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….. Now part of a big « cluster » of universities : « Paris Saclay »

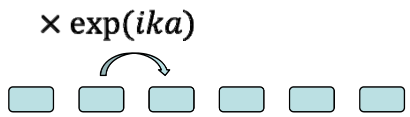
- Course 1 :
  - Semiconductors
  - Basics of coupling : two oscillators... two guides , two waves (à la Haus)

    (What if 3rd oscillator ? … Fano, EIT, …)
  - Mode profiles and active behaviour of waveguide
  - From periodic structures to lasers
    - Edge –emitting DFB laser
    - Vertically emitting VCSEL
  - Mach-Zehnder interferometer for switching.

Course 2 & 3 PT symmetry (transverse & longitudinal)
Course 4 : PT symmetry (advanced)

1.A – Periodicity (for electrons)

Periodicity

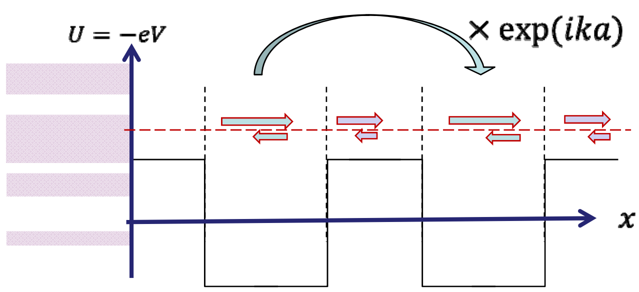


Floquet-Bloch theorem

(plane wave generalization)

$$\boxed{\psi_k(x+a) = \psi_k(x+a)\ e^{ika}}$$

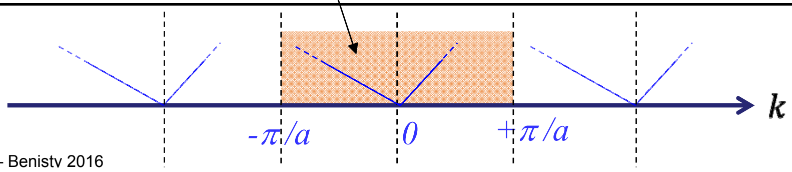
Kronig-Peney example (≡ multilayer in optics)



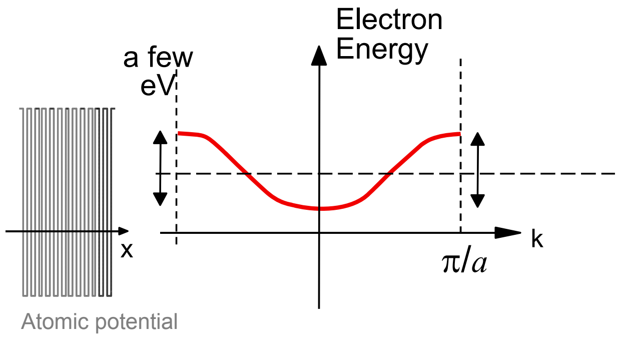
Bloch modes with real k (in bands) never decay
…
(in spite of infinitely many reflection's)

Fourier space : 1st Brillouin Zone, Reciprocal Lattice

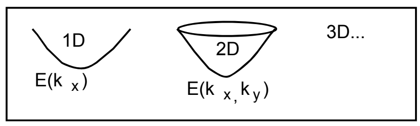
$\exp(i(k+mG_0)a) = \exp(ika)$ if $mG_0 a = m(2\pi)$ → repeats @ $G_0 = 2\pi/a$



- **One band**



BAND EDGES



- **Two bands + a gap**

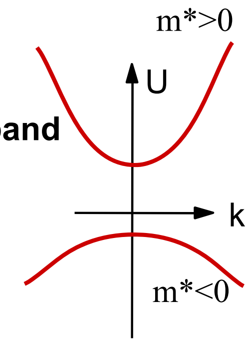
$m^* \sim +0.1\, m_o$

- **Conduction band**

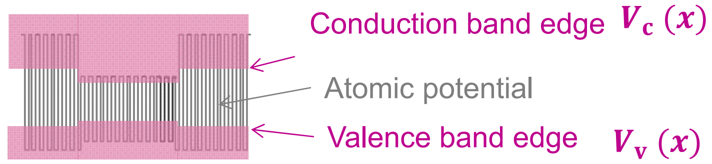
$$U = \frac{\hbar^2 k^2}{2m^*}$$

$m^* \sim -m_o$

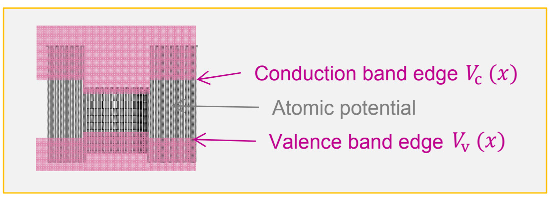
- **Valence band**

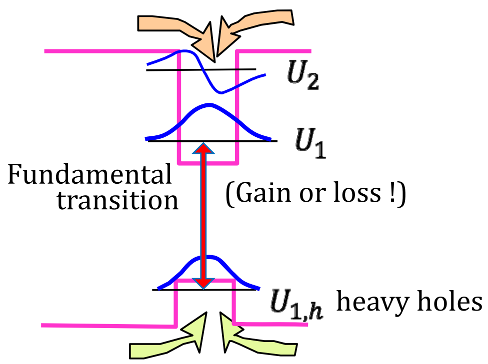


- **Quantum wells**
  Band edges ≡ potential $V_{c,v}(x)$



Quantum well

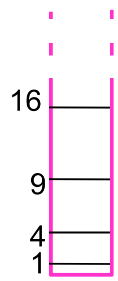




- Levels of infinite well,

Eigenvalue

$$U_n = \frac{\hbar^2}{2m^*}\left(\frac{n\pi}{L}\right)^2$$



$$\left[\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial x^2} + V(x)\right] F_c^{(n)}(x) = U_n F_c^{(n)}(x)$$

Hamiltonian

$$\boldsymbol{r} \equiv (y,z)$$

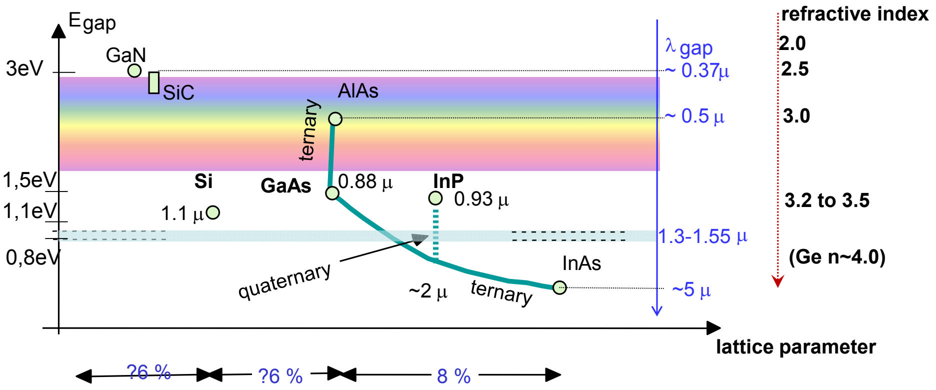
$$\psi_c = F_c(x)\exp(i\,\boldsymbol{k}.\boldsymbol{r})\quad u_{c,k=0}(x,\boldsymbol{r})$$

$$\psi_v = F_v(x)\exp(i\,\boldsymbol{k}.\boldsymbol{r})\quad u_{v,k=0}(x,\boldsymbol{r})$$

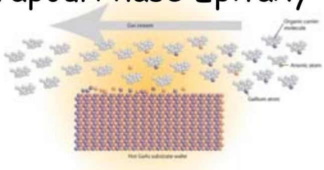
Eigenfunction

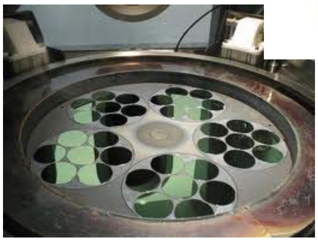
- Mode discretization
  Similar to dielectric waveguide

