

- John M. Jowett (SLAC), Introductory Statistical Mechanics for Electron Storage Rings, AIP Conf.Proc. 153 (1987) 864-970
- J. Jowett (CERN), Dynamic aperture for LEP: Physics and calculations, Conf.Proc. C9401174 (1994) 47-71, In *Chamonix 1994, LEP performance* 47-71
- F. Barbarin, F. C. Iselin and J. M. Jowett, Particle dynamics in LEP at very high-energy, Conf. Proc. C **940627**, 193 (1994).



Comments

- Some tracking plots are similar.
- There was no mentioning of damping turning into raising.

Parameters (radiation ON, no tappering)

- Energy: $E = 45.6$ Gev.
- Tunes: $\nu_s = 0.0413$, $\nu_y = 0.2217$, $\nu_x = 0.1366$
- Damping times [turns]: $\tau_\sigma = 1300$, $\tau_y = 2600$, $\tau_x = 2600$
- Energy loss: $U_0 = 35.96$ MeV/turn
  - $U_d(B, arc) = 3014 \times 12.4$ keV$= U_0$,
  - $U_q(FF, 50\sigma_y) = 4 \times 0.5$ MeV, $U_q(FF, 50\sigma_x) = 4 \times 3$ MeV,
  - $U_q(QF, 50\sigma_y) = 1470 \times 2.5$ eV, $U_q(QF, 50\sigma_x) = 1470 \times 2.8$ keV,
  - $U_q(QD, 50\sigma_y) = 1468 \times 10$ eV, $U_q(QD, 50\sigma_x) = 1468 \times 1$ keV
- Radiation from quadrupoles:

$$U_q = \frac{C_\gamma}{2\pi} E_0^4 \oint K_1^2(x^2 + y^2)ds = E_0 \Gamma \Pi \left[\left\langle K_1^2 \beta_x \right\rangle J_x + \left\langle K_1^2 \beta_y \right\rangle J_y \right]$$

  - $\langle K_1^2 \beta_x \rangle = 4 \times 10^{-3}$ m^{-3}, $\langle K_1^2 \beta_y \rangle = 1.4 \times 10^{-1}$ m^{-3},

Equations of motion: longitudinal

Exact

$$\begin{cases} \sigma' = -K_0 x - \dfrac{p_x^2}{2} - \dfrac{p_y^2}{2} \\ p'_\sigma = \left(-\dfrac{eV_0}{p_0 c}\right) \sin\left[\phi_s + 2\pi\dfrac{\sigma}{\lambda}\right] \delta(s - s_0) \\ \qquad - \Gamma\left[K_0^2(1 + 2p_\sigma) + (2K_0K_1 + K_0^3)x + K_1^2(x^2 + y^2)\right] \end{cases}$$

where $\Gamma = \frac{C_\gamma}{2\pi}\frac{E_0^4}{p_0 c}$

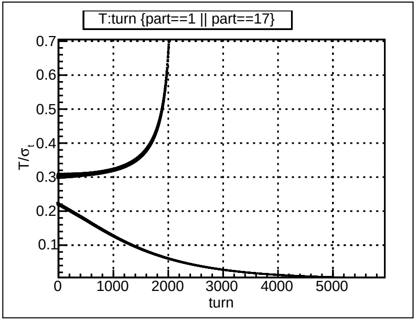
Average, $x_\beta = 0$, $x = \eta p_\sigma$

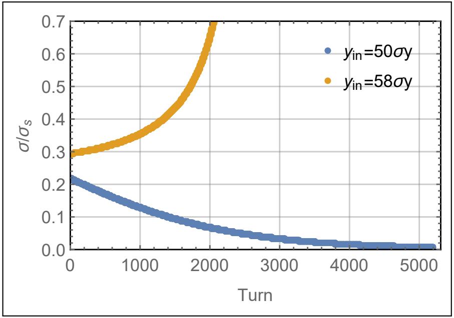
$$\begin{cases} \sigma' = -\alpha p_\sigma - \dfrac{J_y \langle \gamma_y \rangle}{2}, \\ p'_\sigma = \dfrac{k_s^2}{\alpha}\sigma - 2\alpha_\sigma p_\sigma - \Gamma\left\langle K_1^2 \beta_y \right\rangle J_y, \quad k_s = \dfrac{\nu_s}{R} \end{cases}$$

