



THE HONG KONG
POLYTECHNIC UNIVERSITY
DEPARTMENT OF
APPLIED PHYSICS
應用物理系



Presentation in AoE Workshop (2016)
Advanced Concept in Wave Physics
Topology and PT Symmetry

PT symmetries & Non-reciprocity in periodic photonic systems

by

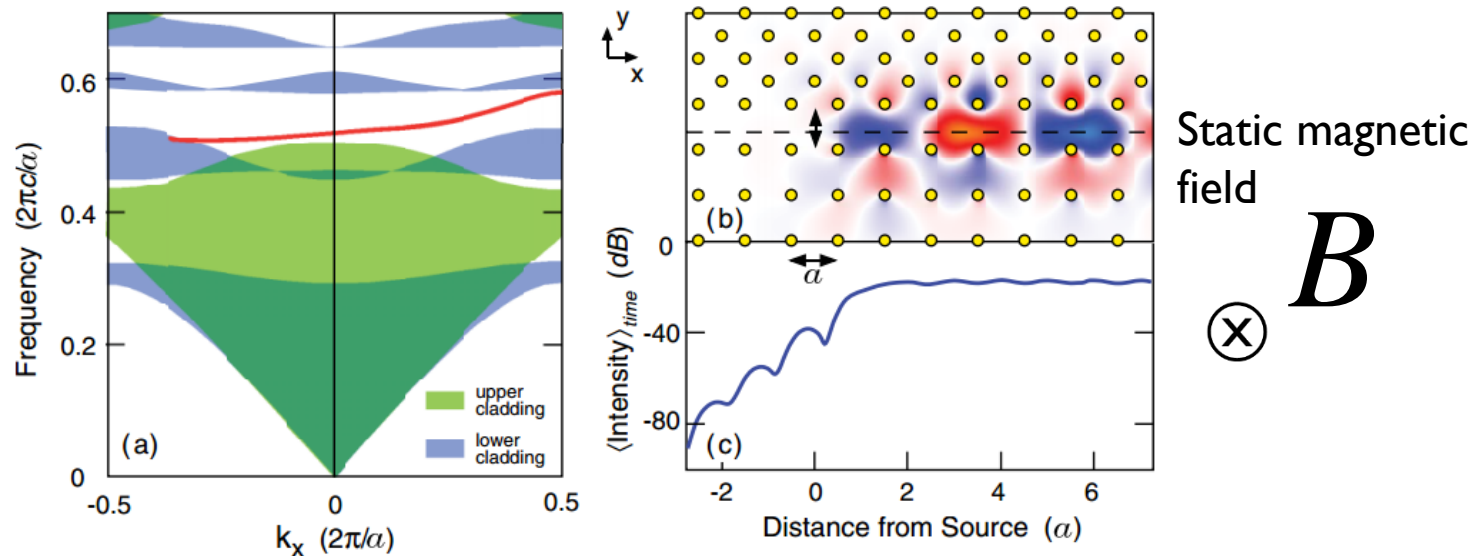
Kin Hung Fung

Department of Applied Physics
The Hong Kong Polytechnic University

One-way propagation in photonic circuit

- Topological Photonics
 - Term often used: *Time-reversal symmetry* (TRS)

Edge mode breaks spectral reciprocity: $\omega(k) \neq \omega(-k)$



Z. Wang et al., PRL 100, 013905

F. D. M. Haldane and S. Raghu, PRL 100, 013904

A recent review:

Topological Photonics

Nat. Photon. by Ling Lu et al.



This talk focuses on breaking spectral reciprocity

$$\omega(k) \neq \omega(-k)$$

There is a difference between spectral reciprocity and Lorentz reciprocity.

- Our recent works related to spatial-temporal symmetries such as PT symmetry are also provided as examples:
 - Asymmetric bands in a “diatomic” plasmon waveguide
 - [Phys. Rev. B 92, 165430 \(2015\)](#)
 - Non-reciprocal μ -near-zero surface modes
 - [Phys. Rev. B 91, 235410 \(2015\)](#)

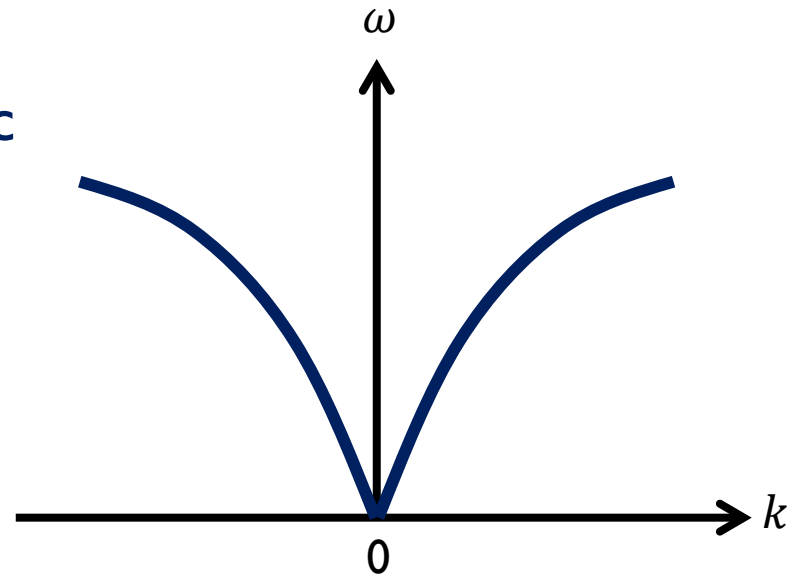


Spectral Reciprocity & PT Symmetries

Spectral reciprocity

- band structure is symmetric

$$\omega(k) = \omega(-k)$$



PT symmetry

- system is invariant by P and T operations together.

T: time reversal

$$(x, y, z, t) \rightarrow (x, y, z, -t)$$

P: spatial inversion

$$(x, y, z, t) \rightarrow (-x, -y, -z, t)$$

P_x : spatial inversion

$$(x, y, z, t) \rightarrow (-x, y, z, t)$$

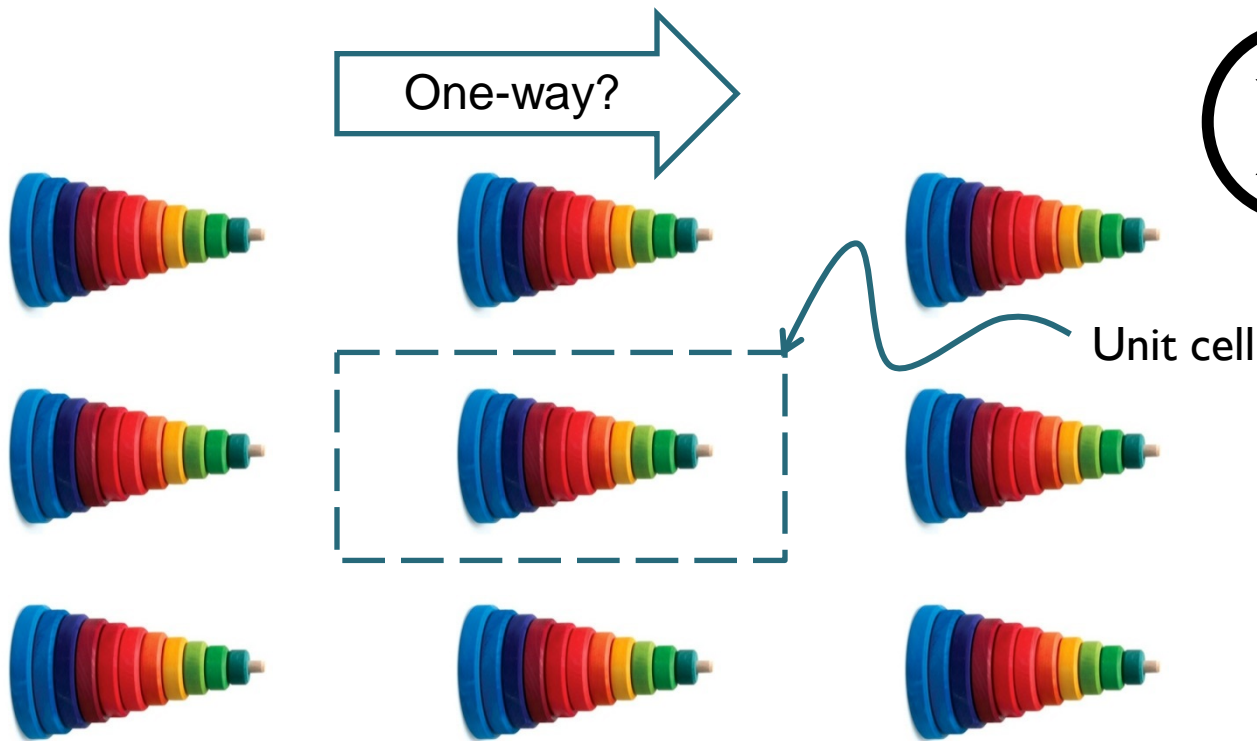
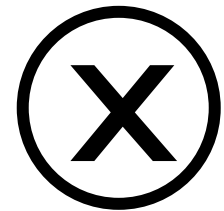