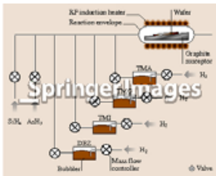
Epitaxy, MOCVD and MBE



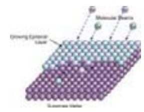
MetalOrganicVapourPhase Epitaxy



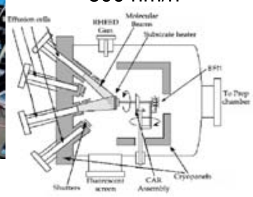




Molecular Beam Epitaxy



~slow growth rate
< 500 nm/h

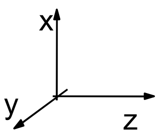


Basics of waveguide

- Field confined along x, free along z
- TE pol. here

$$\boldsymbol{E} = E_o \; f(x) \; \exp(ik_z z - \omega t) \; \boldsymbol{e}_y \; ;$$

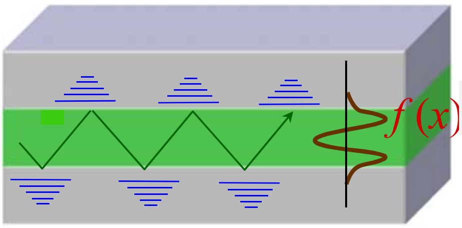
$$\varepsilon(x) = n^2(x)$$



n_{clad}

n_{core}

n_{clad}



- wave picture
- ray picture

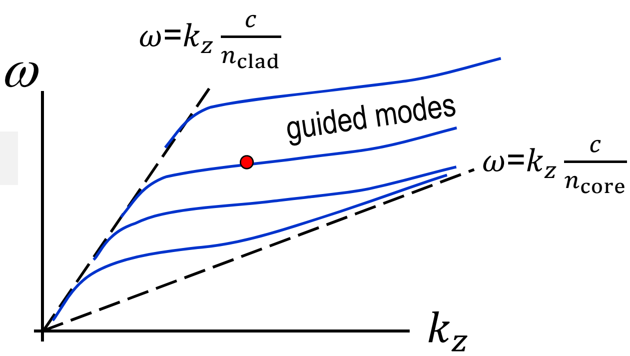
evanescent waves due to TIR (Tot. Int. Refl.)

- Helmholtz eqn. →

$$-\frac{\partial^2}{\partial x^2} f(x) + k_z^2 f(x) = \varepsilon(x) \frac{\omega^2}{c^2} f(x)$$

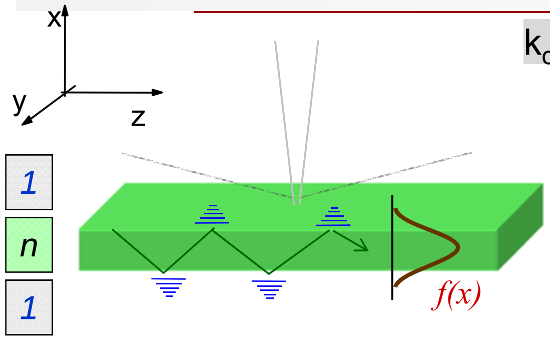
- Similar to Schrödinger eqn. with a swap…

$$-\frac{\partial^2}{\partial x^2} \psi(x) - E\psi(x) = -V(x)\psi(x)$$

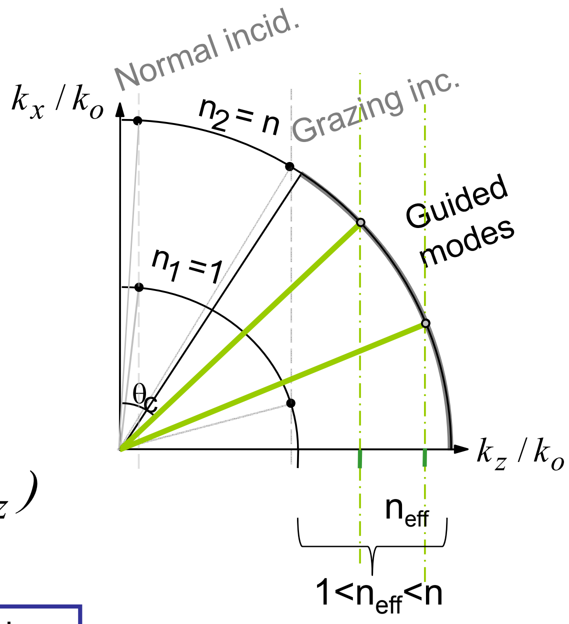


Waveguide mode in k-space

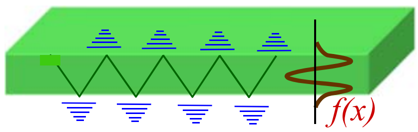
$k_o = (\omega/c)$



$$\boldsymbol{E} = E_o \; f(x) \; \exp(i k_z z) \; \boldsymbol{e}_y$$



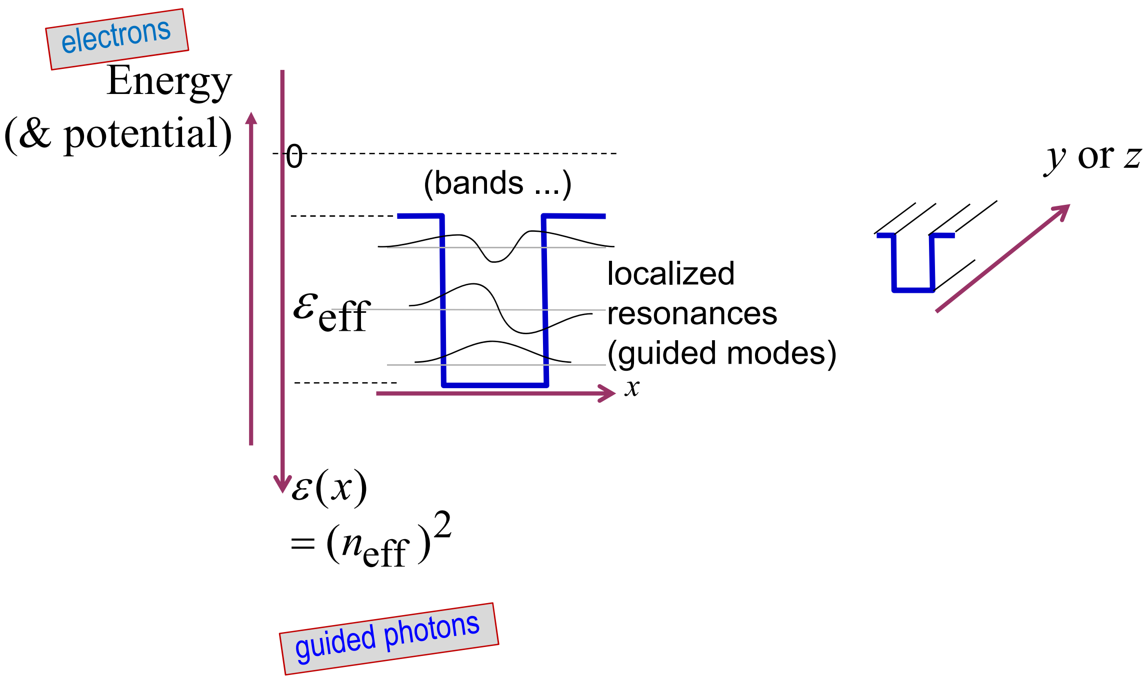
$(\beta \equiv k_z)$



Guided modes

- ***DEFINITION OF EFFECTIVE INDEX*** $k_z = n_{eff}\,(\omega/c)$ i.e. $k_z = n_{eff}\, k_o$
- FUNDAMENTAL MODE ⇔ highest ε_{eff} & n_{eff} (bottom of well)

Equivalence with potential (quantum) well



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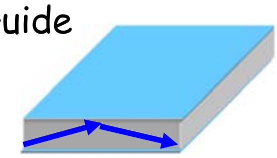
Photonic structures

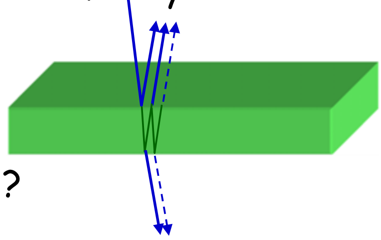
Many of them can be considered as **oscillators**

Guide

L



Fabry-Perot



So what is the simple description ?