No synchrotron oscillations

$$\sigma' = 0\,, \qquad \sigma = -\frac{\alpha_\sigma}{k_s^2} J_y \langle \gamma_y \rangle + \frac{\alpha}{k_s^2} \Gamma \left\langle K_1^2 \beta_y \right\rangle J_y\,,$$

$$p_\sigma' = 0\,, \quad p_\sigma = -\frac{1}{2\alpha} J_y \langle \gamma_y \rangle\,, \quad J_y = \frac{J_0 \exp\left(-2\alpha_y s\right)}{1 - \frac{\beta}{2\alpha_y} J_0 \left(1 - \exp\left(-2\alpha_y s\right)\right)}$$





Equations of motion: longitudinal

Fourie harmonics

$$y = \sqrt{2J_y\beta_y}\cos(\psi_0 + \psi_y(s)) = A_y f_y + A_y^* f_y^*$$

$$\Gamma K_1^2 y^2 = \Gamma K_1^2 \beta_y J_y + \Gamma A_y^2 e^{i\frac{2\nu_y s}{R}} \sum_{n=-\infty}^{\infty} F_{y,n} e^{i\frac{s}{R}n} + c.c.$$

$$F_{y,n} = \frac{1}{\Pi}\int_0^{\Pi} K_1^2(s)\beta_y(s) e^{i\left(2\psi_y(s) - 2\nu_y\frac{s}{R} - \frac{s}{R}n\right)} ds\,;$$

Averaged equations, $x_\beta = 0$

$$\begin{cases} \sigma' = -\alpha p_\sigma - \dfrac{J_y\left\langle\gamma\right\rangle}{2} \\ p_\sigma' = \dfrac{k_s^2}{\alpha}\sigma - 2\alpha_\sigma p_\sigma - \Gamma\left\langle K_1^2\beta_y\right\rangle J_y - \Gamma A_y^2 F_{y,n} e^{i\frac{s}{R}(2\nu_y+n)} - c.c. \end{cases}$$

$n = -[2\nu_y]$ is the negative integer part of the double betatron tune

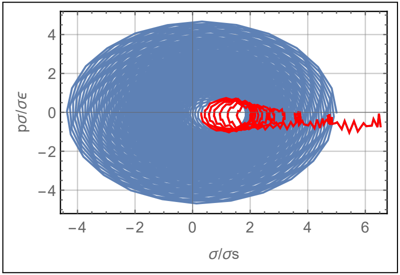
Longitudinal: solution versus tracking

Solution

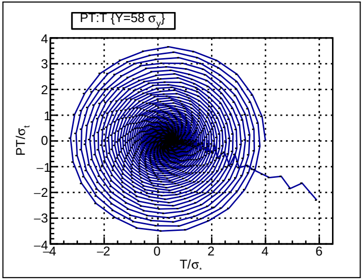
$$p_\sigma = Be^{-\alpha_\sigma s}\cos(k_s s) - \frac{J_y\langle\gamma\rangle}{2\alpha} + iA_y^2\frac{\Gamma F_{y,n}}{(2\nu_y+n)/R}e^{i\frac{s}{R}(2\nu_y+n)} + c.c.$$

where $n = -[2\nu_y]$, $J_y = 2A_yA_y^*$

Solution



Tracking



Longitudinal: solution versus tracking

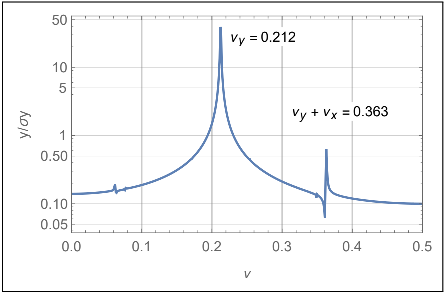
Solution

$$p_\sigma = Be^{-\alpha_\sigma s}\cos(k_s s) - \frac{J_y\langle\gamma\rangle}{2\alpha} + iA_y^2\frac{\Gamma F_{y,n}}{(2\nu_y+n)/R}e^{i\frac{s}{R}(2\nu_y+n)} + c.c.$$

