

Tensor Tomography in Fan-beam Coordinates

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The tensor tomography problem (TTP) consists of assessing what can be reconstructed about a symmetric m -tensor field from knowledge of its integrals along geodesic curves (its so-called X-ray transform) through a manifold. Such a problem finds applications to Computerized Tomography ($m=0$), Ultrasound Doppler Tomography ($m=1$), deformation boundary rigidity ($m=0,2$) and tomography in slightly anisotropic elastic media ($m=4$). In this talk, we will review recent results on the TTP in the case of the Euclidean disk, where straight lines are parametrized in fan-beam coordinates.

We will first discuss an equivalence between the classical moment conditions (due to Gelfand, Graev, Helgason and Ludwig) and a more recent range characterization due to Pestov and Uhlmann, found in the context of simple Riemannian surfaces. Inspired by the latter characterization, the introduction of new operators at the boundary will allow for an extremely efficient way to enforce all moment conditions at once on noisy X-ray transforms of functions ($m=0$). Passing to tensors of arbitrary order, a problem where injectivity does not hold (unlike the case $m=0$), we then propose a suitable representative (other than the traditionally sought-after “solenoidal” candidate) to be reconstructed modulo the kernel of the transform, and an efficient procedure to do so, illustrated numerically. Finally, we will explain how to generalize some of these ideas to the case of the attenuated transform.