One mode ≡ 1 oscillator → frequency, losses

Two modes ≡ 2 oscillators → + phase matching phenomena

Three modes ≡ 3 oscillators → + interaction control (EIT, Fano)

…

Oscillators treated with 1st order ODE

(homog. 1st order ODE)

Oscillator alone

Simplest(*) $\frac{d}{dt}a = i\omega_o a$

losses! $\frac{d}{dt}a = i\omega_o a - \frac{a}{\tau}$

Complex ω $\frac{d}{dt}a = i\tilde{\omega} a$

Eigenvalue $\tilde{\omega} = \omega_o + i/\tau = \omega_o + i\gamma$

Matrix equation $\frac{d}{dt}a = [i\tilde{\omega}]a$

(*) "square root of $\frac{d^2}{dt^2}a = -\omega_0^2 a$"

(inhomog. 1st order ODE)

Response to outside excitation

$$u = u_o \exp(i\omega t)$$

$$\frac{d}{dt}a = i\tilde{\omega} a + \rho u \qquad \text{Coupling } \rho$$

- Harmonic solutions

$a = a_o \exp(i\omega t)$

- Thus

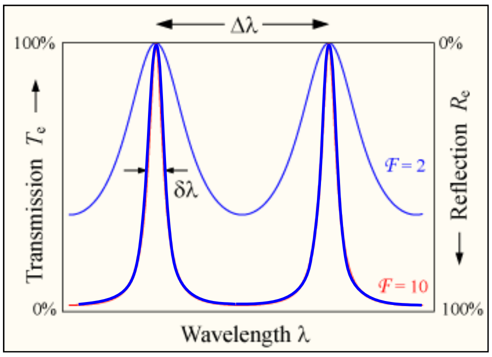
$i\omega a_o = i\tilde{\omega} a_o + \rho u_o$

$$\boxed{a_o = \frac{\rho}{i(\tilde{\omega} - \omega)} u_o}$$

- Damped resonator response

$$\boxed{|a_o|^2 = \frac{\rho^2}{(\omega_o - \omega)^2 + \gamma^2} |u_o|^2}$$

Oscillators Abs and Disp responses



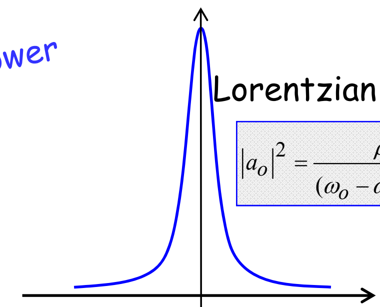
Fabry-Perot **(FP)** response : locally a Lorentzian

$$\boxed{|t|^2 = \frac{1}{1 + m\sin^2(\varphi/2)} \approx \frac{1}{1 + m(\varphi/2 - p\pi)^2}}$$

Internal field
has same denominator as
Transmission
... *A FP is an oscillator !*

Power

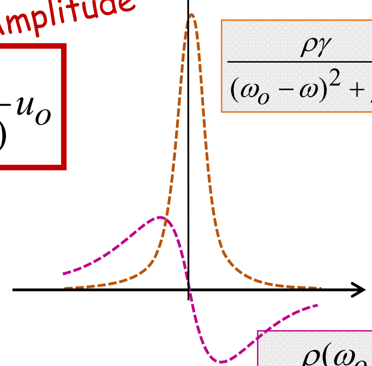
Lorentzian



$$|a_o|^2 = \frac{\rho^2}{(\omega_o - \omega)^2 + \gamma^2} |u_o|^2$$

Amplitude

$$\boxed{a_o = \frac{\rho}{i(\tilde{\omega} - \omega)} u_o}$$



$$\frac{\rho\gamma}{(\omega_o - \omega)^2 + \gamma^2}$$

(Abs)

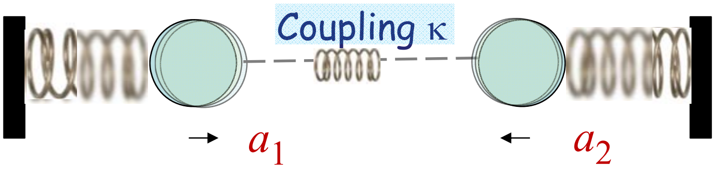
(Disp)

$$\frac{\rho(\omega_o - \omega)}{(\omega_o - \omega)^2 + \gamma^2}$$

Coupled oscillators

$$\frac{d}{dt}a_1 = i\tilde{\omega}_1 a_1 \qquad \frac{d}{dt}a_2 = i\tilde{\omega}_2 a_2$$

without coupling



$$\frac{d}{dt}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} i\tilde{\omega}_1 & -\kappa \\ \kappa & i\tilde{\omega}_2 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = i\begin{pmatrix} \tilde{\omega}_1 & i\kappa \\ -i\kappa & \tilde{\omega}_2 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

with coupling

- Without losses, this is Hermitian evolution (eigenvalues = eigen-ω are real) $\begin{pmatrix} \tilde{\omega}_1 & i\kappa \\ -i\kappa & \tilde{\omega}_2 \end{pmatrix} \mapsto \begin{pmatrix} \omega_1 & i\kappa \\ -i\kappa & \omega_2 \end{pmatrix}$
- Given by solving « secular equation » $\det\begin{pmatrix} \omega_1 - \lambda & i\kappa \\ -i\kappa & \omega_2 - \lambda \end{pmatrix} = 0$

Eigenfrequencies with simplest algebra

- Define detuning δ: $\omega_2 = \omega_1 + \delta$

$\begin{pmatrix} \omega_1 & i\kappa \\ -i\kappa & \omega_2 \end{pmatrix}$ has the same eigenvalues as $\begin{pmatrix} 0 & i\kappa \\ -i\kappa & \delta \end{pmatrix}$ except a shift of ω_1

- Solution :

$(H+\lambda_1 1)\,\psi=(E+\lambda_1)\psi$
(« Energy has arbitrary origin in Hamiltonians »)

$$\det\begin{pmatrix} -\lambda & i\kappa \\ -i\kappa & \delta-\lambda \end{pmatrix} = 0$$

$$\boxed{\lambda^2 - \delta\lambda - \kappa^2 = 0}$$

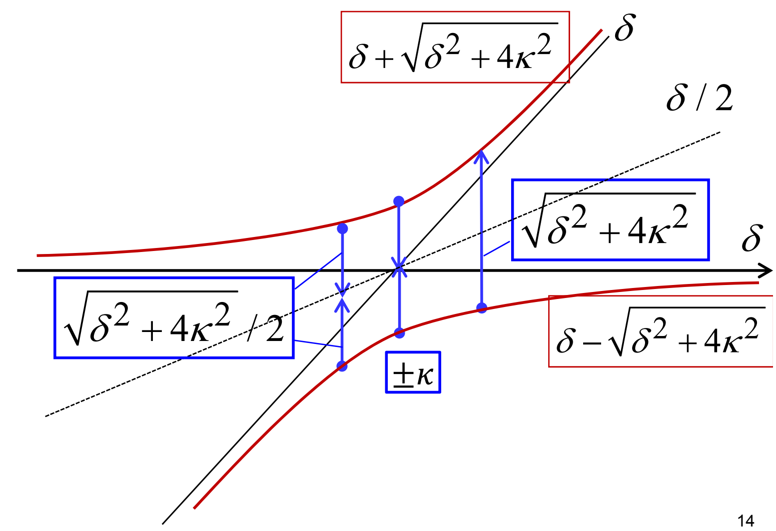
$$\Delta = \delta^2 + 4\kappa^2$$

- Eigenvalues

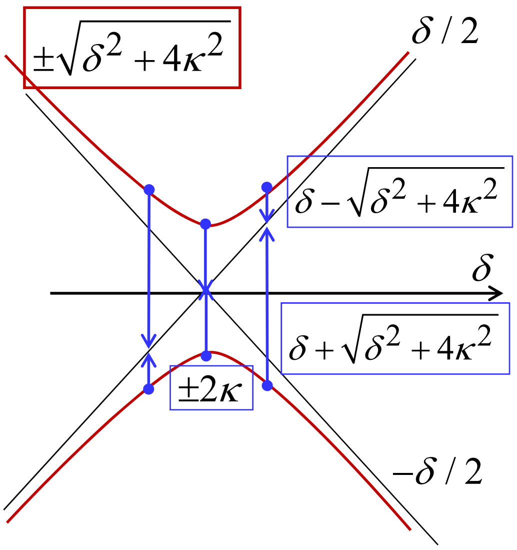
$$\boxed{\lambda = \frac{\delta \pm \sqrt{\delta^2 + 4\kappa^2}}{2}}$$