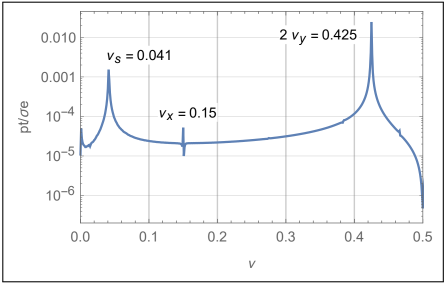
where $n = -[2\nu_y]$, $J_y = 2A_yA_y^*$

Longitudinally adjusted tracking

y spectrum



p_σ spectrum



Equations of motion: vertical

Exact

$$\begin{cases} y' = p_y(1 - p_\sigma) \\ p_y' = K_1 y + K_2 \eta p_\sigma y - \Gamma p_y \left[K_0^2 + p_\sigma D + K_1^2 y^2\right] \end{cases}$$

Parametric oscillator and Van der Pol oscillator

$$y'' - [K_1 - (K_1 - K_2\eta)p_\sigma]\, y + \Gamma \left(K_0^2 + K_1^2 y^2\right) y' = 0$$

Solution

$$p_\sigma = Be^{-\alpha_\sigma s} \cos(k_s s) - \frac{J_y \langle\gamma\rangle}{2\alpha} - J_y \frac{\Gamma |F_{y,n}|}{(2\nu_y + n)/R} \sin\left(\frac{s}{R}(2\nu_y + n) + \chi_0\right)$$

where $n = -[2\nu_y]$, $\Gamma K_1^2 y^2$ is small

Vertical dynamic aperture limit

Solving: parameter variation and averaging

$$y(s) = A_y(s) f_y(s) + A_y(s)^* f_y(s)^* \,, \quad J_y = 2 A_y A_y^*$$

Averaged equation

$$J'_y = -2\alpha_y J_y \mp Im(B_2) J_y^2$$

$$\alpha_y = \frac{1}{2}\left\langle \Gamma K_0^2 \right\rangle \text{ is vertical damping decrement}$$

$$B_2 = \frac{i}{2} \frac{\Gamma F_{y,n}}{(2\nu_y + n)/R} \left\langle (K_1 - K_2\eta)\beta_y e^{i\left(-2\psi_y(s) + \frac{s}{R}(2\nu_y + n)\right)} \right\rangle$$

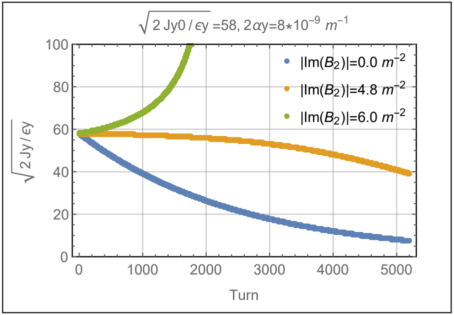
DA limit

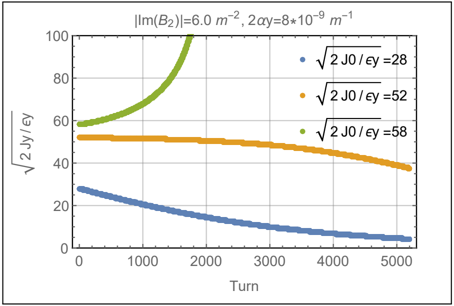
$$J'_y = 0 \quad \Rightarrow \quad J_{y,lim} = \frac{2\alpha_y}{\mp Im(B_2)}$$

Vertical action

Solution

$$J_y(s) = \frac{J_{y,0} e^{-2\alpha_y s}}{1 - J_{y,0} \frac{|Im(B_2)|}{2\alpha_y}\left(1 - e^{-2\alpha_y s}\right)}.$$





Parametric resonance

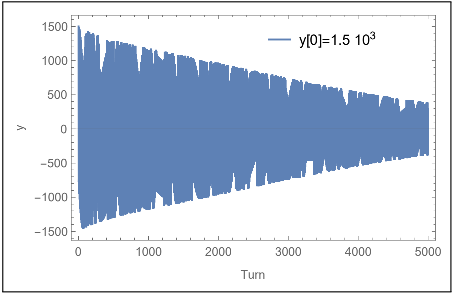
Exact:

