Manifold Learning and an Inverse Problem for a Wave Equation

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We consider following two problems on manifold learning and wave imaging:

1. We study the geometric Whitney problem on how a Riemannian manifold (M,g) can be constructed to approximate a metric space (X,d_X) in the Gromov-Hausdorff sense. This problem is closely related to manifold interpolation (or manifold learning) where a smooth n-dimensional surface S in m-dimensional Euclidean space, where m>n, needs to be constructed to approximate a discrete set of points. These questions are encountered in differential geometry, machine learning, and in many inverse problems encountered in applications. The determination of a Riemannian manifold includes the construction of its topology, differentiable structure, and metric.

2. We study an inverse problem for a wave on a compact manifold (M,g). We assume that we are given an open subset V of the manifold and the source-to-solution map corresponding to the data obtained from measurements made on the set V.
We use this data to construct in a stable way a discrete metric space X that approximates the manifold M in the Gromov-Hausdorff sense.

By combining these results, we obtain that the source-to-solution map in an open set V, determines in a stable way the smooth manifold (M,g).

The results on the first problem are done in collaboration with C. Fefferman, S. Ivanov, Y. Kurylev, and H. Narayanan, and the results on the second problem with R. Bosi and Y. Kurylev.

References:

[1] C. Fefferman, S. Ivanov, Y. Kurylev, M. Lassas, H. Narayanan: Reconstruction and interpolation of manifolds I: The geometric Whitney problem. arXiv:1508.00674

[2] R. Bosi, Y. Kurylev, M. Lassas: Reconstruction and stability in Gel'fand's inverse interior spectral problem. arXiv:1702.07937.