Globally hyperbolic moment model of arbitrary order for special relativistic Boltzmann equation

Huazhong Tang
School of Mathematics Sciences, Peking University, China
School of Mathematics and Computer Science, Xiangtan University, China
Email: hztang@math.pku.edu.cn

We extend the model reduction method by the operator projection to the special relativistic Boltzmann equation. The derivation of globally hyperbolic moment system of arbitrary order is built on our careful study of several families of the complicate Grad type orthogonal polynomials depending on a parameter. We give their recurrence relations, calculate their derivatives with respect to the independent variable and parameter respectively, and study their zeros and coefficient matrices in the recurrence formulas. Some properties of the moment system are also proved. They include the eigenvalues and their bound as well as eigenvectors, hyperbolicity, characteristic fields, linear stability, and Lorentz covariance.

A semi-implicit numerical scheme is presented to solve a 1D Cauchy problem of our hyperbolic moment system in order to verify convergence of the moment method. The results show that the solutions of our hyperbolic moment system converge to the solution of the special relativistic Boltzmann equation as the order of the hyperbolic moment system increases.

It is interesting to develop robust, high order accurate numerical schemes for the moment system, find other basis for the derivation of moment system with some good property, e.g. non-negativity, and investigate the relativistic effects by using the moment system.

[1] Y.Y. Kuang & H.Z. Tang, J. Stat. Phys, 167(5), 2017, 1303-1353.
[2] Y.Y. Kuang & H.Z. Tang, arXiv: 1705.03990, 2017.