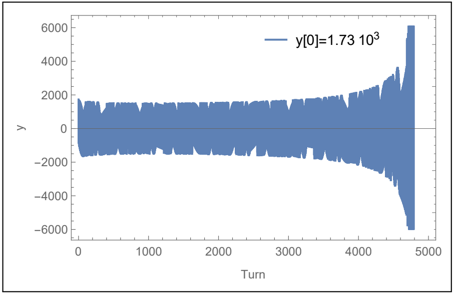
$$y'' - [K_1 - (K_1 - K_2\eta)p_\sigma]\, y + \Gamma\left(K_0^2 + K_1^2 y^2\right) y' = 0$$

$$p_\sigma = J_y \frac{\Gamma\,|F_{y,n}|}{(2\nu_y + n)/R} \sin\left(\frac{s}{R}(2\nu_y + n)\right)$$

Illustration:

$$y'' + k_y^2\left(1 - F_1 y^2 \cos(2k_y s)\right) y + 2\alpha y' = 0$$



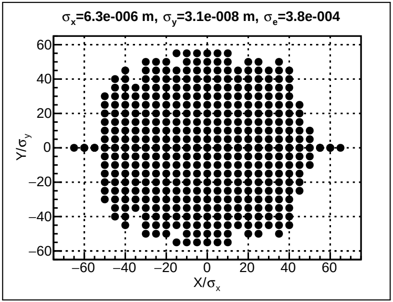


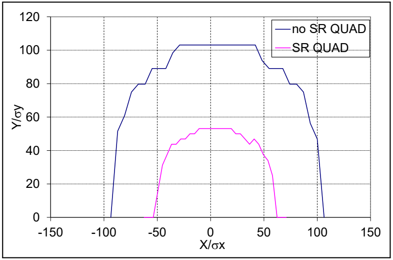
Conclusion for vertical plane at 45.6 GeV

1. Observed and studied a new effect limiting dynamic aperture.
2. Radiation from FF quadrupoles modulates p_σ at double betatron frequency.
3. Parametric resonance in vertical motion changes damping. It is observed in tracking and obtained by equations.
4. Estimations with some assumptions predict dynamic aperture limit $J_{y,limit} \approx 51\sigma_y$, tracking gives $58\sigma_y$.
5. $\pi/2$ phase advance between quadrupoles will decrease p_σ modulation at double betatron frequency and eliminate parametric resonance.

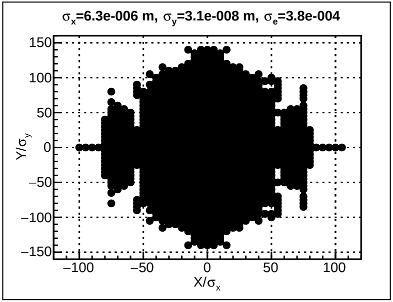
6d (SR from BEND, QUAD) and 6d tracking: *XY*

