A Generalized MBO Diffusion Generated Motion for Orthogonal Matrix-valued Fields

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MBO diffusion generated motion is a method introduced by Merriman, Bence, and Osher for evolving the boundary of a set by mean curvature flow. The method consists of two simple steps, which are iterated until convergence. The first is the time evolution of an indicator function of the set by the diffusion equation for a short time. The second is the point-wise thresholding of this function to obtain a new indicator function. Over the last 25 years, this method has been further analyzed, developed, and employed in a variety of interesting applications.

In this talk, We consider the problem of finding stationary points of the Dirichlet energy for orthogonal matrix-valued fields. Following the Ginzburg-Landau approach, this energy is relaxed by penalizing the matrix-valued field when it does not take orthogonal matrix values. A generalization of the MBO diffusion generated motion is introduced that effectively finds local minimizers of this energy by iterating two steps until convergence. In the first step, as in the original method, the current matrix-valued field is evolved by the diffusion equation. In the second step, the field is point-wise reassigned to the closest orthogonal matrix, which can be computed via the singular value decomposition. We extend the Lyapunov function of Esedoglu and Otto to show that the method is non-increasing on iterates and hence, unconditionally stable. We also prove that spatially discretized iterates converge to a stationary solution in a finite number of iterations. The algorithm is implemented using the closest point method and non-uniform fast Fourier transform. We conclude with several numerical experiments on flat tori and closed surfaces, which, unsurprisingly, exhibit classical behaviors from the Allen-Cahn and complex Ginzburg-Landau equations, but also new phenomena.

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