High-order time-stepping schemes for time-fractional diffusion equations

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The time-fractional diffusion, which has received much attention in recent years, describes a diffusion process in which the mean square displacement of a particle grows slower (sub-diffusion) than that in the normal diffusion process. The solution of the fractional diffusion often exhibits a singular layer, provided that the source data is not compatible with the initial data, which makes the numerical treatment and analysis challenging. We develop a systematic strategy to the starting k-1 steps in order to restore the desired kth-order convergence rate of the k-step BDF convolution quadrature for the time-fractional equations. The desired kth-order convergence rate can be achieved even if the solution is nonsmooth. Extensive numerical schemes will be provided to illustrate the theoretical results.

References:

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