6d(SR): $R_x = 65\sigma_x$ $R_y = 55\sigma_y$

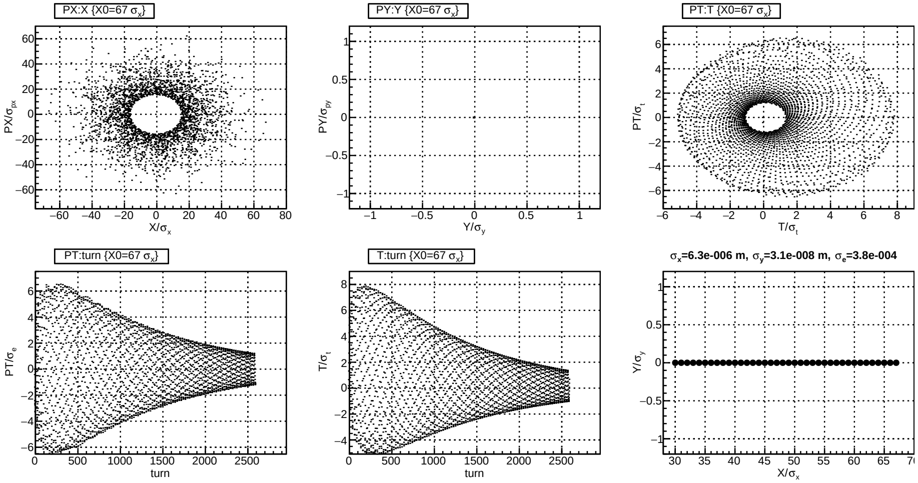




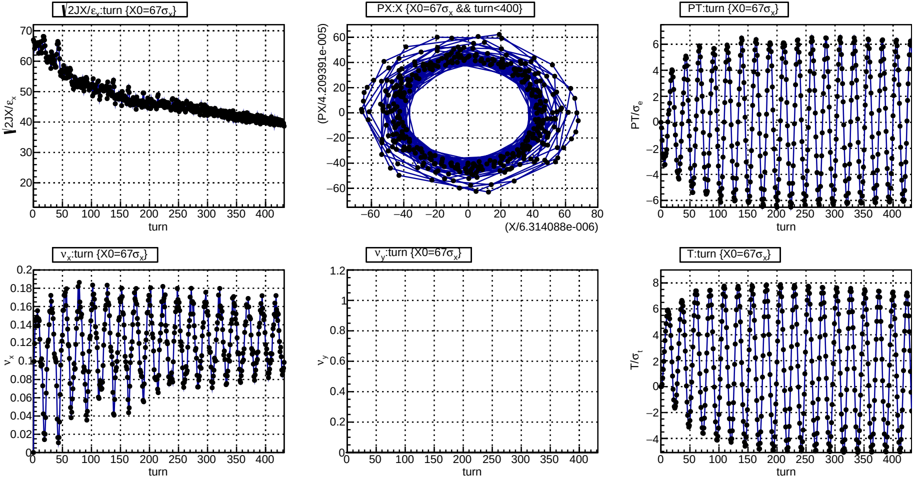
6d: $R_x = 109\sigma_x$ $R_y = 142\sigma_y$



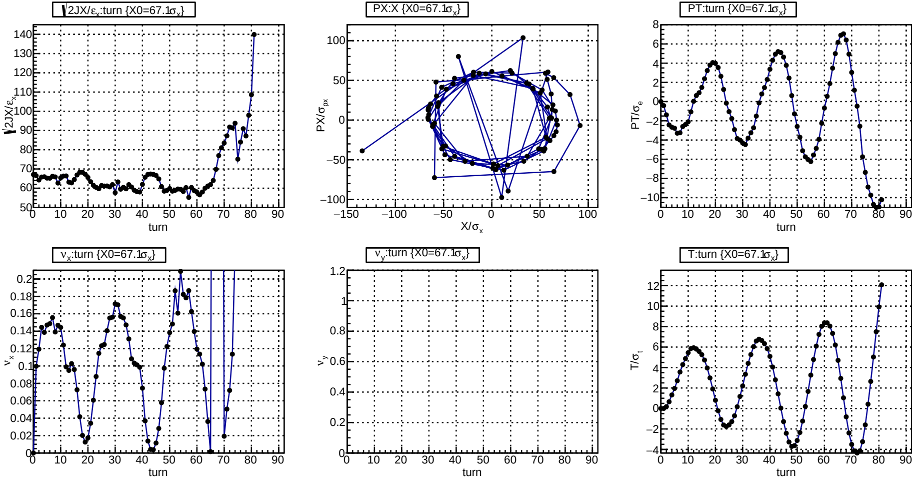
6d (SR from BEND, QUAD; last stable): $X_0 = 67\sigma_x$



6d (SR from BEND, QUAD; last stable): $X_0 = 67\sigma_x$



6d (SR from BEND, QUAD; first unstable): $X_0 = 67.1\sigma_x$



Equations of motion: longitudinal

Exact

$$
\begin{cases}
\sigma' = -K_0 x - \dfrac{p_x^2}{2} \\
p'_\sigma = \left(-\dfrac{eV_0}{p_0 c}\right) \sin\left[\phi_s + 2\pi\dfrac{\sigma}{\lambda}\right] \delta(s - s_0) \\
\qquad - \Gamma\left[K_0^2(1+2p_\sigma) + (2K_0K_1 + K_0^3)x + K_1^2 x^2\right]
\end{cases}
$$

where $\Gamma = \frac{C_\gamma}{2\pi}\frac{E_0^4}{p_0 c}$

Average, $x = x_\beta + \eta p_\sigma$, $y = 0$

$$
\begin{cases}
\sigma' = -\alpha p_\sigma - \dfrac{J_x \langle\gamma_x\rangle}{2} - \left\langle \eta'^2 \right\rangle \dfrac{p_\sigma^2}{2}\,, \\
p'_\sigma = \dfrac{k_s^2}{\alpha}\sigma - 2\alpha_\sigma p_\sigma - \Gamma\left\langle K_1^2 \beta_x \right\rangle J_x - \Gamma\left\langle K_1^2 \eta^2 \right\rangle p_\sigma^2\,, \quad k_s = \dfrac{2\pi\nu_s}{\Pi}
\end{cases}
$$