Quiz I

- Can this periodic system support asymmetric band $\omega(k) \neq \omega(-k)$?

Static
magnetic field

B

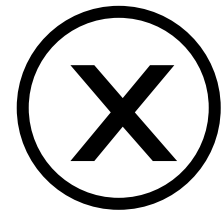


Quiz 2

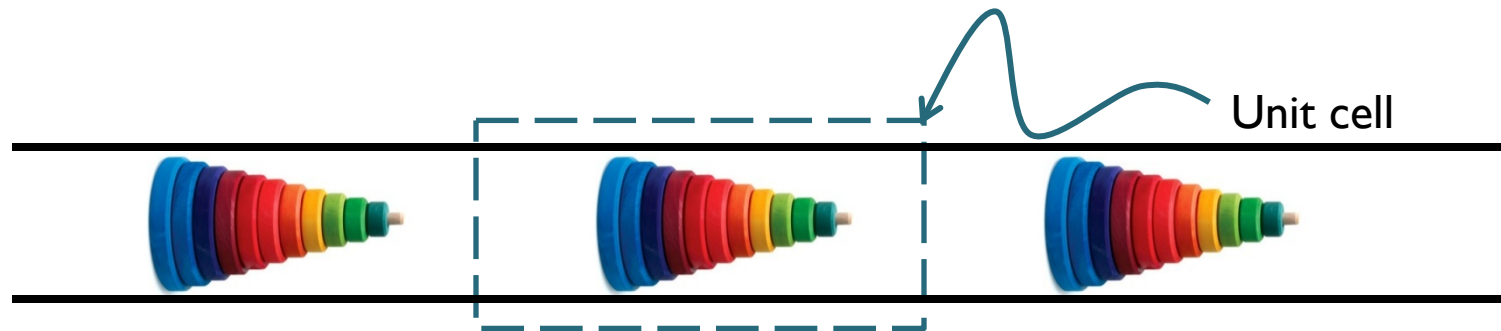
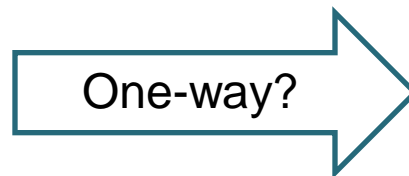
Static
magnetic field



B



- Can this **ID** periodic system support asymmetric band $\omega(k) \neq \omega(-k)$?



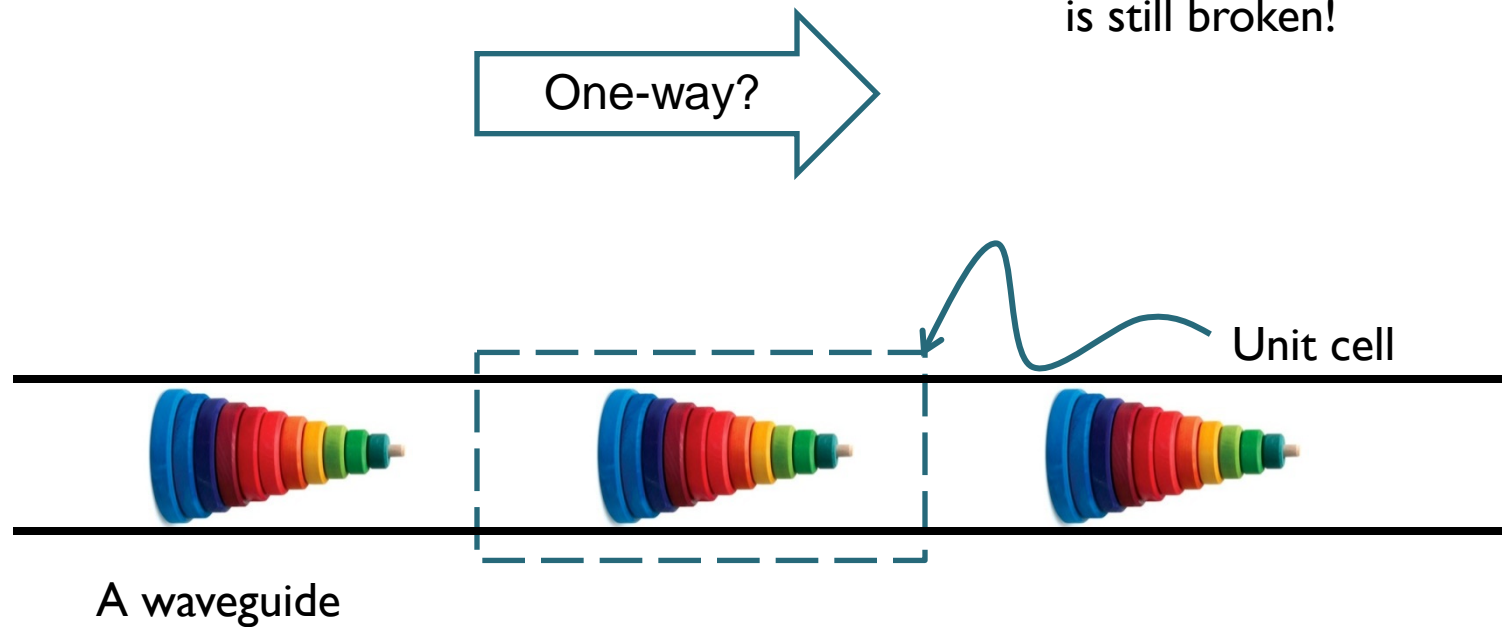
A waveguide



Quiz 3

- What if the materials have small gain/loss instead of magnetic field?

The time reversal symmetry is still broken!



Symmetries and spectral reciprocity

- We need to break enough symmetries to achieve spectral non-reciprocity

$$\omega(k) \neq \omega(-k)$$

- Well-known examples of symmetries to break
 - P: spatial inversion symmetry
 - T: time reversal symmetry (TRS)



What is TRS for EM waves?