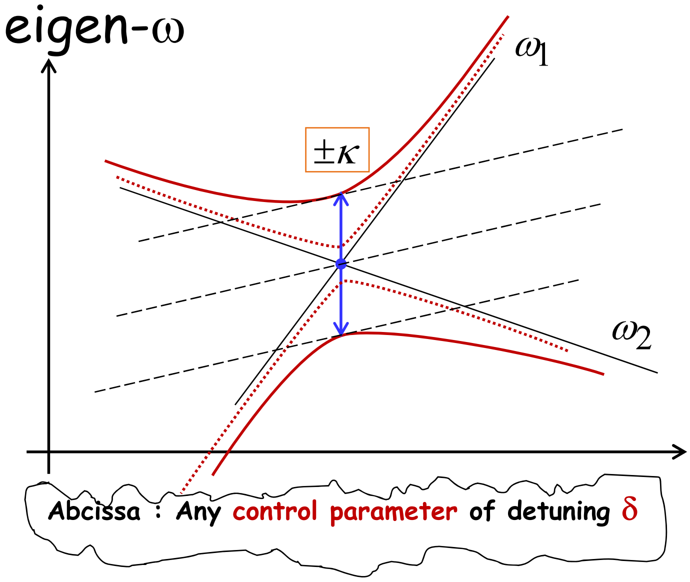
$$\boxed{\lambda(\delta = 0) = \pm\kappa}$$

- A simple plot shows core of anticrossing

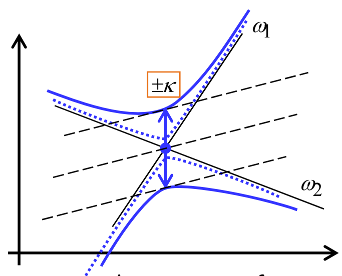


generality of splitting & anticrossing





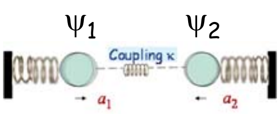
Exciting the coupled system

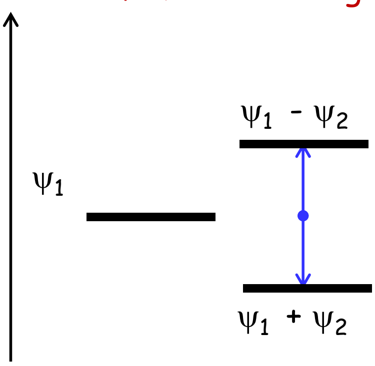


Any control parameter of detuning delta

- Only δ=0 case tractable
- i.e. the two oscillators are degenerate, symmetric

- New eigenmodes = the famous "binding" and "antibinding" « orbitals » :



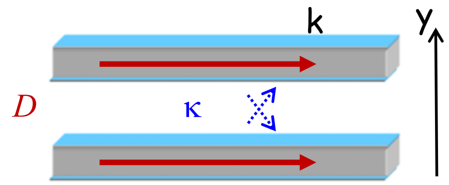


- means 1 and 2 in opposition $a_2=-a_1$
- means 1 and 2 in phase $a_2=+a_1$

- Also called « supermodes »

Application to coupled waveguides in space

Two Guides

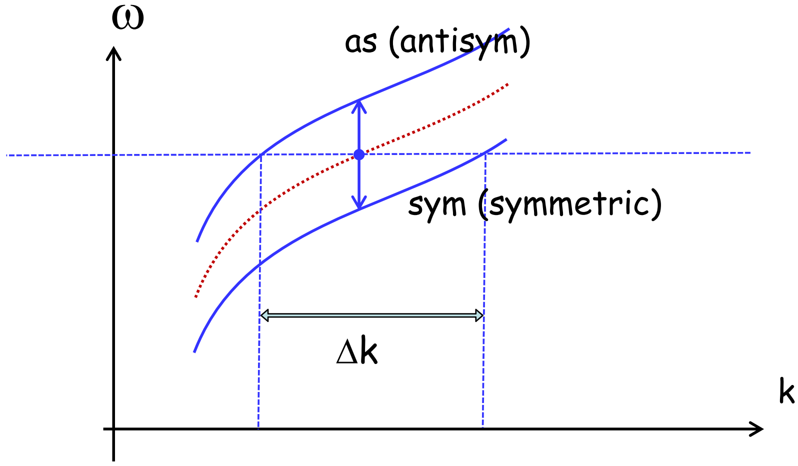


$E_1 = f_1(y)e^{ikx}e^{-i\omega t}$

$E_2 = f_2(y)e^{ikx}e^{-i\omega t}$

A guide ≡ a time-dependent oscillator

- Adding space to time → the control parameter is « k »,… it becomes « free »



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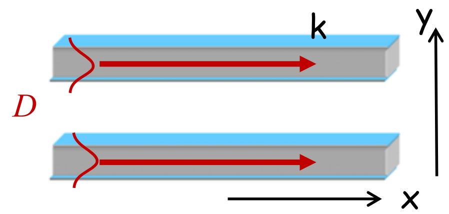


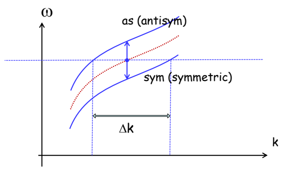
$E_1 = f_1(y)e^{ikx}e^{-i\omega t}$

$E_2 = f_2(y)e^{ikx}e^{-i\omega t}$

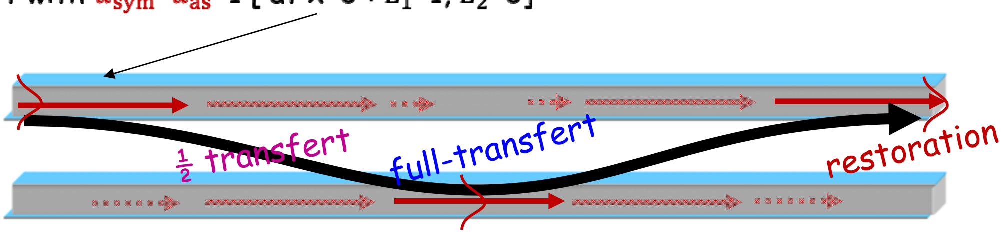
A guide ?=? a time-dependent oscillator

- Subsequently, supermode components evolve « diagonally »

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \alpha_{\text{sym}} e^{ik_{\text{sym}}x} e^{-i\omega t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \alpha_{\text{as}} e^{ik_{\text{as}}x} e^{-i\omega t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$



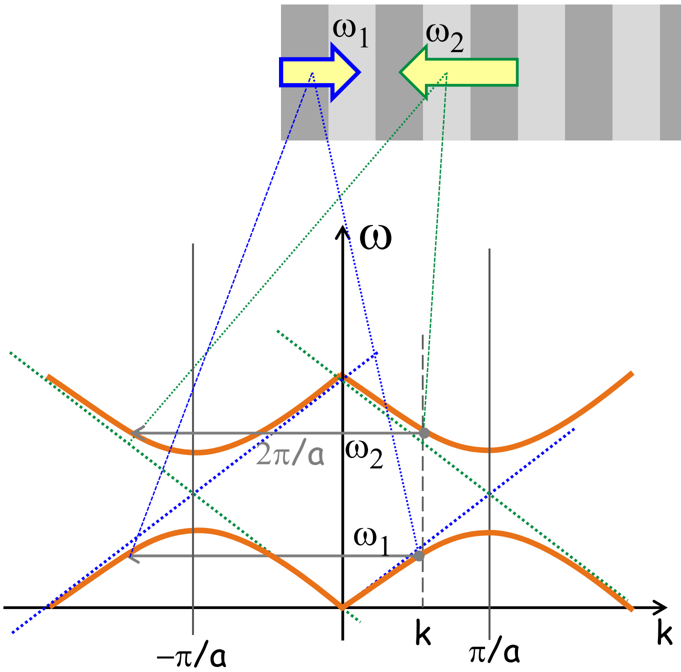
- **If start with α_{sym}=α_{as}=1 [at x=0 : E_1=1, E_2=0]**



- So :beat length
  - full restoration after $\Delta x = 2\pi/\Delta k$
  - full transfert after $\Delta x = \pi/\Delta k$
  - half transfert after $\Delta x = \pi/2\Delta k$ (≡ *beam-splitter* !)

