Bootstrap Unit Root Inference for Short-memory Linear Processes with GARCH(1,1) Noises

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Abstract: Recently there has been an interest in unit root inference in the presence of infinite-variance noises. Various re-sampling methods are proposed. For instance, Cavaliere, Georgiev and Taylor (2016) consider three sieve bootstraps for i.i.d. infinite-variance noises; Cavaliere, Georgiev and Taylor (2018) consider a wild bootstrap for infinite-variance noises possibly with unconditional heteroscedasticity. While these procedures may serve as a pre-test for robust estimations of heavy-tailed series (such as the self-weighted and local quasi-maximum likelihood estimators pioneered by Ling, 2007), most if not all papers do not consider conditional heteroscedasticity such as IGARCH(1,1). This paper assumes the error follows a short-memory linear process of GARCH(1,1) noises which tail-index, $\alpha \in (0,2)$, $\alpha = 2$, and $\alpha \in (2, \infty)$ (corresponding to the heavy-tailed Garch, Igarch and the finite variance Garch respectively). The limiting distributions of our unit root tests are functionals of two stable processes when $\alpha \in (0,2)$ and they are functionals of a standard Brownian motion when $\alpha \in [2,\infty)$. The limiting distributions may contain some nuisance parameters which are difficult, if not impossible, to estimate. We show, theoretically, that an m-out-of-n centered residual-based block bootstrap (RBB) can be applied not only to finite-variance series, but also to infinite-variance series. Despite our test statistics are non-pivotal, some simulation studies show encouraging performance of this RBB approach.

Keywords and phrases: Conditional heteroscedasticity, index of regular variation, infinite variance, non-pivotal, nuisance parameter, residual-based block bootstrap (RBB)