We say that a system of given $\epsilon(\mathbf{x})$ and $\mu(\mathbf{x})$ has TRS if
The macroscopic Maxwell's equations and the constitutive relations for the same ϵ and μ are still satisfied by time-reversing the oscillating fields,

Original:

$$\begin{aligned}\nabla \times \mathbf{E} &= i\omega\mu \cdot \mathbf{H} \\ \nabla \times \mathbf{H} &= -i\omega\epsilon \cdot \mathbf{E}\end{aligned}$$

$$\epsilon^* = \epsilon$$

$$\mu^* = \mu$$

To have TRS, we want the following
after time-reversal of fields:

$$\begin{aligned}\nabla \times (\mathbf{E}^*) &= i\omega\mu \cdot (-\mathbf{H}^*) \\ \nabla \times (-\mathbf{H}^*) &= -i\omega\epsilon \cdot (\mathbf{E}^*)\end{aligned}$$

These new equations may
NOT be satisfied.

If they are satisfied, then we have
these conditions on ϵ and μ .



Consequence of TRS on Band Structures

Ref: Optical Properties
of Photonic Crystals
by K. Sakoda

If $E^*(x)e^{i(kx-\omega t)}$ is a solution,

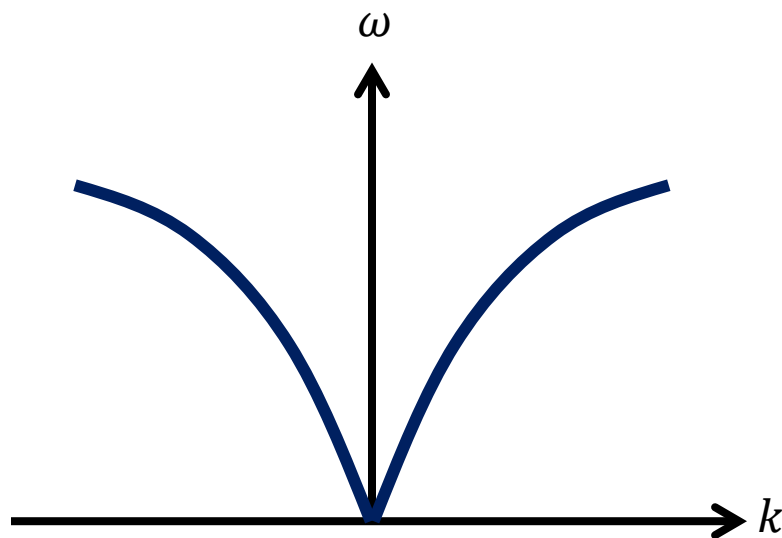
$E(x)e^{i(-k^*x-\omega t)}$ is also a solution.

Symmetry in band structure

$$\omega(k^*) = \omega(-k)$$

For pass band with real k ,
we have $\omega(k) = \omega(-k)$

even when there is no spatial
symmetry other than periodicity



Symmetries and spectral reciprocity

- We need to break enough symmetries to achieve spectral non-reciprocity

$$\omega(k) \neq \omega(-k)$$

- Well-known examples of symmetries to break
 - P: spatial inversion symmetry
 - T: time reversal symmetry (TRS)

Enough?



Symmetries and spectral reciprocity

- We need to break enough symmetries to achieve spectral non-reciprocity

$$\omega(k) \neq \omega(-k)$$

- Well-known examples of symmetries to break
 - P: spatial inversion symmetry
 - T: time reversal symmetry
 - Symmetric permittivity and permeability tensor



Lorentz Reciprocity



Lorentz Reciprocity

Lorentz reciprocity
can be written as

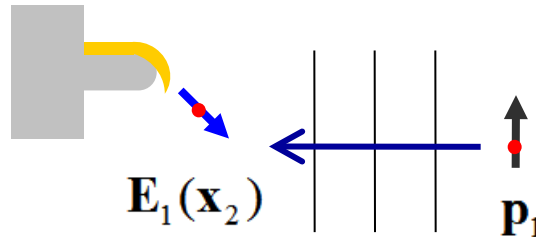
$$\int \mathbf{J}_1 \cdot \mathbf{E}_2 dV = \int \mathbf{J}_2 \cdot \mathbf{E}_1 dV$$

$$\mathbf{p}_1 \cdot \mathbf{E}_2(\mathbf{x}_1) = \mathbf{p}_2 \cdot \mathbf{E}_1(\mathbf{x}_2)$$

or symmetry in Green's Function

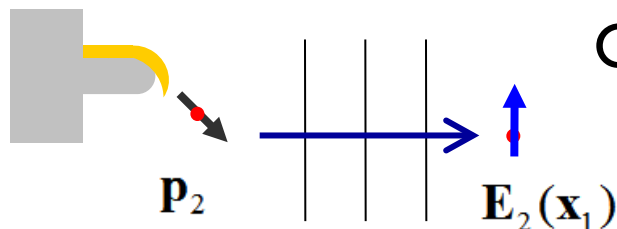
$$\vec{\mathbf{G}}(\mathbf{x}_1, \mathbf{x}_2) = \vec{\mathbf{G}}^T(\mathbf{x}_2, \mathbf{x}_1)$$

Case A:



Source \leftrightarrow *Receiver*

Case B:



Conditions of reciprocal medium:

$$\boldsymbol{\varepsilon}^T = \boldsymbol{\varepsilon}$$

$$\boldsymbol{\mu}^T = \boldsymbol{\mu}$$



A static magnetic field breaks both

- 1) T reversal symmetry (TRS) &
- 2) Symmetry in ϵ and μ (Lorentz reciprocity)

Observe the difference

e.g., gyromagnetic materials

$$\boldsymbol{\mu} = \begin{pmatrix} \mu & i\kappa_m & 0 \\ -i\kappa_m & \mu & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$

$$\boldsymbol{\epsilon} = \begin{pmatrix} \epsilon & i\kappa_e & 0 \\ -i\kappa_e & \epsilon & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

(representation in frequency domain)

Broken TRS

$$\boldsymbol{\epsilon}^* \neq \boldsymbol{\epsilon}$$

$$\boldsymbol{\mu}^* \neq \boldsymbol{\mu}$$

Broken Lorentz reciprocity

$$\boldsymbol{\epsilon}^T \neq \boldsymbol{\epsilon}$$

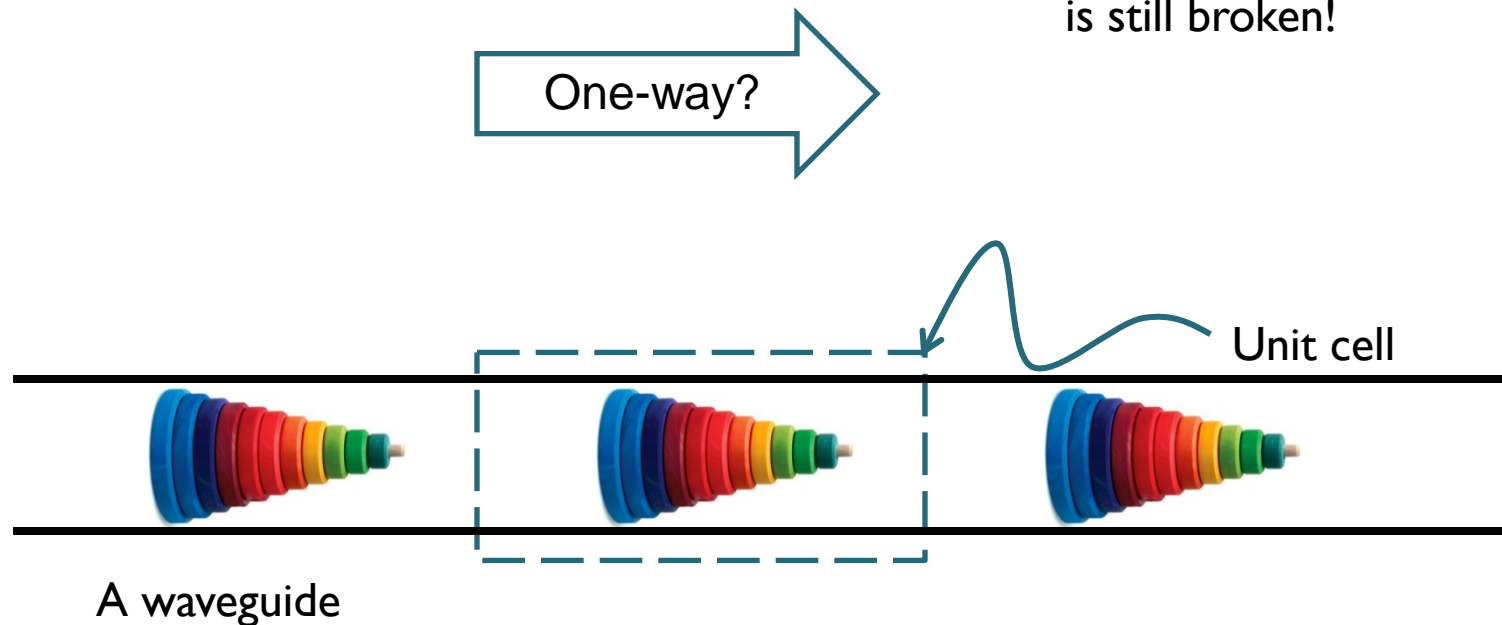
$$\boldsymbol{\mu}^T \neq \boldsymbol{\mu}$$