Application to coupled plane waves by periodicity

Result = dispersion in a periodic system

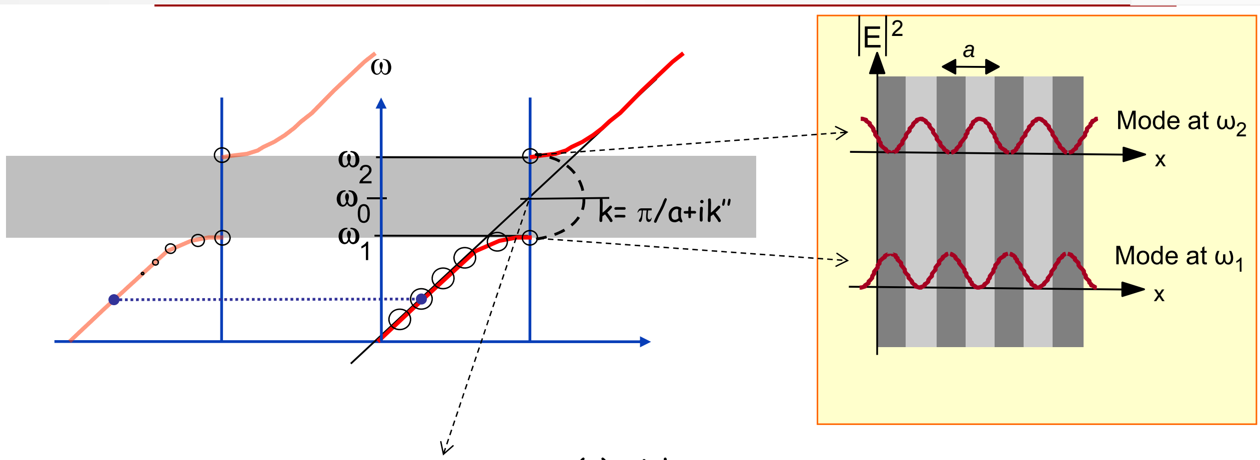


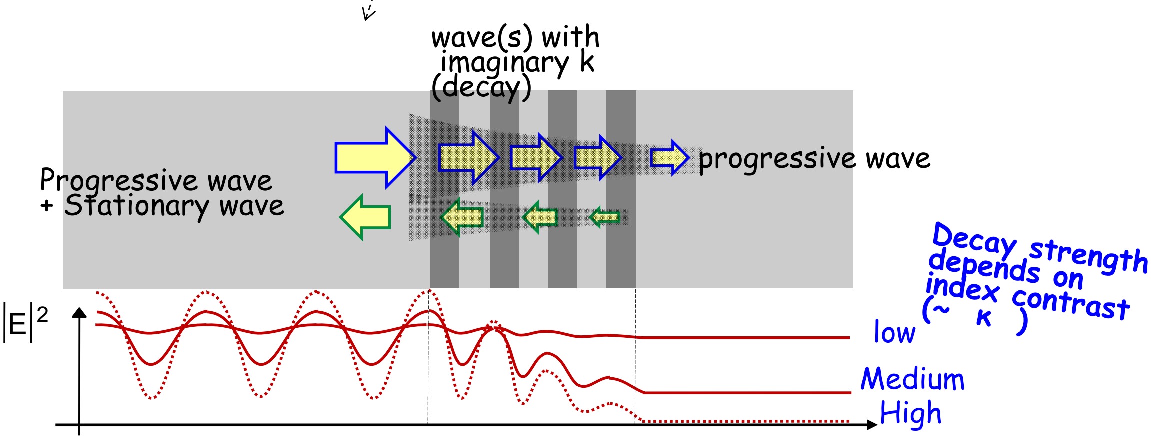
Step 1 : forward&backward modes
Step 2 : shift by $2\pi/a$
Step 3: form anticrossing

Many comments !

- Initial waves are plane waves
- Origin of κ : Fresnel reflections
- k modulo **G** cf. slide 2.
- waves : mix of → and ←
- Appearance of band gap (« *photonic band gap* »)
- solutions = {sym} & {as} at π/a (*Brillouin zone edge*)
- Stationary waves at this point
- « slow light » near these edges
- ...

Use in bandgap : partial Bragg reflector





General treatment of weak periodicity

Coupled Mode Theory

We have seen one version :

$$\frac{d}{dt}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = i\begin{pmatrix} \tilde{\omega}_1 & i\kappa \\ -i\kappa & \tilde{\omega}_2 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

- Time version
(not so common :
Hermann Haus
microwave people)

... swapping control parameter (k) and time's roles ...

$$\frac{d}{dz}\begin{pmatrix} a_1(z) \\ a_2(z) \end{pmatrix} = i\begin{pmatrix} \delta_1(\omega) & i\kappa \\ -i\kappa & \delta_2(\omega) \end{pmatrix}\begin{pmatrix} a_1(z) \\ a_2(z) \end{pmatrix}$$

- Spatial version
(Kogelnik, BTSJ)
(Yariv, Tamir)

δ and κ in cm^{-1}

(Obtained from Maxwell equations at constant frequency, by taking out slowly varying « envelope » terms of the field.)

- The spatial dephasing (spatial coherence) now plays a big role instead of temporal effects... but fundamentally same story

Distributed Bragg reflector

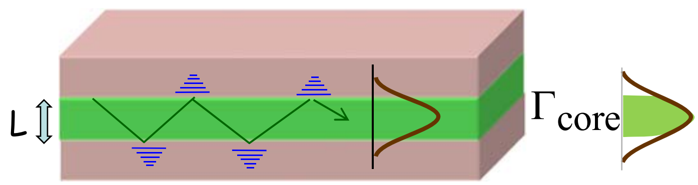
- Origin of coupling by periodicity κ : Fresnel reflections
- Strength ~ $\Delta n/2n$ depends on INDEX CONTRAST

		Penetration length [# of periods] ~ [<n>/ Δn]
Very high Δn	Air/Silicon: 1.0/3.5	1 to 2
Quite high Δn	SiO_2/TiO_2: 1.5/2.5	2 to 3
Medium Δn	AlAs/GaAs, SiO_2/Al_2O_3 3.0/3.5 1.5/1.72	4 to 6
Low Δn	InP/InGaAsP 3.30/3.17	8 to 16
Very low Δn	SiO_2/Ge:SiO_2 1.5/1.51	40 to 200
Very very low Δn	(Holograms) 1.5/1.501	200 to 2 000
Very very very low Δn	X-rays : 1.0 / 0.999 98	> 20 000

- Course 1 :
  - Semiconductors
  - Basics of coupling : two oscillators... two guides , two waves (à la Haus)