Synchronous phase

No synchrotron oscillations

Neglecting $\left\langle \eta'^2 \right\rangle \frac{p_\sigma^2}{2}$ and $\Gamma \left\langle K_1^2 \eta^2 \right\rangle p_\sigma^2$

$$\begin{cases} \sigma' = 0 & \Rightarrow \quad p_\sigma = -\frac{1}{2\alpha} J_x \left\langle \gamma_x \right\rangle \\ p'_\sigma = 0 & \Rightarrow \quad \sigma = -\frac{\alpha_\sigma}{k_s^2} \left\langle \gamma_x \right\rangle J_x + \frac{\alpha}{k_s^2} \Gamma \left\langle K_1^2 \beta_x \right\rangle J_x \end{cases}$$

Energy deviation amplitude

$$p_{\sigma,max} = J_x \sqrt{\frac{\left\langle \gamma_x \right\rangle^2}{4\alpha^2} + \left(-\frac{\alpha_\sigma \left\langle \gamma_x \right\rangle}{\alpha k_s} + \frac{\Gamma \left\langle K_1^2 \beta_x \right\rangle}{k_s} \right)^2}$$

$\{X_0, Y_0\}$	$\{1\sigma_x, 0\}$	$\{67\sigma_x, 0\}$	$\{0, 1\sigma_y\}$	$\{0, 58\sigma_y\}$
$p_{\sigma,max}$	$10^{-3}\sigma_\delta$	$4\sigma_\delta$	$10^{-4}\sigma_\delta$	$0.26\sigma_\delta$

Betatron tune modulation

Tune derivatives

$\dfrac{\partial \nu_x}{\partial J_x}$	-5×10^4
$\dfrac{\partial^2 \nu_x}{\partial J_x \partial \delta}$	-6.8×10^7

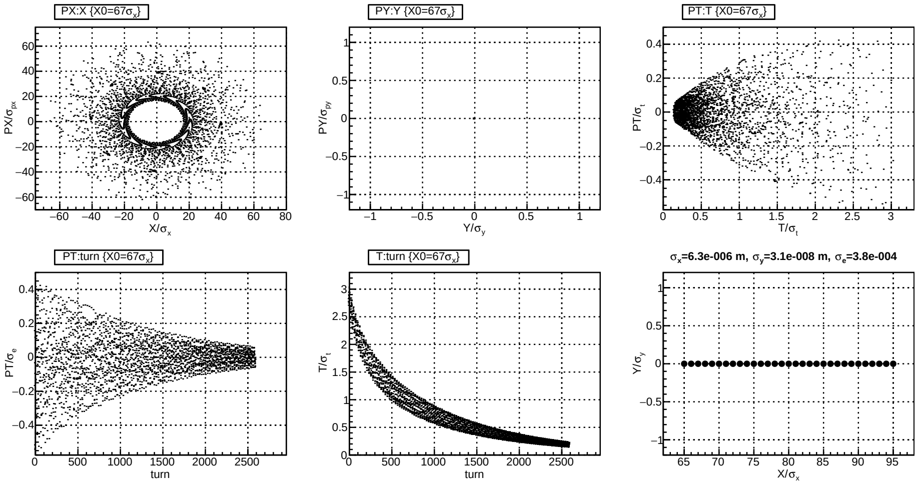
J_x	$67^2 \varepsilon_x / 2$
p_σ	$7\sigma_\delta$
$\Delta\nu_x = \dfrac{\partial \nu_x}{\partial J_x} Jx$	-0.03
$\Delta\nu_x = \dfrac{\partial^2 \nu_x}{\partial J_x \partial \delta} J_x p_\sigma$	-0.11
ν_x	0.14