Quiz 3 (simple)

- What if the materials have small gain/loss instead of magnetic field?

The time reversal symmetry is still broken!



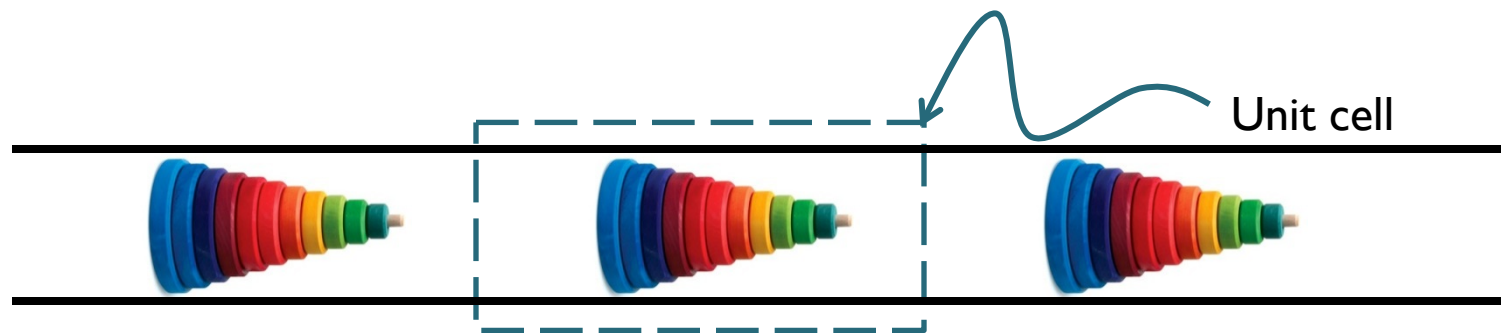
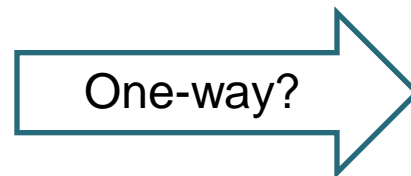
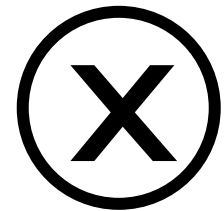
NO because of Lorentz reciprocity.



Quiz 2 (not very simple) Static magnetic field

- Can this **ID** periodic system support asymmetric band $\omega(k) \neq \omega(-k)$?

B



A waveguide



Symmetries and spectral reciprocity

- We need to break enough symmetries to achieve spectral non-reciprocity

$$\omega(k) \neq \omega(-k)$$

- Well-known examples of symmetries to break
 - P: spatial inversion symmetry
 - T: time reversal symmetry
 - Symmetric permittivity and permeability tensor



Lorentz Reciprocity

Enough now?

The answer is still NO!

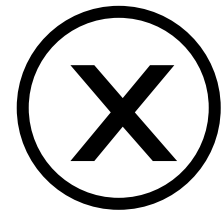


Quiz 2

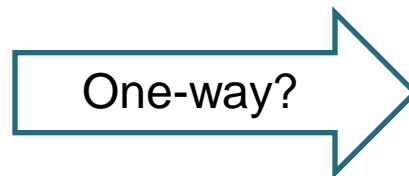
Static
magnetic field



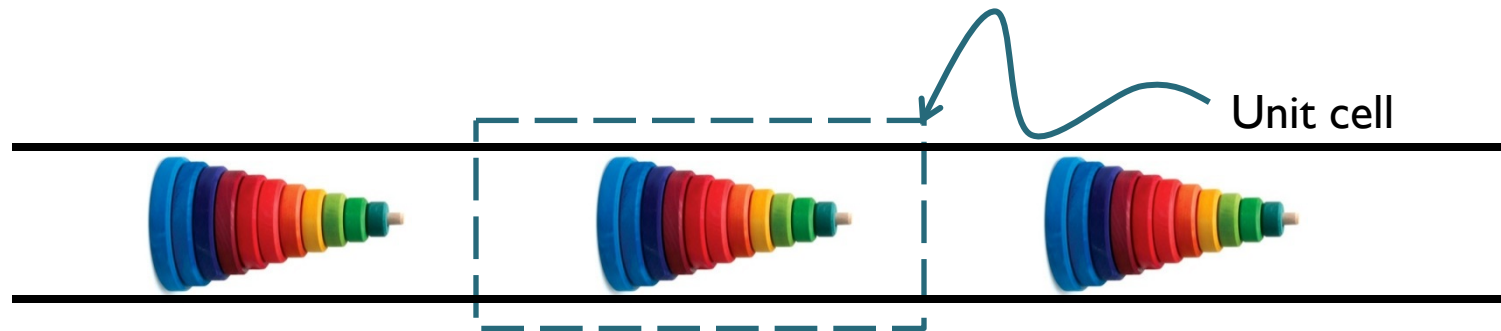
B



- Can this **ID** periodic system support asymmetric band $\omega(k) \neq \omega(-k)$?



NO



A waveguide

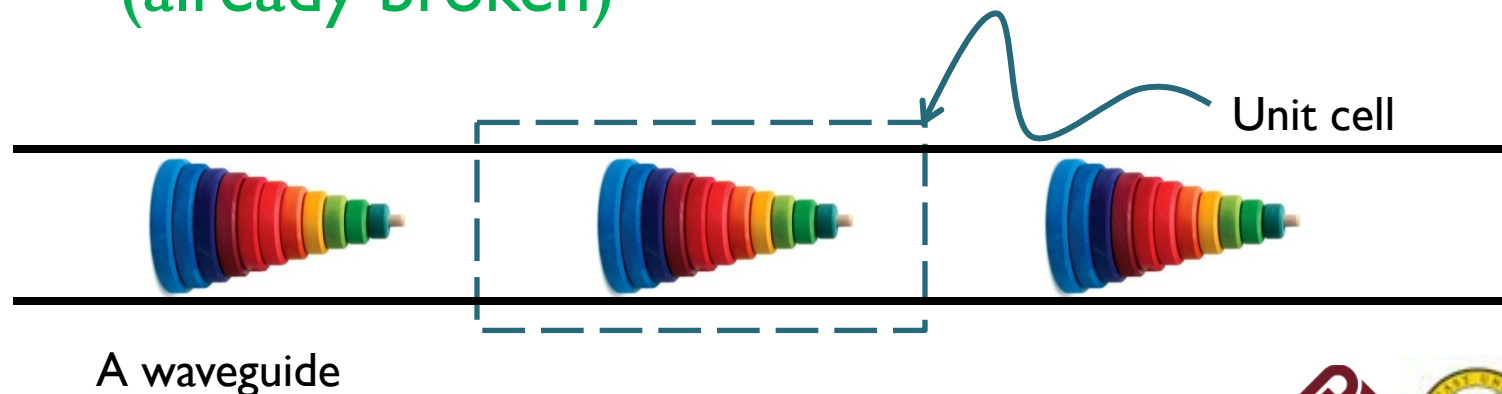
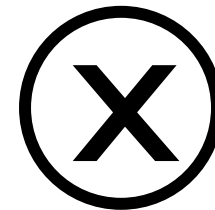


Analysis for Quiz 2

- To break
 - P: spatial inversion symmetry
(already broken)
 - T: time reversal symmetry
(already broken)
 - Symmetric ϵ and μ
(already broken)

Static
magnetic field

B



Why is the answer still NO?