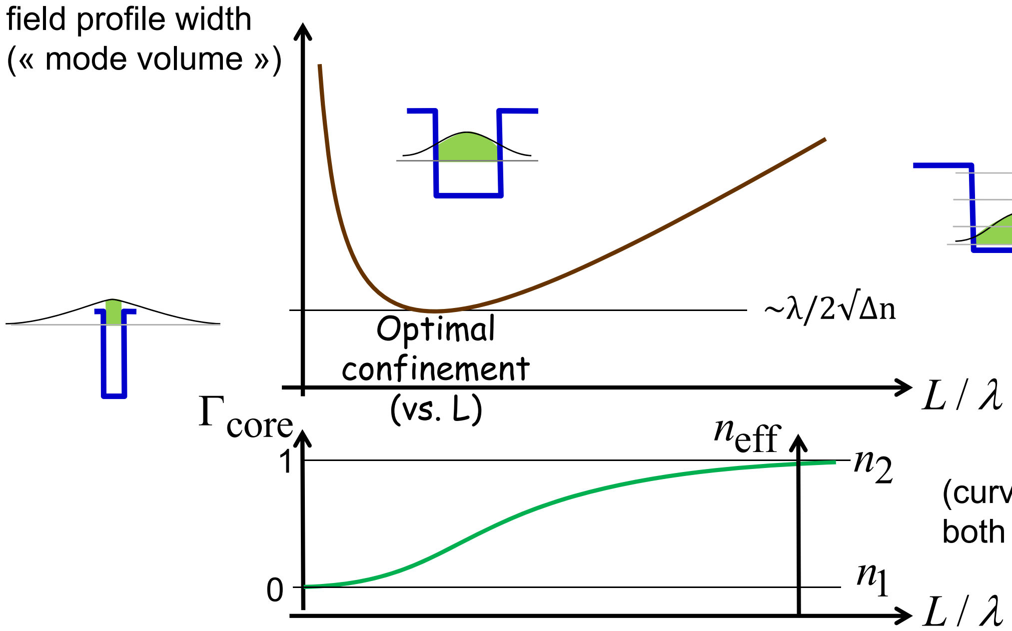
    (What if 3rd oscillator ? … Fano, EIT, …)
  - Mode profiles and active behaviour of waveguide
  - From periodic structures to lasers
    - Edge –emitting DFB laser
    - Vertically emitting VCSEL
  - Mach-Zehnder interferometer for switching.

OVERLAP : CONFINEMENT FACTOR Γ



$$\Gamma_{layer} = \frac{\int_{x_{min}}^{x_{max}} |E|^2 dx}{\int_{-\infty}^{+\infty} |E|^2 dx}$$

$$\Gamma_{bottom\_clad} + \Gamma_{core} + \Gamma_{top\_clad} = 1$$



(curve ~ valid for both Γ & n scales)

- Course 1 :
  - Semiconductors
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    (What if 3rd oscillator ? … Fano, EIT, …)
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INDEX CONTRAST ←→ size (by materials.)

		optimal size @1 µm
Very high Δn	Air/Silicon: 1.0/3.5	~ 140 nm
Quite high Δn	SiO_2/TiO_2: 1.5/2.5	~ 150 nm
Medium Δn	AlAs/GaAs, SiO_2/Al_2O_3 3.0/3.5 1.5/1.72	200 , 350 nm
Low Δn	InP/InGaAsP 3.30/3.17	~ 450 nm
Very low Δn	$Si0_2$/Ge:SiO_2 1.5/1.51	3 µm
Very very low Δn	(Holograms) 1.5/1.501	10 µm
Very very very low Δn	X-rays : 1.0 / 0.999 98	100 µm ?

DFB laser example

Distributed Bragg reflector

×

Waveguide

×

Gain

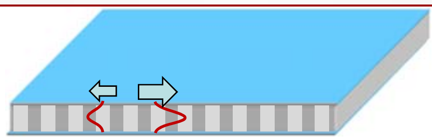
×

QW

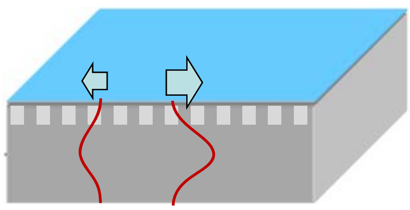
=

DFB SCH laser

DFB FP laser

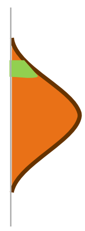
- Ideally this …  … but not feasible

- Rather this extra step :
  thin layer periodically etched away and replaced by another (epitaxially grown) medium

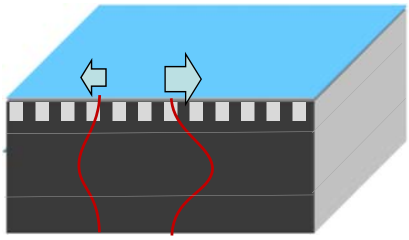


- So Confinement factor of grating is very small

$$\Gamma_{\text{layer}} = \frac{\int_{x_{\text{min grating}}}^{x_{\text{max grating}}} |E|^2 dx}{\int_{-\infty}^{+\infty} |E|^2 dx}$$

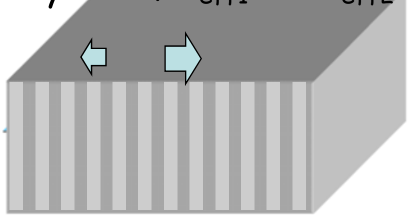
$\Gamma_{\text{grating layer}}$ 

DFB FP laser n_{eff} model



- Reality
- Each section seen as a fraction of a waveguide
  → 2 effective indices n_{eff1} and n_{eff2} .

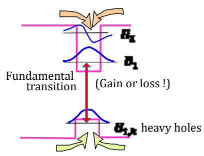
Layers of n_{eff1} and n_{eff2}

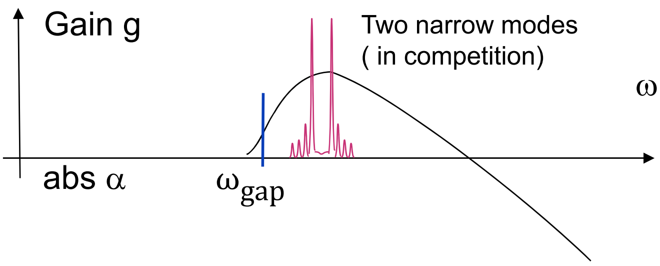


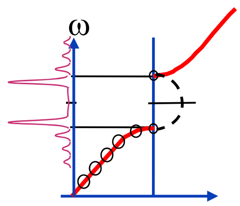
- **Model** : laser considered as a multilayer with index n_{eff1} and n_{eff2} . (and gain, abs)

- **Because of small** Γ n_{eff1} and n_{eff2} only differ by ~ 0.1%
- So the actual stop band is ~1-3 nm , same order as FSR
- So lasing become much better on any of the two band edges (the slowest allowed modes)

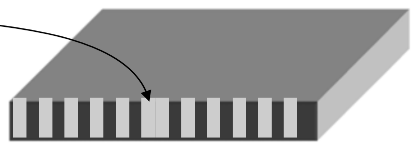
DFB FP laser spectrum





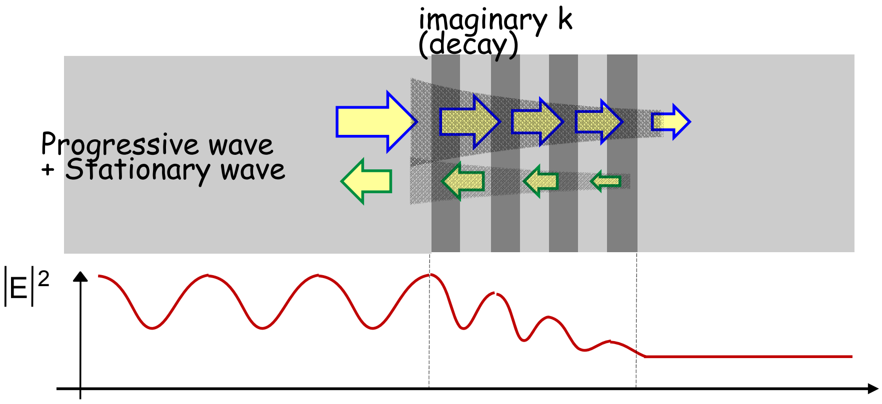


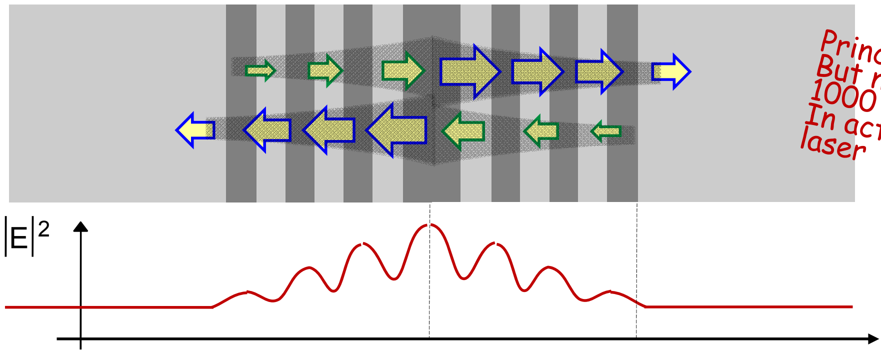
- Still two modes left… how to get one ?
- With a modified grating : grating with a $\lambda/2$ defect among $\lambda/4$ steps
- Phase-shifted DFB laser



