

# **Overcoming the Curse of Dimensionality for Hamilton-Jacobi equations with Applications to Control and Differential Games**

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It is well known that certain Hamilton-Jacobi partial differential equations (HJ PDE's) play an important role in analyzing control theory and differential games. The cost of standard numerical algorithms for HJ PDE's is exponential in the space dimension and time, with huge memory requirements. Here we propose and test methods for solving a large class of these problems without the use of grids or significant numerical approximation. We begin with the classical Hopf and Hopf-Lax formulas which enable us to solve state independent problems via variational methods originating in compressive sensing with remarkable results. We can evaluate the solution in  $10^{-4}$  to  $10^{-8}$  seconds per evaluation on a laptop. The method is embarrassingly parallel and has low memory requirements.

Recently, with a slightly more complicated, but still embarrassingly parallel, we have extended this in great generality to state dependent HJ equations, apparently, with the help of parallel computers, overcoming the curse of dimensionality for these problems.

The term, "curse of dimensionality" was coined by Richard Bellman in 1957 when he did his classic work on dynamic optimization