My answer is:
Nature is happy with symmetric bands.

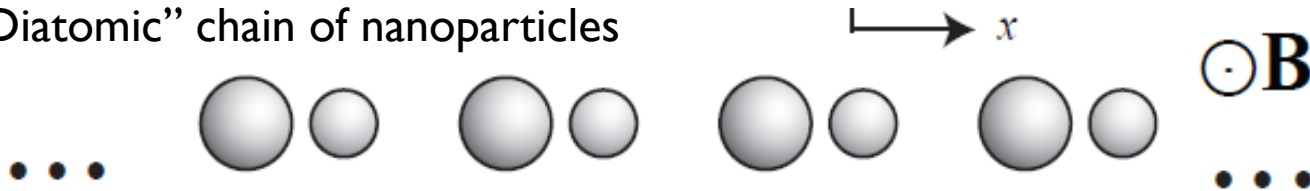


Let us consider the simplest example in plasmonics.



Example: Plasmonic nanoparticle chain

“Diatomic” chain of nanoparticles



$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon & i\kappa_e & 0 \\ -i\kappa_e & \varepsilon & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}$$

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}$$

$$\kappa_e = -\frac{\omega_c}{\omega} \frac{\omega_p^2}{\omega^2 - \omega_c^2}$$

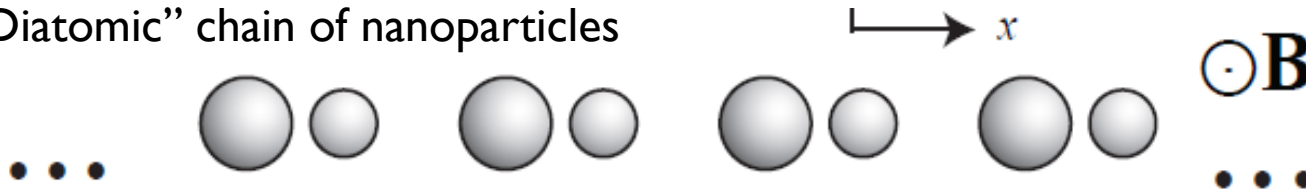
ω_p : Plasma frequency

ω_c : Cyclotron frequency



Example: Plasmonic nanoparticle chain

“Diatomic” chain of nanoparticles



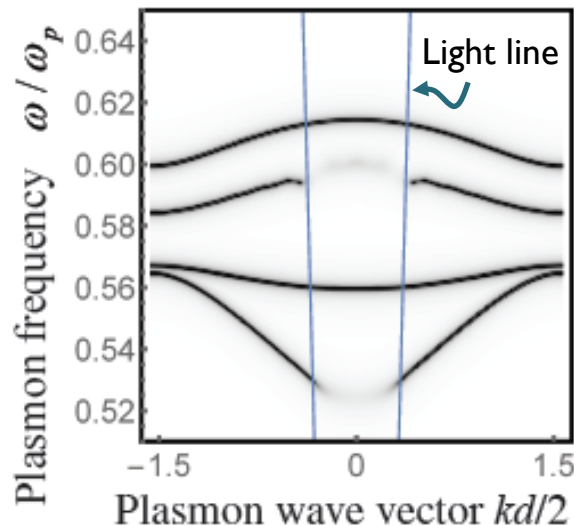
$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon & i\kappa_e & 0 \\ -i\kappa_e & \varepsilon & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}$$

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}$$

ω_p : Plasma frequency

ω_c : Cyclotron frequency

$$\kappa_e = -\frac{\omega_c}{\omega} \frac{\omega_p^2}{\omega^2 - \omega_c^2}$$



Notes:

4 bands due to
4 degrees of freedom
in unit cell.

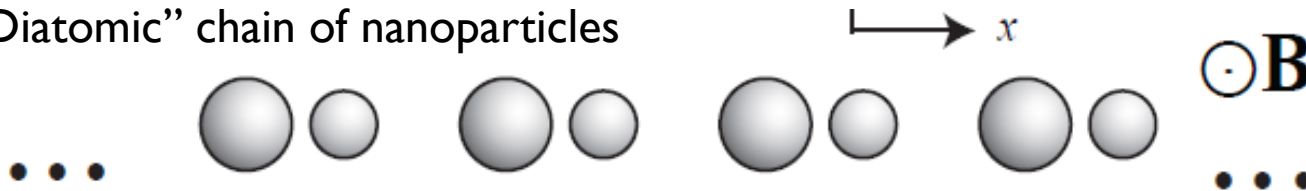


We want this
asymmetric angry face!



Why are the bands still symmetric in k ?

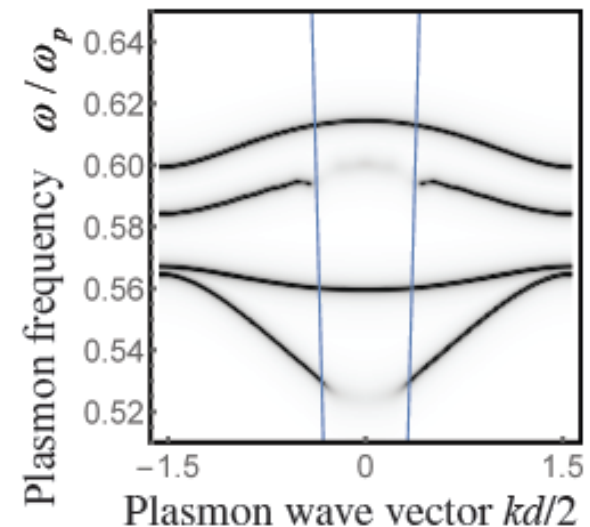
“Diatomic” chain of nanoparticles



Protected by RT symmetry

T = time reversal

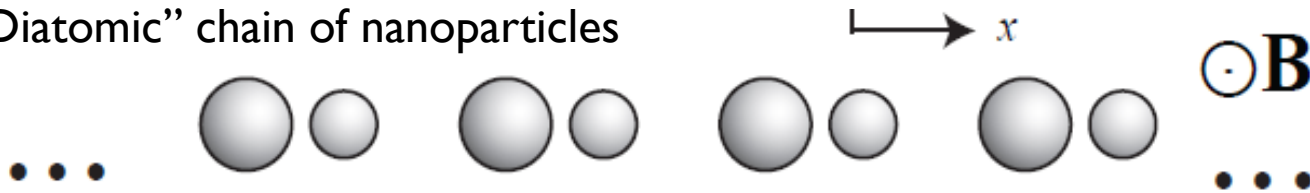
R = 180° rotation about x -axis



We need to break this RT symmetry too!

Why are the bands still symmetric in k ?