DFB with a defect ("phase-shifted") :

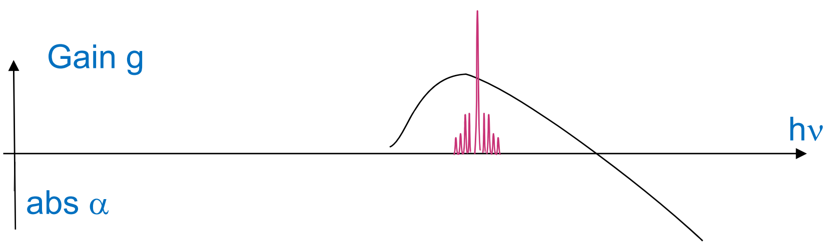
It's a laser with two ("elongated") mirrors





Principle OK
But not at scale
1000 periods
In actual DFB
laser

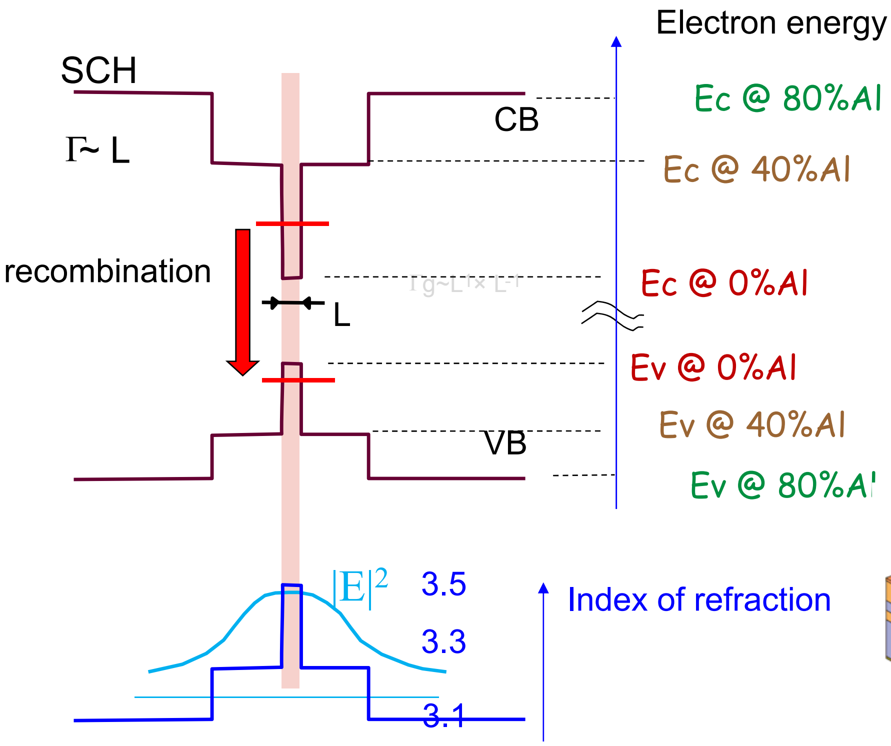
DFB with phase-shift : spectrum

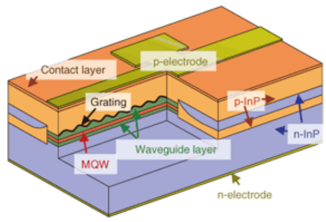


- One frequency only is selected
- Need not be at peak gain

Maximizing Light-Matter interaction: Separate Confinement Heterostructure (SCH)

- Use ~ half of "alloy span" to produce photon confinement
- Use remaining half to achieve electron confinement

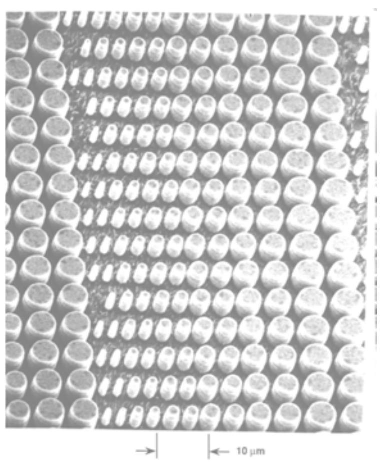


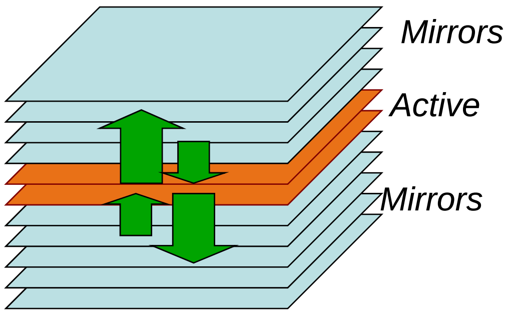


The VCSEL

- Need to parallelize optical links

Vertical Surface Emitting Laser





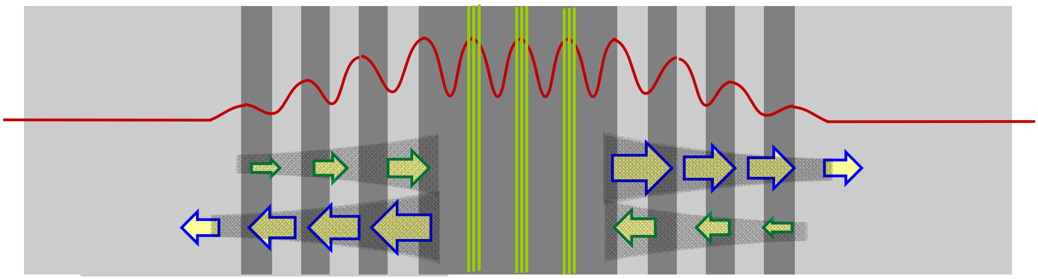
→ *So business as usual ?*

$$g_{th} = \frac{1}{L} Ln(\frac{1}{r_1 r_2}) + \alpha_{scatt}$$

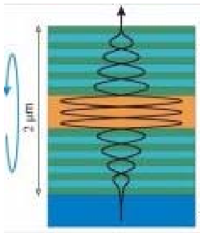
- *Extreme R required > 99.5%*
- *Quantum well location vs. (anti) node critical*

VCSEL QW placement

- Having QW only where fmode field is large
  → A new concept is naturally introduced :
- a periodic gain medium with ~$\lambda/2n$ periodicity



(admittedly, in practice 850 nm VCSELs can « unfortunately » operate with 3QWs and no period, but lower gain ones or high power ones have to work so)



