A Martingale Approach to Limit Results for Spatial Causal Linear Processes

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Let $(X_{i;j} : i, j \in \mathbf{Z})$ be a stationary random field on the lattice. Let F be the distribution function of $X_{i,j}$. We are interested in the behaviour of the empirical distribution function

$$H_{m,n}(x) := \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} I(X_{i,j} \leq x).$$

A random field is said to have short memory if and only if its covariance function is absolutely summable, otherwise it is said to have long memory. While there are many results for the empirical process associated with long memory, stationary random fields the results for short memory fields typically assume mixing or association. For short memory, stationary, stochastic processes these assumptions can be avoided by using martingale methods. In general the martingale techniques do not extend to spatial processes due to the lack of a total order on the lattice.

An exception is provided by the short memory, causal, linear process

$$X_{s,t} = \sum_{i \geq 0} \sum_{j \geq 0} a_{ij} \; \xi_{s-i,t-j}, \quad s, t \in \mathbf{Z},$$

where $\{\xi_{u,v} \;\; u,v \in \mathbf{Z}\}$ is an array of independent and identically distributed random variables with mean 0 and variance 1 and $\{a_{i,j}\}$ is an array of constants with $\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} |a_{i,j}| < \infty$. By judicious choice of σ–fields and element enumeration, the structure of this process allows us to exploit one-dimensional martingale arguments, similar to those of Doukhan and Surgailis (1998) for short memory linear processes, to establish a functional limit theorem for the empirical process.

This is joint work with Gail Ivanoff, University of Ottawa.

Reference

Doukhan, P. and Surgailis, D. (1998). Functional central limit theorem for the empirical process of short memory linear processes, *C.R. Acad. Sci. Paris*, **326**, 87-92.