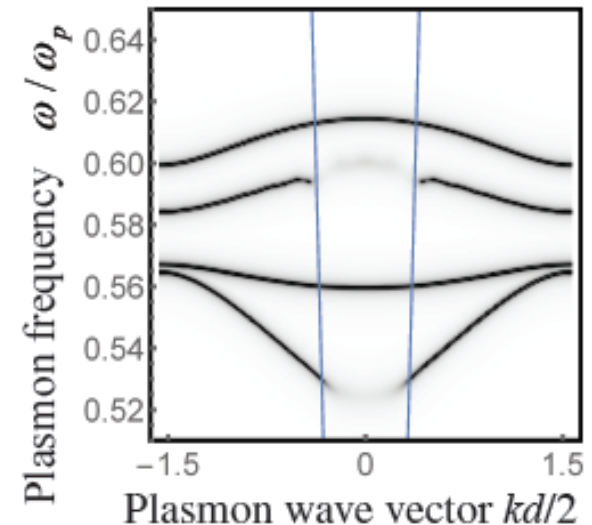
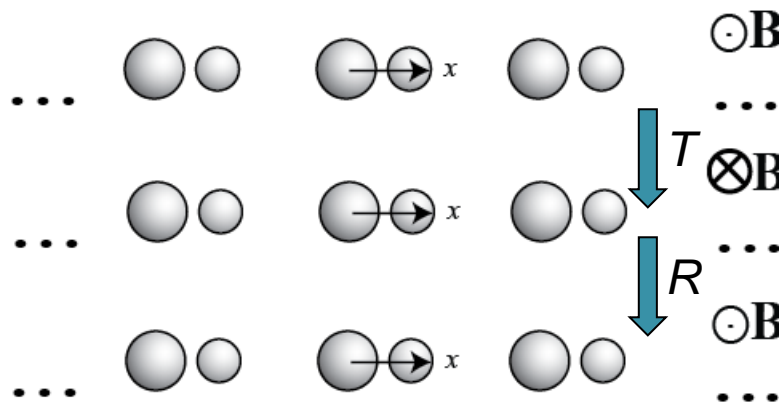
“Diatomic” chain of nanoparticles



Protected by **RT symmetry**

T = time reversal

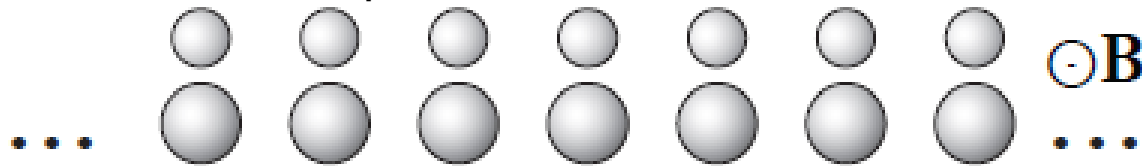
R = 180° rotation about x-axis



We need to break this RT symmetry too!

What about this one? (Case B)

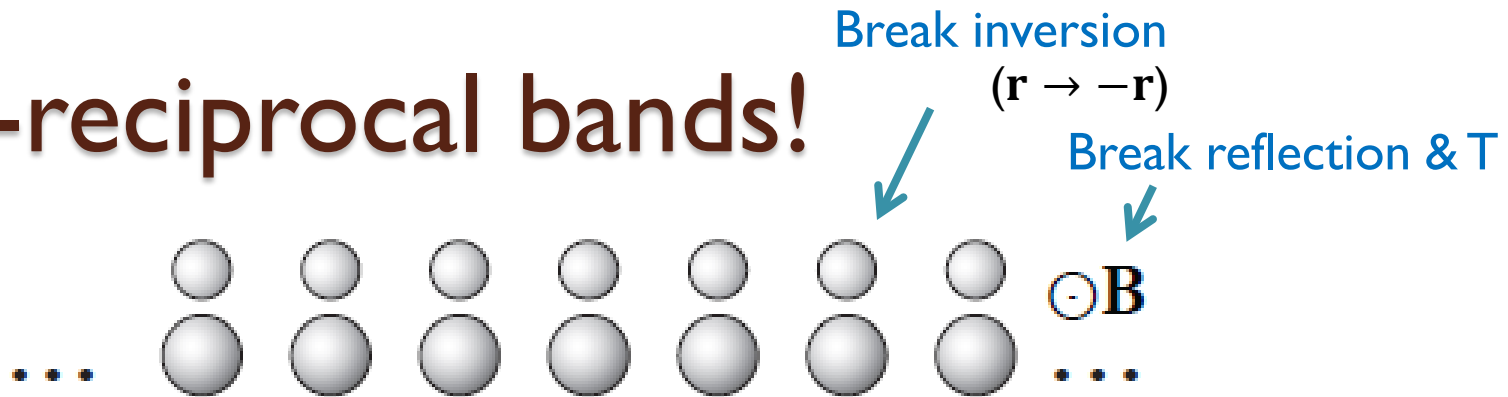
“Diatomic” chain of nanoparticles



Did we break enough
symmetries?

Yes.

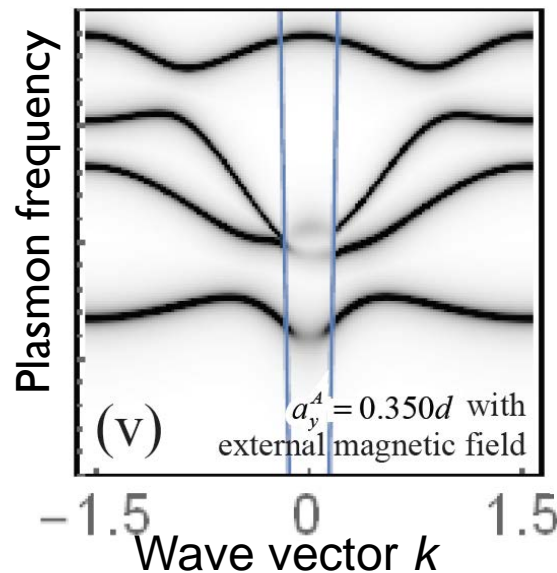
Non-reciprocal bands!



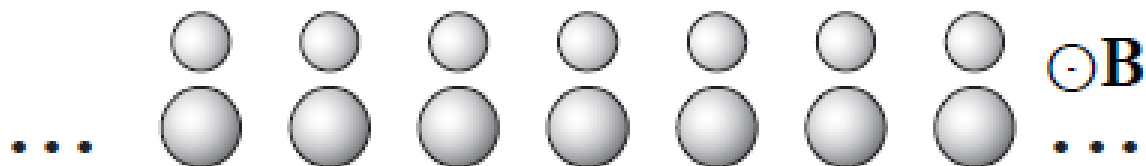
Result: **Non-reciprocal bands**

Did we break enough symmetries?

Yes,
It breaks P, T, RT, ...



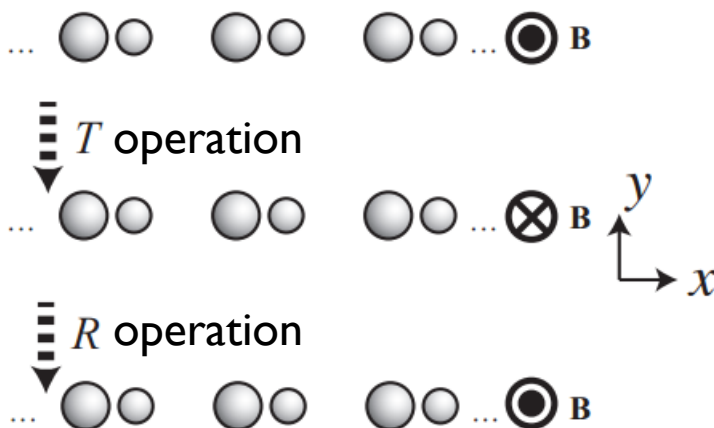
Why?