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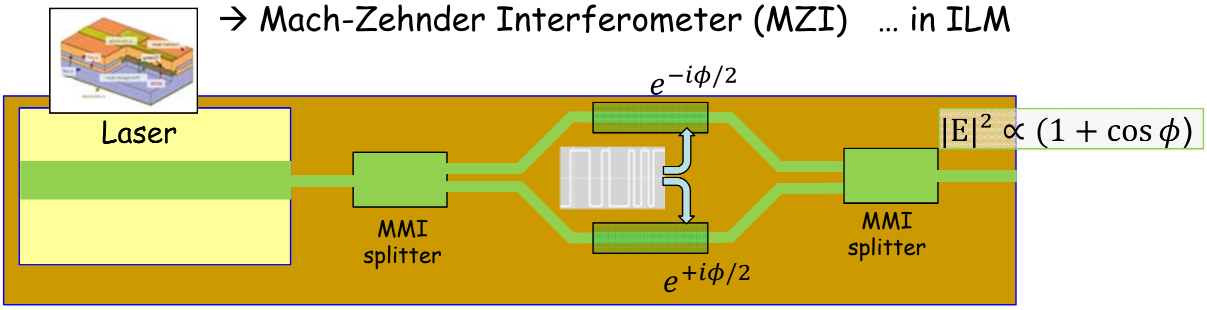
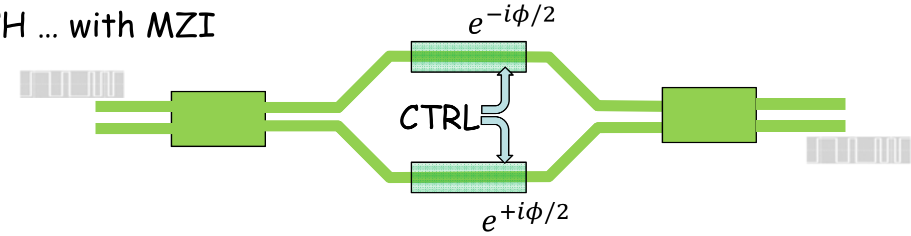
Integrated laser modulator(ILM) & Switching

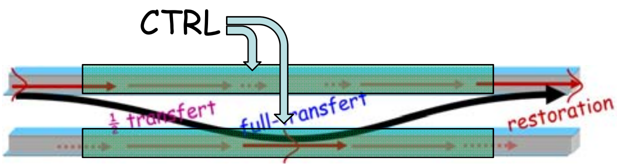
→ Mach-Zehnder Interferometer (MZI) … in ILM



→ 2x2 SWITCH … with MZI

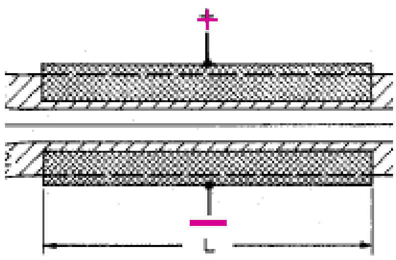


→ Or with coupled wg's



E. Kogelnik's 1976 "on couplers"

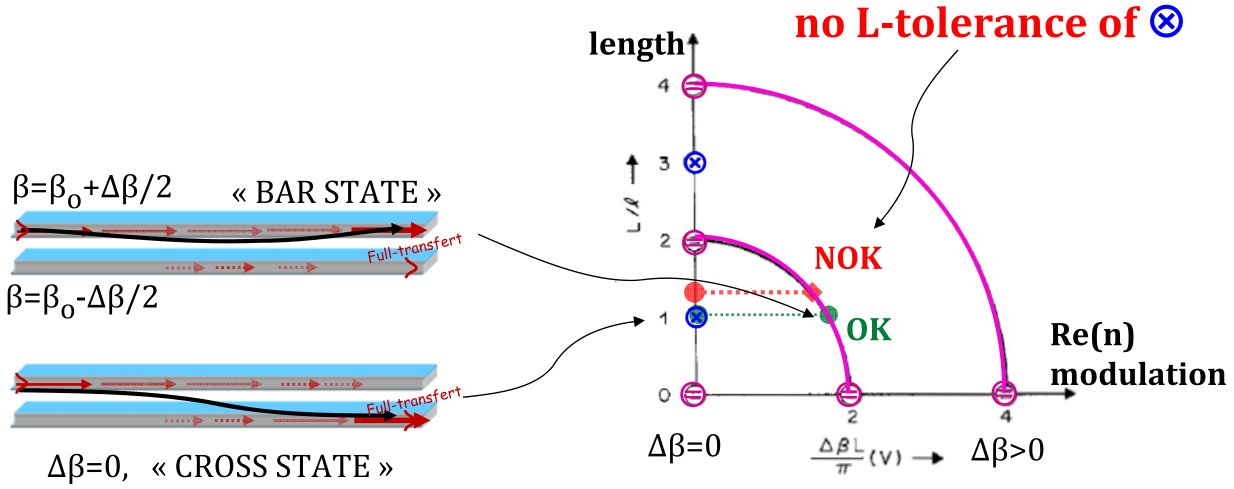
Kogelnik, IEEE JQE, 1976



Electro-optic modulation

Two parameters :
→ Length L
→ Propagation constant modulation Δβ… by Re(n)

Single-section :
no L-tolerance of ⊗

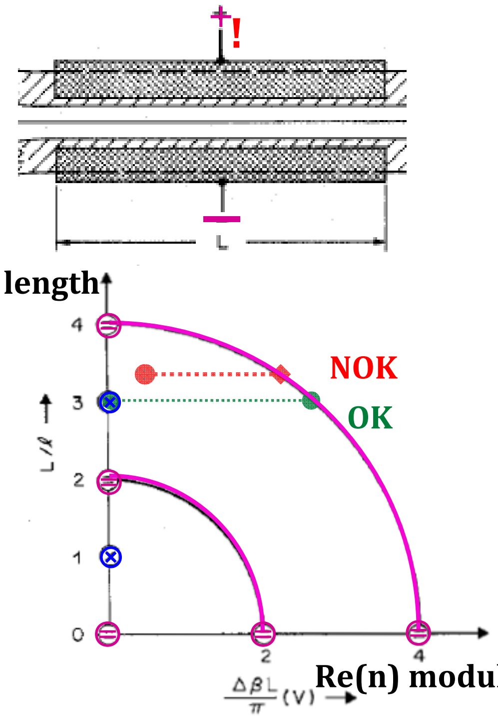


E. Kogelnik’s 1976 “alternation” idea for couplers

Kogelnik, IEEE JQE, 1976

Single-section :

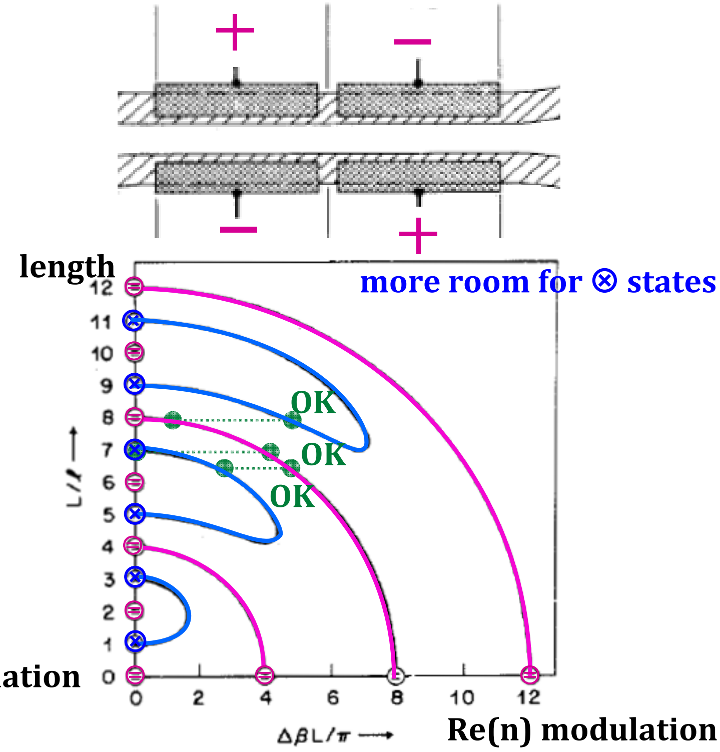
no L-tolerance of ⊗



Re(n) modulation

Alternate sections :

L-tolerance of ⊗

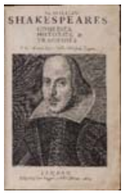


Re(n) modulation

CONCLUSION : Still time for clever gain…

What a nice job in the 20th century !
But …

« Much ado about … real index »
(guide & DFB, SCH, couplers)



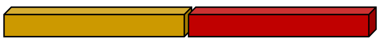
Gain made better by QW

Charles (« Chuck ») Henry

VCSEL as first engineered periodic gain media… but not the commonly shared description

Kenichi Iga

Switching (Telecom) optics developed with « functional » view point :
gain and lasing are here, index tuning is there, separately



→ Still room & food for thought for a clever use of distributed gain