Tune variation from sextupoles

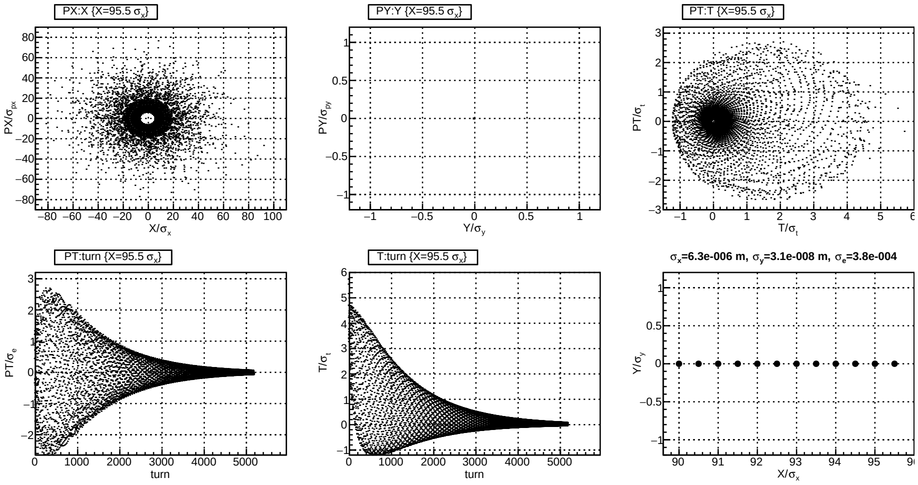
$$\frac{\partial \nu_x}{\partial J_x} = -\frac{1}{64\pi} \oint ds \oint d\tau K2(s) K2(\tau) \beta_x^{\frac{3}{2}}(s) \beta_x^{\frac{3}{2}}(\tau)$$
$$\times \left[\frac{3\cos\left(\pi\nu_x + |\varphi_x(s) - \varphi_x(\tau)|\right)}{\sin(\pi\nu_x)} + \frac{\cos\left(3\pi\nu_x + 3\,|\varphi_x(s) - \varphi_x(\tau)|\right)}{\sin(3\pi\nu_x)} \right]$$

$\dfrac{\partial^2 \nu_x}{\partial J_x \partial p_\sigma}$ comes from beta chromaticity $\dfrac{d\beta_x}{dp_\sigma}$

6d (SR from BEND, QUAD; longitudinally adjusted): $X_0 = 67\sigma_x$

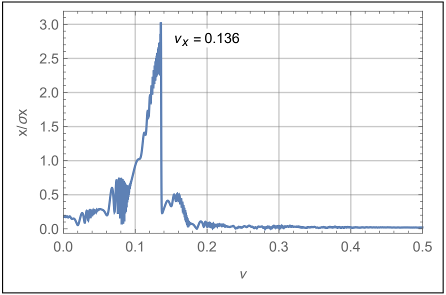


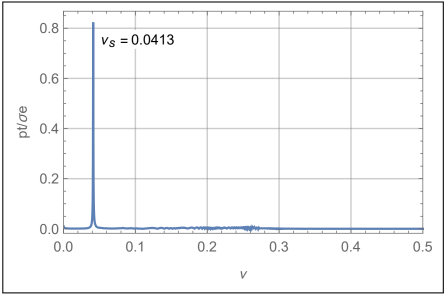
6d (SR from BEND, QUAD; longitudinally adjusted): $X_0 = 95.5\sigma_x$