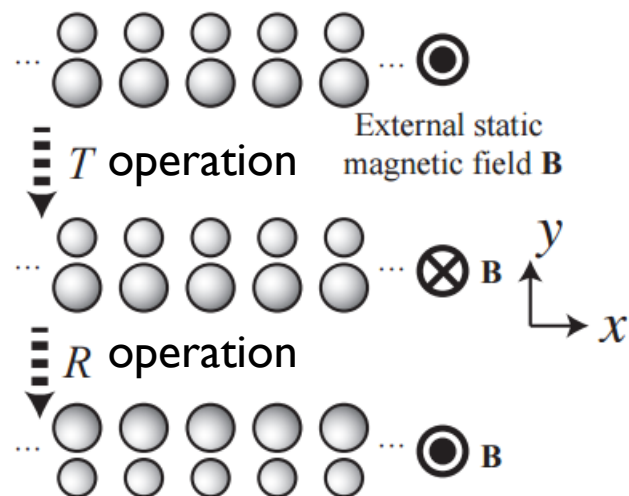
Did we break enough symmetries?

Yes.

Case A



Case B



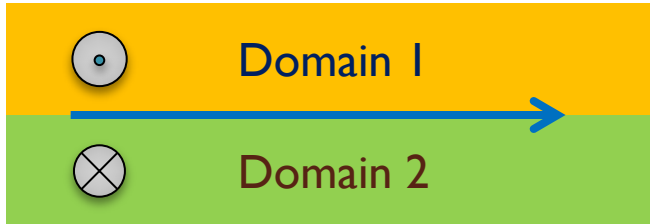
This talk focuses on **breaking spectral reciprocity &** its relation to PT or RT symmetries

- Conclusion of part A:
 - We need to break a lot of spatial temporal symmetries to achieve non-reciprocity
 - CW Ling et al. *Physical Review B* 92 165430 (2015)
- **Can we keep PT symmetry while having non-reciprocal bands? Yes.**
- Next part:
 - Non-reciprocal μ -near-zero surface modes
 - PT symmetric magnetic domains



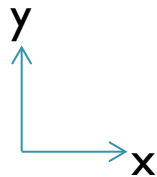
We now consider this

We will focus on interface modes



$$\vec{\mu}_I = \begin{pmatrix} \mu_1 & -i\mu_2 & 0 \\ i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu'_1 \end{pmatrix}$$

$$\vec{\mu}_{II} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu'_1 \end{pmatrix}$$



The surface mode dispersion has been considered before:
H. Zhu and C. Jiang, Opt. Express **18**, 6914 (2010)
but there is something missing...



This one has PT symmetry

T: time reversal

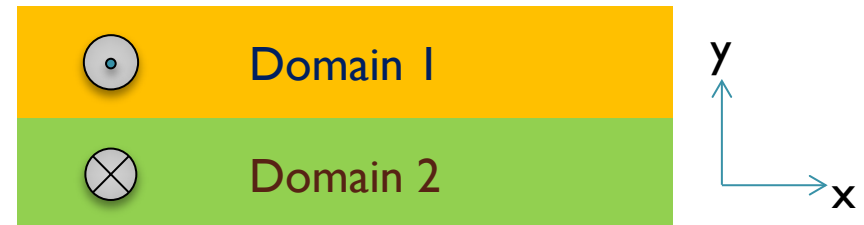
$$(x, y, z, t) \rightarrow (x, y, z, -t)$$

P: spatial inversion

$$(x, y, z, t) \rightarrow (-x, -y, -z, t)$$

P_y : spatial inversion in y

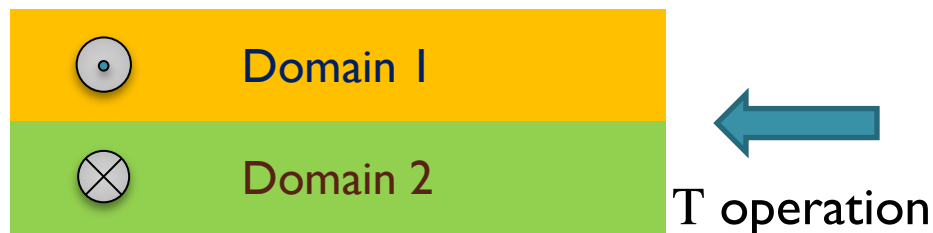
$$(x, y, z, t) \rightarrow (x, -y, z, t)$$



P operation



The system has PT but not $P_y T$ symmetry.

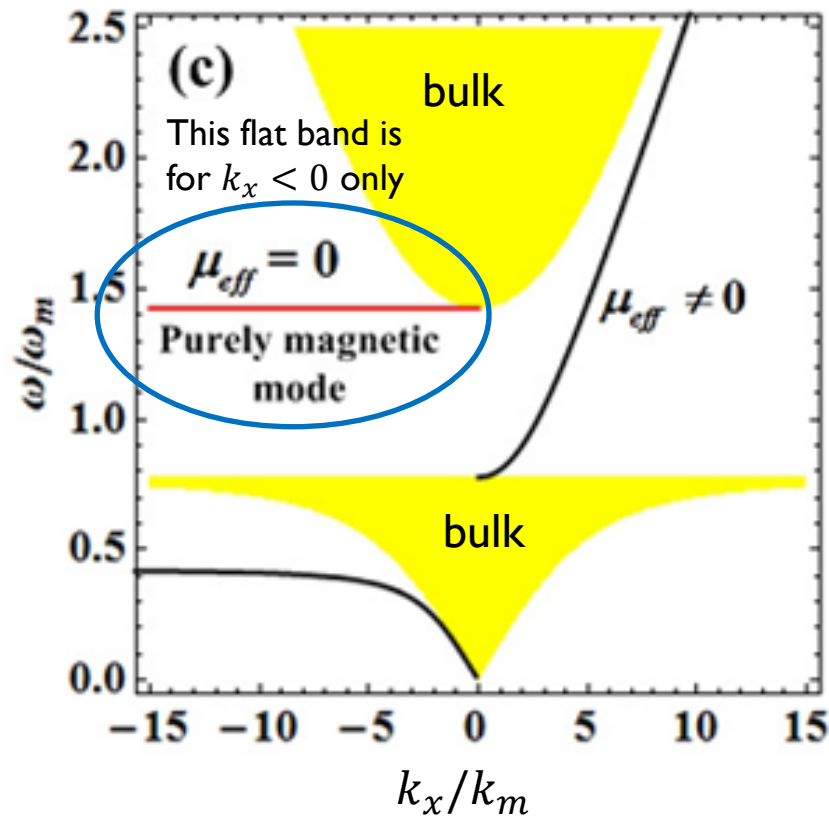


We will focus on interface modes

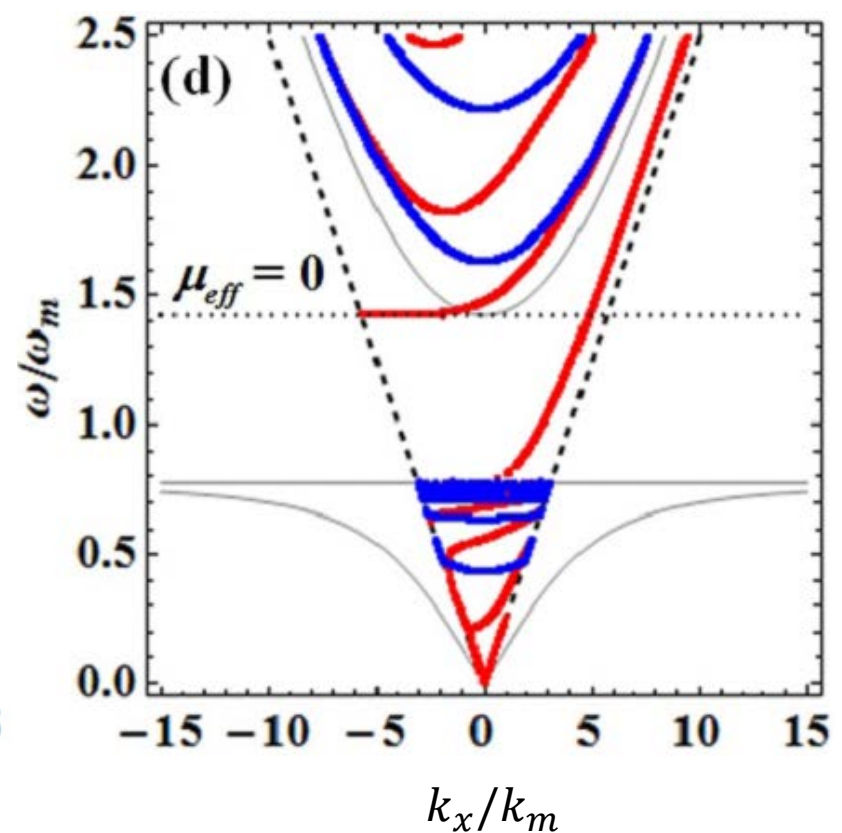


PT symmetric magnetic domains

Semi-infinite



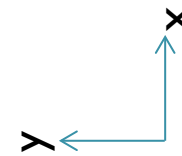
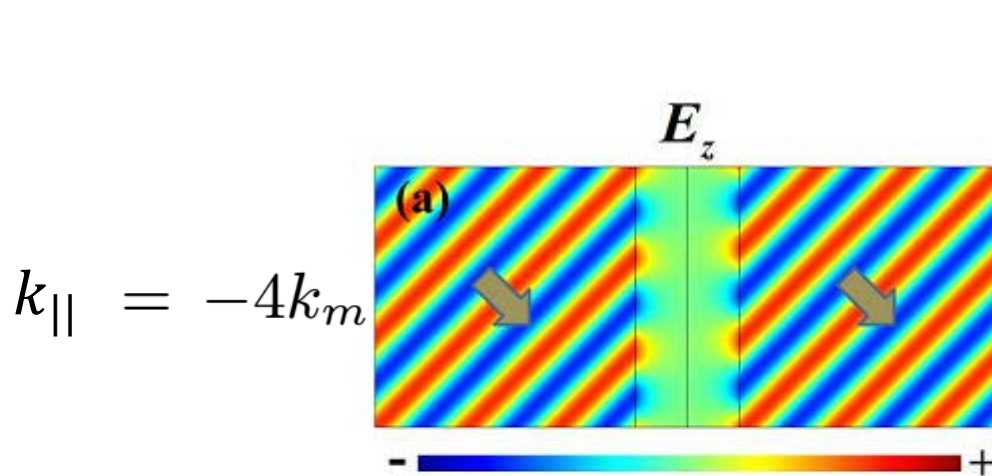
Finite



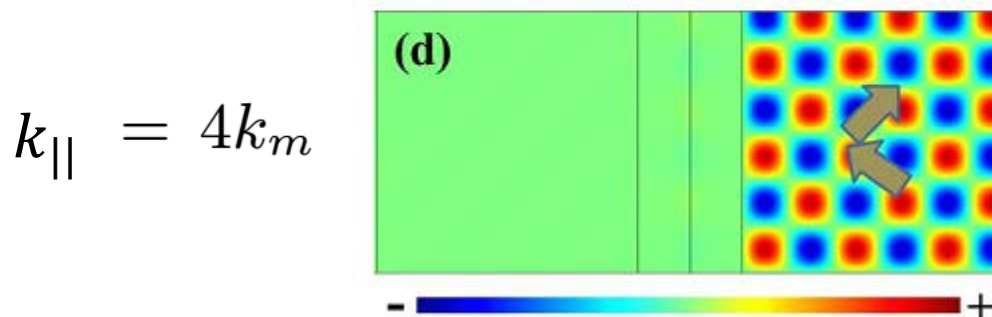
Jin Wang et al. Physical Review B **91** 235410 (2015)



Oblique incidence – one way tunneling



This ideal system gives on-way tunneling with perfect transmission.



Notes:

$P_y T$ (or RT) symmetry is broken to give $\omega(k) \neq \omega(-k)$.
 PT symmetry is kept to give perfect transmission mode.

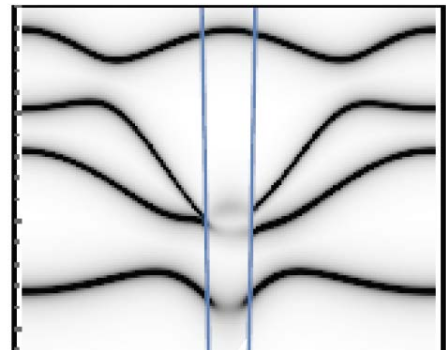
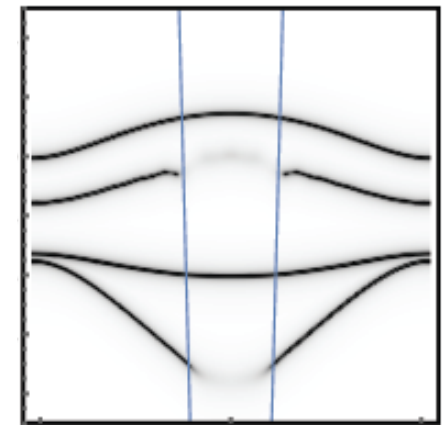
Conclusion

- We demonstrated that
 - RT (or PyT) symmetry may protect the spectral reciprocity (in bands)

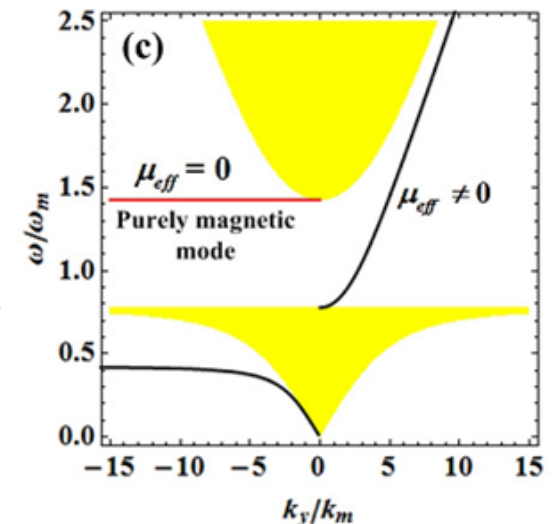
CW Ling et al. PRB 92 165430 (2015)

- We found
 - a non-reciprocal μ -near-zero surface modes in PT-symmetric magnetic domains

Jin Wang et al. PRB 91 235410 (2015)



Asymmetric Bands



Thank you!

Contact info:

Kin Hung Fung*

khfung@polyu.edu.hk

<http://ap.polyu.edu.hk/apkhfung>

Dept. of Applied Physics

Hong Kong Polytechnic University



CW Ling et al. PRB **92** 165430 (2015)

Jin Wang et al. PRB **91** 235410 (2015)

Other PT / Topology – related stuff

PT and zero extinction (see poster no. 10)

Topological plasmon chain:

C.W. Ling et. al. Optics Express 23, 2021(2015)

Other collaborators involved in this work:

C.W. Ling

(my PhD student)



The Hong Kong Polytechnic
University (PolyU)

Jin Wang



Southeast University
(China)

C. T. Chan



Hong Kong University of
Science & Technology (HKUST)

Supported by Hong Kong RGC through the Area of Excellence Scheme
& The Hong Kong Polytechnic University