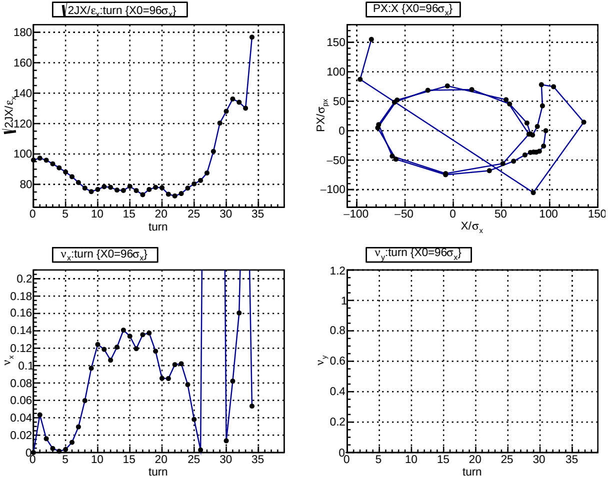
Spectra: $X_0 = 95.5\sigma_x$

$\nu_x = 269.14$ $\nu_y = 267.22$ $\nu_s = 0.0413$

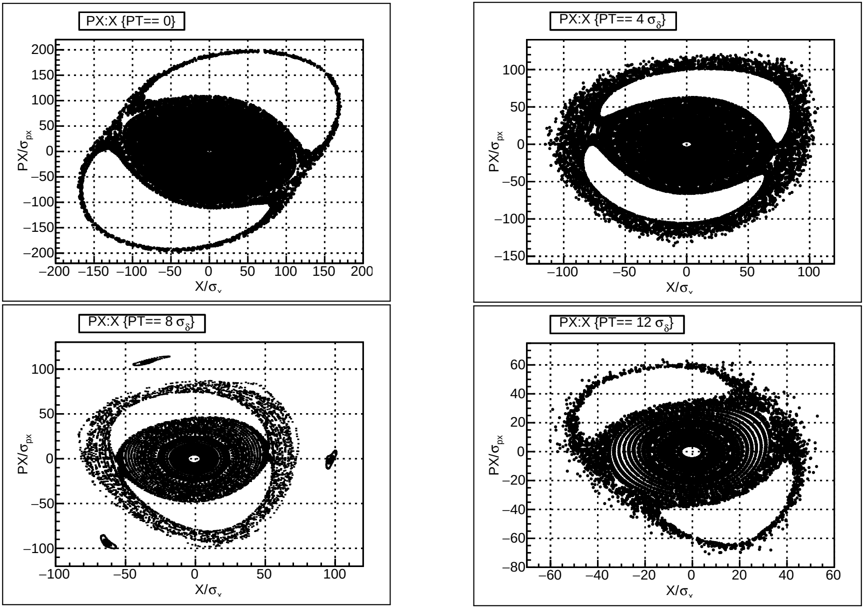




6d (SR from BEND, QUAD; longitudinally adjusted): first unstable, $X_0 = 96\sigma_x$



5d tracking: *PX* : *X*



Integer resonance: $PX : X$

$$\begin{pmatrix} \cos(\mu) & \beta\sin(\mu) \\ -\frac{1}{\beta}\sin(\mu) & \cos(\mu) \end{pmatrix} \begin{pmatrix} x \\ p_x \end{pmatrix} + \begin{pmatrix} 0 \\ kx^3 \end{pmatrix} = \begin{pmatrix} x \\ p_x \end{pmatrix}$$

Solution

$$\{x, p_x\} = \{0, 0\}$$

$$\{x, p_x\} = \left\{ \frac{\sqrt{2}}{\sqrt{k\beta}} \left(\tan\frac{\mu}{2}\right)^{\frac{1}{2}}, \frac{\sqrt{2}}{\sqrt{k\beta^3}} \left(\tan\frac{\mu}{2}\right)^{\frac{3}{2}} \right\}$$

$$\{x, p_x\} = \left\{ -\frac{\sqrt{2}}{\sqrt{k\beta}} \left(\tan\frac{\mu}{2}\right)^{\frac{1}{2}}, -\frac{\sqrt{2}}{\sqrt{k\beta^3}} \left(\tan\frac{\mu}{2}\right)^{\frac{3}{2}} \right\}$$

History

J. Jowett (CERN), Dynamic aperture for LEP: Physics and calculations, Conf.Proc. C9401174 (1994) 47-71, In *Chamonix 1994, LEP performance* 47-71

”Here I shall briefly describe a new effect which I propose to call Radiative Beta-Synchrotron Coupling (RBSC). It is a non-resonant effect. A particle with a large betatron amplitude make an extra energy loss by radiating in quadrupoles. ...you can say that its ”effective stable phase angle" will change to reflect the greater energy loss. The particle will tend to oscillate about a displaced fixed point in the synchrotron phase plane. This results in a growth of the oscillation amplitude which may eventually lead the particle outside the stable region in synchrotron phase space.”

Conclusion for horizontal plane at 45.6 GeV

1. Radiation from quadrupoles shifts the synchronous phase and energy proportional to the horizontal action,
2. therefore synchrotron oscillations arise.
3. Horizontal tune dependence on action and energy of the particle shifts the tune to integer resonance.
4. Minimization of beta function chromaticity will reduce $\frac{\partial^2 \nu_x}{\partial J_x \partial p_\sigma}$ and enhance horizontal dynamic aperture.